FAULT DETECTION AND IDENTIFICATION FOR UNCERTAIN LINEAR NEUTRAL DELAY SYSTEMS

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Abstract: This paper presents a sensor fault detection and identification scheme for uncertain linear time-invariant neutral delay systems. Modeling uncertainty, disturbance and noise are represented as unknown bounded inputs appearing in the state and output equations. An adaptive observer scheme extending Vemuri and Polycarpou (1997)'s work is utilized to detect and identify the fault. Robustness to the unknown inputs is analysed rigorously. It is proved that the fault estimate and the errors of state and output estimation are uniformly bounded. A numerical example is included for illustration.

Keywords: fault detection and identification, neutral delay systems, adaptive observers, uncertain systems, sensor faults, linear matrix inequalities (LMIs)

1. INTRODUCTION

Neutral time-delay systems are a particular class of delay systems that the delay argument occurs also in the derivative of the state variable. Many engineering systems can be modeled by these systems, such as steam or water pipelines, heat exchangers and loss-less transmission lines (LC circuits) in electrical engineering (Ivănescu *et al.*, 2003). In recent years, studies on these systems have received more attention, for example stability analysis (Verriest and Niculescu, 1998; Ivănescu *et al.*, 2003), controller (Mahmoud, 2000) and observer design (Wang *et al.*, 2002). Their particularity different from that of the familiar retarded type systems (which have delays only in the state) implies an increased complexity for investigation.

On the other hand, fault diagnosis is becoming an important issue in modern engineering systems due to the rising demands of product quality, effectiveness and safety. On the contrary to the intensively investigated researches of robust fault diagnosis for uncertain linear systems and fault diagnosis for nonlinear systems (Frank *et al.*, 2001), the studies on fault diagnosis for delay systems (retarded type or neutral type) are very few. Yang and Saif (1998) first proposed a scheme of actuator and sensor fault diagnosis for linear state-retarded systems using an unknown input observer combined with a technique of input estimation. Jiang *et al.* (2002) developed an adaptive observer scheme to estimate abrupt component fault for linear (nonlinear) systems with delays in the state. For systems with constant delays in the input and output, Zhang *et al.* (2002) presented a component fault detection method based on parity equations. As for neutral type delay systems, under the assumption that the system was stable, Zhong *et al.* (2002) designed a robust fault detection filter without delays in the state guaranteeing both sensitivity to the fault and insensitivity to the disturbance and control input. In most cases, large delay can make the closed-loop system unstable and deteriorate the performance, and most methods of fault diagnosis for linear systems modeled by ordinary differential equations are not applicable to systems with delays. So far, the fault diagnosis problem for linear delay systems, especially neutral delay systems, has not been fully investigated and remains to be important and challenging.

This paper addresses the problem of sensor fault detection and identification for a class of uncertain linear time-invariant neutral delay systems. The system investigated has two different constant delays in the state and state's derivative. Modeling uncertainty, disturbance and measurement noise are represented as unknown inputs with bounded magnitudes appearing in the state and output equations. An adaptive observer scheme extending the one of Vemuri and Polycarpou (1997) is utilized to detect and identify the sensor fault, and the fault estimate can be used in further fault diagnosis and fault tolerant control. It is proved that the fault detection algorithm is robust against the unknown inputs with a zero false-alarm ratio, and the fault estimate and the errors of state and output estimation are uniformly bounded under the assumption of formal stability (Richard, 2003) of the system.

Throughout this paper, $\mathcal{C}([a,b],\mathbb{R}^n)$ is the Banach space of continuous functions mapping the interval $[a,b]$ into $\mathbb{R}^n$ with the topology of uniform convergence, $|\cdot|$ denotes the Euclidean vector norm or induced matrix 2-norm, $\|\cdot\|_\infty$ denotes the Lebesgue infinity norm for time functions defined by $\|\eta(t)\|_\infty \triangleq \sup_{t\geq 0}|\eta(t)|$, or the $\mathcal{H}_\infty$ norm for transfer functions, and the Frobenius matrix norm is denoted by $|M|_{\mathrm{F}}^2 \triangleq \sum_{ij}|m_{ij}|^2 = \mathrm{trace}(MM^T)$. The arguments of a function will be omitted in the analysis when no confusion can arise.

