FINITE-FREQUENCY IDENTIFICATION: SELFTUNING OF TEST SIGNAL

A.G. Alexandrov

Lab.7, Institute for Control Science RAS
65 Profsojusnaya str., 117806 Moscow, Russia
e-mail: alex7@ipu.rssi.ru

Abstract: A linear stable plant with unknown coefficients in the presence of an unknown-but-bounded disturbance is considered. The finite-frequency technique for identification of the plant makes use of a test signal with minimal number of harmonics (this value is equal to the plant state-space dimension). It is shown that if frequencies of the test signal are chosen outside of a natural frequencies band (where log magnitude of the plant has corner frequencies), then identification results may very strongly depend on errors of the frequency characteristics determination. In order to find an estimate of the boundaries of the natural frequencies band, a procedure of selftuning of test signal frequencies is given. Test signal amplitudes are selftuned as well. It provides the prescribed boundaries of the input and output plant.

Keywords: Identification, unknown-but-bounded disturbance, frequency response, test signal, self-tuning.

1. INTRODUCTION

In the last decades, the various identification methods were developed for control plants subjected to unknown-but-bounded external disturbances and noises. These are the instrumental-variable method (Wong and Polak, 1967;Tsypkin and Poznyak, 1989;Ljung, 1999), finite-frequency identification (Alexandrov, 1994), randomized algorithms (Polyak and Granichin, 2003;Bunich and Bakhtatze,2003). In these methods, a test signal is used. It is based on *a priori* information about the parameters of the plant, disturbance, and noise. The test signal has to be independent on disturbance and noise and the plant output has to be in the prescribed limits.

Since the form and the parameters of a test signal affect essentially the accuracy and duration of identification, selftuning of this signal in the process of identification is required when *a priori* information is little.

In finite-frequency identification, the test signal is a sum of harmonics with given amplitudes and frequencies; their number is equal to the state-space plant dimension. In order to provide the limits on the input and output plant, selftuning of the amplitudes is required. Selftuning of test frequencies is needed as well, since it is intuitively clear that they have to be chosen from an interval (natural frequency band), where corner frequencies of log magnitude of the plant are located; however this interval is not known. A method of the experimental estimation of the natural frequency band was proposed in (Alexandrov, 2001).It is the basis of a procedure of the frequencies selftuning described here.

The paper consists of two parts. In the first part, it is shown that if the test frequencies are taken outside of the natural frequency band, then the results of identification may depend heavily on errors in the determination of the frequency characteristics. In the second part, the procedure of selftuning for test signal is proposed.

The paper is organized as follows. First, the finite-frequency identification technique is described. Next, in Section 3, the problem of test signal selftuning is formulated. Section 4 is devoted to the analysis of sensitivity of the identification results to the choice of test frequencies. Finally, the procedure of test signal selftuning is given in Section 5.

2. PRELIMINARIES

Consider a completely controllable, asymptotically stable plant is described by the following equation:

$$\begin{gathered} d_n y^{(n)} + \cdots + d_1 \dot{y} + y = \\ k_\gamma u^{(\gamma)} + \cdots + k_1 \dot{u} + k_0 u + f, \quad t \geq t_0, \end{gathered} \tag{1}$$

where $y(t)$ is the measured output, $u(t)$ is the input to be controlled, $y^{(i)}$, $u^{(j)}$ $\left(i = \overline{1,n},\ \ j = \overline{1,\gamma}\right)$ are the derivatives of these functions, $f(t)$ is unknown-but-bounded disturbance. The coefficients d_i and k_j $\left(i = \overline{1,n},\quad j = \overline{0,\gamma}\right)$ are some unknown numbers; n and γ are known, and $\gamma < n$.

The identification problem is to find estimates $\hat{d}_i$ and $\hat{k}_j$ $\left(i = \overline{1,n},\ \ j = \overline{0,\gamma}\right)$ of the plant coefficients such that the identification errors satisfy the following relations:

$$\hat{d}_i \div d_i \leq \varepsilon_i^d, \quad \hat{k}_j \div k_j \leq \varepsilon_j^k, \quad i = \overline{1,n} \quad j = \overline{0,\gamma}, \tag{2}$$

where ε_i^d and ε_j^k $\left(i = \overline{1,n},\quad j = \overline{0,\gamma}\right)$ are given numbers, and the symbol $\div$ means: $a \div b = |a - b|/|b|$ if $b \neq 0$ and $a \div b = |a|$ otherwise.

Let us consider the finite-frequency identification technique, which gives a solution to this problem.

A set of $2n$ numbers

$$\alpha_k = \operatorname{Re} w(j\omega_k), \quad \beta_k = \operatorname{Im} w(j\omega_k), \quad k = \overline{1,n}, \tag{3}$$

where

$$w(s) = \frac{k_\gamma s^\gamma + \cdots + k_0}{d_n s^n + d_{n-1} s^{n-1} + \cdots + 1} \tag{4}$$

is called the *frequency domain parameters* (FDP).

The FDP estimates are found experimentally as follows: the plant (1) is excited by the test signal

$$u = \sum_{k=1}^{n} \rho_k \sin \omega_k (t - t_0), \qquad t \geq t_0, \tag{5}$$

where the amplitudes ρ_k $\left(k = \overline{1,n}\right)$ and test frequencies ω_k $\left(k = \overline{1,n}\right)$ are specified positive numbers,then its output is fed to the Fourier filters, whose outputs give the following FDP estimates:

$$\begin{aligned} \hat{\alpha}_k &= \alpha_k(\tau) = \frac{2}{\rho_k \tau} \int_{t_F}^{t_F+\tau} y(t) \sin \omega_k (t - t_0) dt, \\ & \qquad\qquad\qquad\qquad\qquad\qquad k = \overline{1,n}, \\ \hat{\beta}_k &= \beta_k(\tau) = \frac{2}{\rho_k \tau} \int_{t_F}^{t_F+\tau} y(t) \cos \omega_k (t - t_0) dt, \end{aligned} \tag{6}$$

where τ is a filtering time and $t_F \