2. PROBLEM FORMULATION

Consider an uncertain linear time-invariant neutral time-delay system described as

$$\begin{cases} \dot{x} - E\dot{x}(t-\tau) = Ax + A_d x(t-d) + Bu + \eta_x \\ \qquad\qquad\quad y = Cx + Du + \mathcal{B}(t-T)f + \eta_y \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^l$ and $y(t) \in \mathbb{R}^m$ are respectively the state, input and output of the system. $0 < \tau, d < \infty$ are two constant delays. A, A_d, B, E, C, D are known constant matrices with compatible dimensions. The initial condition is $x(\theta) = \phi(\theta)$ for $\theta \in [-\overline{\tau}, 0]$, where $\overline{\tau} \triangleq \max\{\tau, d\}$ and $\phi(\theta) \in \mathcal{C}([-\overline{\tau},0],\mathbb{R}^n)$.

$\mathcal{B}(t-T)f$ denotes the sensor fault, where $f \in \mathbb{R}^m$ is an unknown constant vector representing the fault in the stable mode. $\mathcal{B}(t-T)$ is a time profile function defined by

$$\mathcal{B}(t-T) \triangleq \mathrm{diag}\{\beta_1, \cdots, \beta_m\} \tag{2}$$

$$\beta_i = \begin{cases} 0 & \text{if} \quad t < T \\ 1 - \mathrm{e}^{-\alpha_i(t-T)} & \text{otherwise} \end{cases} \quad (i = 1, \cdots, m)$$

where T is the unknown time of fault occurrence, and $\alpha_i > 0$ $(i = 1, \cdots, m)$ are unknown scalars. This form of fault representation can describe either slowly developing (incipient) fault with a small α_i or abrupt fault with a large α_i.

$\eta_x(t)$ and $\eta_y(t)$ are the unknown inputs representing modeling uncertainty, disturbance and measurement noise.

For system (1), the following assumptions are made.

A1: The values of the delays τ and d are known.
A2: The unknown fault is bounded, i.e. $|f| \leq c$, where c is known.
A3: The magnitudes of the unknown inputs are bounded, i.e. $\|\eta_x(t)\|_\infty = \overline{\eta}_x$ and $\|\eta_y(t)\|_\infty = \overline{\eta}_y$, with the bounds $\overline{\eta}_x$ and $\overline{\eta}_y$ available.
A4: The system (1) is formally stable (Richard, 2003), i.e. the difference equation $x(t) = Ex(t-\tau)$ is stable for all $0 < \tau < \infty$.

Assumption 4 ensures that the number of unstable eigenvalues of system (1) is finite, and this condition is necessary for the observer design proposed in this paper.

Now, the objective of this paper is to detect and estimate the sensor fault for the uncertain linear neutral system defined by equation (1).

3. ROBUST FAULT DETECTION AND IDENTIFICATION SCHEME

To address the proposed problem, a robust adaptive observer is constructed in this section, which is defined by

$$\begin{cases} \dot{\hat{x}} - E\dot{\hat{x}}(t-\tau) = A\hat{x} + A_d\hat{x}(t-d) \\ \qquad\qquad +Bu + K[y - \hat{y}] \\ \qquad\qquad -\Omega\dot{\hat{f}} + E\Omega(t-\tau)\dot{\hat{f}}(t-\tau) \\ \qquad\qquad -E\dot{\Omega}(t-\tau)[\hat{f} - \hat{f}(t-\tau)] \\ \qquad\qquad -A_d\Omega(t-d)[\hat{f} - \hat{f}(t-d)] \\ \qquad \hat{y} = C\hat{x} + Du + \hat{f} \end{cases} \tag{3}$$

$$\dot{\Omega} - E\dot{\Omega}(t-\tau) = (A-KC)\Omega + A_d\Omega(t-d) + K \tag{4}$$

where $\hat{x}(t) \in \mathbb{R}^n$, $\hat{y}(t) \in \mathbb{R}^m$ and $\hat{f}(t) \in \mathbb{R}^m$ are the estimates of the state, output and fault, respectively. The initial conditions are $\hat{x}(\theta) = x(\theta)$, $\Omega(\theta) = 0$ for $\theta \in [-\overline{\tau}, 0]$. $K \in \mathbb{R}^{n\times m}$ is the gain matrix of the observer, and $\Omega(t) \in \mathbb{R}^{n\times m}$ is the filtered version of K. The auxiliary filter (4) is used to guarantee the stability of the fault estimation scheme (Vemuri and Polycarpou, 1997). Here, the assumption $\hat{x}(\theta) = x(\theta)$ is only for simplicity, since the influence of the initial error will decrease to zero asymptotically.

The selection of K should ensure that the following system

$$
\begin{aligned}
\dot{\zeta} - E\dot{\zeta}(t-\tau) &= (A-KC)\zeta + A_d\zeta(t-d) \\
&\quad +K\omega \qquad (5) \\
\xi &= C\zeta \qquad (6)
\end{aligned}
$$

is asymptotically stable and there exists a $\gamma > 0$ such that the transfer function $G_{\omega\xi}(s)$ from the noise $\omega(t)$ to the output $\xi(t)$ satisfies $\|G_{\omega\xi}(s)\|_\infty < \gamma$. For the system (5)-(6), $\zeta(t) \in \mathbb{R}^n$, $\xi(t) \in \mathbb{R}^m$, $\omega(t) \in \mathbb{R}^m$, and the initial condition is $\zeta(\theta) = \varphi(\theta)$ for $\theta \in [-\overline{\tau}, 0]$, where $\varphi(\theta) \in \mathcal{C}([-\overline{\tau}, 0], \mathbb{R}^n)$. The additional noise attenuation requirement herein is to guarantee the performance of fault estimation.

Using the Lyapunov synthesis method in adaptive control (Ioannou and Sun, 1996), we derive the following adaptive law to update the fault estimate

$$
\dot{\hat{f}} = \mathcal{P}\left[-\Gamma(C\Omega - I)^T\mathcal{D}(e_y)\right], \qquad (7)
$$

where $e_y(t) \triangleq y(t) - \hat{y}(t)$, $\Gamma = \Gamma^T > 0$ is the learning rate matrix. The initial fault estimate satisfies $\hat{f}(\theta) = 0$ for $\theta \in [-\overline{\tau}, 0]$. $\mathcal{P}$ is the projection operator, which is used to prevent the phenomenon of parameter drift in the presence of bounded disturbances (Ioannou and Sun, 1996). The adaptive law (7) with the projection operator can be further expressed as (Ioannou and Sun, 1996)

$$
\dot{\hat{f}} = -\left[I - \mathcal{I}\frac{\Gamma\hat{f}\hat{f}^T}{\hat{f}^T\Gamma\hat{f}}\right]\Gamma(C\Omega - I)^T\mathcal{D}(e_y),
$$

where the indicator function $\mathcal{I}(t)$ is defined by

$$
\mathcal{I} = \begin{cases} 0 \text{ if } |\hat{f}| < c \text{ or} \\ \quad [|\hat{f}| = c \text{ and } \hat{f}^T\Gamma(C\Omega - I)^T\mathcal{D}(e_y) \geq 0] \\ 1 \text{ if } |\hat{f}| = c \text{ and } \hat{f}^T\Gamma(C\Omega - I)^T\mathcal{D}(e_y) < 0 \end{cases}
$$

It was proved that this projection operator guarantees $|\hat{f}(t)| \leq c \ \forall t \geq 0$ if $|\hat{f}(0)| \leq c$ and $|f| \leq c$ (Ioannou and Sun, 1996). $\mathcal{D}(\cdot)$ is the dead-zone operator with definition (Ioannou and Sun, 1996)

$$
\mathcal{D}(e_y) = \begin{cases} 0 & \text{if } |e_y| < \epsilon \\ e_y & \text{otherwise} \end{cases} \qquad (8)
$$

The threshold of this dead zone can be selected as $\epsilon \triangleq (\mu/\rho)|C|\overline{\eta}_x + [(\mu/\rho)|C||K|+1]\overline{\eta}_y$, where $\mu, \rho > 0$ should satisfy $|\mathrm{X}(t)| \leq \mu\,\mathrm{e}^{-\rho t}$ $(t \geq 0)$. $\mathrm{X}(t)$ is the basic matrix solution of the neutral type functional differential equation (5) for $\omega(t) = 0$. As stated before, the system (5) can be made asymptotically stable through selecting an appropriate K, and Hale and Lunel (1993, theorem 3.2, pp. 271) proved that these constants $\mu, \rho > 0$ always existed. The operator $\mathcal{D}(\cdot)$ is used to guarantee robustness with respect to the unknown inputs. It keeps the fault estimate invariant when the output estimation error is smaller than ϵ. After the detection of a fault it becomes unnecessary and can be disabled.

In the adaptive law (7), the fault estimate is set to zero in the initial and will be kept zero when no fault occurs. Whenever $|e_y(t_f)| \geq \epsilon$ at $t_f > 0$, fault estimation is activated and $\hat{f}(t_f) \neq 0$. So fault can be detected either by distinguishing whether the fault estimate is nonzero or by distinguishing whether $|e_y(t)|$ is larger than ϵ, and t_f is the instant when the fault is detected. Thus, $|e_y(t)|$ can be used as the residual for fault detection, while ϵ is the threshold.

The following lemma gives a sufficient condition for the asymptotic stability with $\mathcal{H}_\infty$ noise attenuation of the system (5)-(6) and also a parameterization of K.

Lemma 1. If there exist symmetric matrices $P > 0$, $S > 0$, $Q \in \mathbb{R}^{n\times n}$, matrix $Y \in \mathbb{R}^{n\times m}$ and a scalar $\gamma > 0$ such that the following linear matrix inequality (LMI) holds

$$
\Pi \triangleq \begin{bmatrix} \Pi_{11} & \Pi_{12} & PA_d & Y \\ * & \Pi_{22} & 0 & 0 \\ * & * & -Q & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0
$$

where

$$
\begin{aligned}
\Pi_{11} &= PA + A^TP - YC - C^TY^T + S + Q + C^TC \\
\Pi_{12} &= (PA - YC + S + Q + C^TC)E \\
\Pi_{22} &= E^T(S + Q + C^TC)E - S
\end{aligned}
$$

then the system (5)-(6) is asymptotically stable for any $0 < \tau, d < \infty$ and $\|G_{\omega\xi}(s)\|_\infty < \gamma$. The corresponding observer gain is $K = P^{-1}Y$.

Proof. In order to prove the system (5)-(6) is asymptotically stable for any $\tau, d > 0$ with $\gamma > 0$ being the level of noise attenuation, we only need to prove the accompanied Hamiltonian $H[\zeta(t), \omega(t), t]$ satisfies the following relation under the condition of lemma 1

$$
H(\zeta, \omega, t) \triangleq |C\zeta|^2 - \gamma^2|\omega|^2 + \dot{V}_\zeta < 0.
$$

The Lyapunov-Krasovskii functional $V_\zeta(t)$ can be selected as

$$
\begin{aligned}
V_\zeta &= [\zeta - E\zeta(t-\tau)]^TP[\zeta - E\zeta(t-\tau)] \\
&\quad + \int_{t-\tau}^{t} \zeta^TS\zeta\,\mathrm{d}\theta + \int_{t-d}^{t} \zeta^TQ\zeta\,\mathrm{d}\theta
\end{aligned}
$$

Differentiate $V_\zeta(t)$ with respect to t along the trajectory of (5), and let $\mathcal{M}(\zeta) \triangleq \zeta(t) - E\zeta(t-\tau)$, after some matrix manipulation and completing squares, it follows that

$$
\begin{aligned}
H(\zeta,\omega,t) \leq\ & \mathcal{M}^T[P(A-KC) + (A-KC)^TP \\
& +S + Q + C^TC + \gamma^{-2}PKK^TP]\mathcal{M} \\
& +2\mathcal{M}^T[P(A-KC) + S + Q + C^TC]E\zeta(t-\tau) \\
& +\zeta(t-\tau)^T[E^T(S+Q+C^TC)E - S]\zeta(t-\tau) \\
& +2\mathcal{M}^TPA_d\zeta(t-d) - \zeta(t-d)^TQ\zeta(t-d)
\end{aligned}
$$

Let $Y = PK$, if $\Pi < 0$ then using Schur complement there exists $H[\zeta(t), \omega(t), t] < 0$. On the other hand, from $\Pi_{22} < 0$ (guaranteed by $\Pi < 0$) it is derived that $E^T SE - S < 0$, so the system (5) is formally stable. Furthermore, $P(A-KC)+(A-KC)^T P < 0$ can be derived from $\Pi < 0$, so $A - KC$ is Hurwitz. From the conclusions above, lemma 1 is proved. □

Based on lemma 1, we can select an appropriate gain K to guarantee the asymptotical stability of the system (5)-(6) by solving a feasibility problem of LMI. Suppose the resulting system (5)-(6) has a guaranteed $\gamma_0 > 0$ level of noise attenuation performance.

4. ROBUSTNESS TO UNKNOWN INPUTS

In the fault detection and identification algorithm designed above, robustness to unknown inputs is achieved by using the dead-zone operator $\mathcal{D}(\cdot)$. The following theorem indicates that, before fault occurrence, i.e. when the system is only driven by the unknown inputs, the residual $|e_y(t)|$ is always less than the threshold ϵ, and fault estimate remains at zero. So no false alarms will be generated.

Theorem 2. The robust fault detection and identification scheme (3), (4) and (7) guarantees that $\hat{f}(t) = 0$ for $t \in [0, T]$ before the occurrence of a sensor fault.

Proof. The proof can be derived similarly to (Trunov and Polycarpou, 2000). Suppose there is a time instant t_ϵ (where $0 < t_\epsilon < T$) such that $|e_y(t)| < \epsilon$ for $t \in [0, t_\epsilon)$ and $|e_y(t_\epsilon)| = \epsilon$. Using the adaptive law (7), we can conclude that $\hat{f}(t) = 0$ in the interval $t \in [0, t_\epsilon)$. So the dynamics of the state and output estimation errors in the interval $t \in [0, t_\epsilon)$ satisfies

$$\begin{aligned} \dot{e}_x - E\dot{e}_x(t-\tau) &= (A-KC)e_x + A_d e_x(t-d) \\ &\quad + \eta_x - K\eta_y \\ e_y &= Ce_x + \eta_y \end{aligned} \tag{9}$$

where $e_x(t) \triangleq x(t) - \hat{x}(t)$, and the initial condition is $e_x(\theta) = x(\theta) - \hat{x}(\theta) = 0$ for $\theta \in [-\overline{\tau}, 0]$. Suppose $\mathrm{X}(t)$ is the fundamental matrix solution of the neutral type functional differential equation (9) for $\eta_x(t) = 0$, $\eta_y(t) = 0$. Using the formula (1.16) given by Hale and Lunel (1993, pp. 260), the solution of equation (9) is $e_x = \int_0^t \mathrm{X}(t-\tau)[\eta_x(\tau) - K\eta_y(\tau)]\,\mathrm{d}\tau$. Because of $|\mathrm{X}(t-\tau)| \le \mu\,\mathrm{e}^{-\rho(t-\tau)}$ $(t \ge \tau)$, we can derive that

$$\begin{aligned} |e_x(t)| &\le \int_0^t |\mathrm{X}(t-\tau)|\,\mathrm{d}\tau(\overline{\eta}_x + |K|\overline{\eta}_y) \\ &\le \int_0^t \mu\,\mathrm{e}^{-\rho(t-\tau)}\,\mathrm{d}\tau(\overline{\eta}_x + |K|\overline{\eta}_y) \\ &= \frac{\mu}{\rho}(\overline{\eta}_x + |K|\overline{\eta}_y)(1 - \mathrm{e}^{-\rho t}). \end{aligned}$$

So when $t \in [0, t_\epsilon)$,

$$\begin{aligned} |e_y(t)| &= |Ce_x + \eta_y| \le |C||e_x| + \overline{\eta}_y \\ &\le \frac{\mu}{\rho}|C|(\overline{\eta}_x + |K|\overline{\eta}_y)(1 - \mathrm{e}^{-\rho t}) + \overline{\eta}_y. \end{aligned}$$

Then using the continuity of $e_y(t)$, we can obtain

$$\begin{aligned} |e_y(t_\epsilon)| &< \frac{\mu}{\rho}|C|(\overline{\eta}_x + |K|\overline{\eta}_y) + \overline{\eta}_y \\ &= \frac{\mu}{\rho}|C|\overline{\eta}_x + (\frac{\mu}{\rho}|C||K| + 1)\overline{\eta}_y = \epsilon \end{aligned}$$

which contradicts the assumption. So we conclude that $|e_y(t)| < \epsilon$ for $t \in [0, T]$ and the fault estimate is zero in this interval with the adaptive law (7).□

On the premise of no false alarms, selecting an appropriate threshold, which can be obtained by choosing μ and ρ properly, can also reduce the missing alarm ratio of the scheme.

5. PERFORMANCE OF FAULT ESTIMATION

Now the main theorem of this paper is presented which demonstrates that the fault estimate, the state and the output estimation errors in the dynamics of robust estimation scheme are all uniformly bounded.

Theorem 3. In the presence of a sensor fault the robust fault detection and identification scheme (3), (4) and (7) guarantees that $e_x(t)$, $e_y(t)$ and $\hat{f}(t)$ are uniformly bounded, and for any finite time $t_f > 0$ there exist two constants $\kappa_1, \kappa_2 > 0$ and a bounded function $\zeta(t)$ depending on the unknown inputs such that the output estimation error satisfies

$$\int_T^{T+t_f} |e_y(t)|^2\,\mathrm{d}t \le \kappa_1 + \kappa_2 \int_T^{T+t_f} |\zeta(t)|^2\,\mathrm{d}t \tag{10}$$

Proof. Let $\overline{e}_x(t) \triangleq e_x(t) + \Omega(t)\tilde{f}(t)$ (Vemuri and Polycarpou, 1997) where $\tilde{f}(t) \triangleq f - \hat{f}(t)$, from the equations of the system (1) and the adaptive observer (3) and (4), we can obtain

$$\begin{aligned} \dot{\overline{e}}_x - E\dot{\overline{e}}_x(t-\tau) &= (A-KC)\overline{e}_x + A_d\overline{e}_x(t-d) \\ &\quad + (\eta_x - K\eta_y) + K\Phi f \end{aligned}$$

$$e_x = \overline{e}_x - \Omega\tilde{f} \tag{11}$$

$$e_y = C\overline{e}_x - C\Omega\tilde{f} + \tilde{f} + \eta_y - \Phi f \tag{12}$$

where $\Phi(t) \triangleq I - \mathcal{B}(t-T)$. From (2), it can be derived that

$$\dot{\Phi} = -\Lambda\Phi \qquad t \ge T \tag{13}$$

where $\Phi(T+\theta) = I$ for $\theta \in [-\overline{\tau}, 0]$ and $\Lambda = \mathrm{diag}\{\alpha_1, \cdots, \alpha_m\}$. Let $\overline{e}_x(t) = \zeta_1(t) + \zeta_2(t)$, and decompose the dynamics of $\overline{e}_x(t)$ when $t \ge T$ into two parts (Vemuri and Polycarpou, 1997)

$$\dot{\zeta}_1 - E\dot{\zeta}_1(t-\tau) = (A-KC)\zeta_1 + A_d\zeta_1(t-d) + (\eta_x - K\eta_y) \quad (14)$$

$$\dot{\zeta}_2 - E\dot{\zeta}_2(t-\tau) = (A-KC)\zeta_2 + A_d\zeta_2(t-d) + K\Phi f, \quad (15)$$

where $\zeta_1(T+\theta) = 0$ and $\zeta_2(T+\theta) = \overline{e}_x(T+\theta)$ for $\theta \in [-\overline{\tau}, 0]$. If $|e_y(t)| < \epsilon$ for $t \geq T$ then $\hat{f}(t) = 0$, and the theorem holds trivially. To prove stability after the fault is detected we can choose a Lyapunov-Krasovskii functional candidate

$$V(t) = V_1(t) + V_2(t) + V_3(t) \quad (16)$$

where

$$\begin{aligned} V_1 &= \tilde{f}^T\Gamma^{-1}\tilde{f} \\ V_2 &= (2+\gamma_0^2)c^2\mathrm{trace}(\Phi\Lambda^{-1}\Phi) \\ V_3 &= 2\Big\{[\zeta_2 - E\zeta_2(t-\tau)]^T P[\zeta_2 - E\zeta_2(t-\tau)] \\ &\quad + \int_{t-\tau}^{t} \zeta_2^T S\zeta_2 \,\mathrm{d}\theta + \int_{t-d}^{t} \zeta_2^T Q\zeta_2 \,\mathrm{d}\theta\Big\} \end{aligned}$$

The time derivative of $V_1(t)$ along the solution of (7) is given by

$$\begin{aligned} \dot{V}_1 &= 2\tilde{f}^T(C\Omega - I)^T e_y \\ &\quad -2\tilde{f}^T\mathcal{I}\frac{\hat{f}\hat{f}^T}{\hat{f}^T\Gamma\hat{f}}\Gamma(C\Omega-I)^T e_y \end{aligned}$$

Considering that (Ioannou and Sun, 1996)

$$\tilde{f}^T\mathcal{I}\frac{\hat{f}\hat{f}^T}{\hat{f}^T\Gamma\hat{f}}\Gamma(C\Omega-I)^T e_y \geq 0,$$

we can derive that $\dot{V}_1(t) \leq 2\tilde{f}^T(t)[C\Omega(t) - I]^T e_y(t)$. Thus, from (12) and (13) the time derivative of $V(t)$ can be derived as

$$\begin{aligned} \dot{V} &\leq -2|e_y|^2 - 2(2+\gamma_0^2)c^2|\Phi|_{\mathrm{F}}^2 + \dot{V}_3 \\ &\quad +2e_y^T(C\zeta_1 + \eta_y + C\zeta_2 - \Phi f) \end{aligned}$$

Considering the fact that $|\Phi(t)| \leq |\Phi(t)|_{\mathrm{F}}$, after completing squares we obtain

$$\begin{aligned} \dot{V} &\leq -\tfrac{1}{4}|e_y|^2 + |C\zeta_1 + \eta_y|^2 \\ &\quad -[|e_y|^2 - 2e_y^T(C\zeta_1+\eta_y) + |C\zeta_1+\eta_y|^2] \\ &\quad -[\tfrac{1}{2}|e_y|^2 - 2e_y^T C\zeta_2 + 2|C\zeta_2|^2] \\ &\quad -[\tfrac{1}{4}|e_y|^2 - 2c|e_y||\Phi| + 4c^2|\Phi|^2] \\ &\quad +2|C\zeta_2|^2 - 2\gamma_0^2c^2|\Phi|^2 + \dot{V}_3 \\ &\leq -\tfrac{1}{4}|e_y|^2 + |C\zeta_1+\eta_y|^2 \\ &\quad +2|C\zeta_2|^2 - 2\gamma_0^2c^2|\Phi|^2 + \dot{V}_3 \\ &\leq -\tfrac{1}{4}|e_y|^2 + |C\zeta_1+\eta_y|^2 \\ &\quad +2[|C\zeta_2|^2 - \gamma_0^2|\Phi f|^2 + \tfrac{1}{2}\dot{V}_3] \end{aligned}$$

Now let $\omega(t) = \Phi(t)f$, from the selection of K and the proof of lemma 1 there exists $|C\zeta_2(t)|^2 - \gamma_0^2|\Phi(t)f|^2 + (1/2)\dot{V}_3(t) < 0$, so

$$\dot{V} \leq -\tfrac{1}{4}|e_y|^2 + |C\zeta_1+\eta_y|^2. \quad (17)$$

Therefore $\dot{V}(t) \leq 0$ whenever $|e_y(t)| > 2|C\zeta_1(t) + \eta_y(t)|$.

As stated in section 3, the projection operator $\mathcal{P}(\cdot)$ guarantees $|\hat{f}(t)| \leq c$, so $\tilde{f}(t), \hat{f}(t) \in \mathcal{L}_\infty$. It is obvious that $\Phi(t) \in \mathcal{L}_\infty$ from (13), therefore from assumption 2 and 3, observing (14) and (15), $\zeta_1(t), \zeta_2(t) \in \mathcal{L}_\infty$ can be derived. Observing (4) $\Omega(t) \in \mathcal{L}_\infty$ can also be obtained. Thus from (11) and (12), conclusions can be drawn that $e_x(t), e_y(t) \in \mathcal{L}_\infty$, and the first part of this theorem holds true.

When inequality (17) is integrated over the interval $[T, T+t_f]$, it is derived that

$$\begin{aligned} \int_T^{T+t_f} |e_y(t)|^2 \,\mathrm{d}t &\leq 4[V(T) - V(T+t_f)] \\ &\quad + 4\int_T^{T+t_f} |C\zeta_1(t) + \eta_y(t)|^2 \,\mathrm{d}t \end{aligned}$$

So the second part of this theorem is proved with $\kappa_1 = 4\sup_{t_f\geq 0}[V(T) - V(T+t_f)]$ and $\kappa_2 = 4$. □

The inequality (10) implies that the performance of fault estimation is limited by the extended $\mathcal{L}_2$ norm of $\zeta(t)$, which corresponds to the filtered value of the unknown inputs $\eta_x(t)$ and $\eta_y(t)$.

6. SIMULATIONS

Consider the uncertain linear time-invariant neutral delay system given by (1) with

$$A = \begin{bmatrix} 2.5 & -0.5 \\ 0 & -3 \end{bmatrix}, A_d = \begin{bmatrix} 0.1 & -0.05 \\ 0.03 & 0.1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$E = \mathrm{diag}\{0.1, 0.1\}$, $C = [\,1\ 0\,]$, $D = 0$, $\tau = 0.3$s and $d = 0.5$s. The initial condition is $x(\theta) = [\,-2\ -2\,]^T$ for $\theta \in [-0.5, 0]$. Since this system is unstable, a simple proportion integral controller (with the proportional and integral parameters set to 10 and 4, respectively) is used to keep the output of the system being at 10. Using the LMI Toolbox in Matlab, $K = [\,8.6221\ -0.0743\,]^T$ can be selected that guarantees the system (5)-(6) stable with $\gamma_0 = 36.5381$. The unknown inputs in the state and the output equations are described respectively as

$$\eta_x = \begin{bmatrix} \sin(0.25t) \\ \sin(0.25t + 0.5\pi) \end{bmatrix}, \ \eta_y = \sin(0.25t + \pi/3)$$

which satisfy $\|\eta_x(t)\|_\infty = \|\eta_y(t)\|_\infty = 1$ clearly. In the simulation investigated here the threshold ϵ of the dead-zone is selected as 0.4, and the fault magnitude is supposed to be not larger than 100.

An incipient sensor fault will be investigated which is defined by

$$\mathcal{B}(t-T)f = \begin{cases} 0 & \text{if} \quad t < 40\text{s} \\ 15[1 - \mathrm{e}^{-0.05(t-40)}] & \text{if} \quad t \geq 40\text{s} \end{cases}$$

The learning rate matrix is chosen as $\Gamma = 10$. Figure 1 shows that the fault estimate remains at zero before 40s, and starts deviating from zero at 40.039s when the output estimation error is first

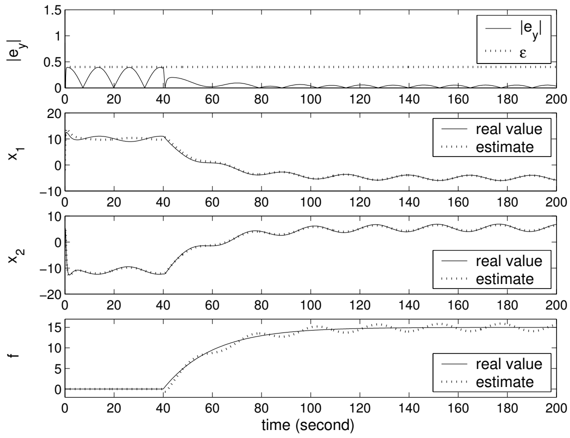

Fig. 1. Simulation results

larger than the threshold, which means that the fault is detected at 0.039s after it occurs. After that the fault estimation unit begins to work, and the fault estimate tracks the real value in a desired manner.

Therefore, conclusions can be drawn that the robust fault detection and identification scheme designed in this paper prevents false alarms in the presence of unknown inputs and provides a good approximation of the fault that can be used in further diagnosis and fault-tolerant control.

7. CONCLUSIONS

Based on an adaptive observer, a robust fault detection and identification scheme is proposed for a class of uncertain linear time-invariant neutral delay systems with unknown inputs in the state and output equations. Sensor fault, either incipient or abrupt, is considered. Theoretical analysis and simulation results demonstrate that the fault detection and identification scheme is robust to the bounded unknown inputs without any false alarm and is capable of estimating the fault with desired accuracy.

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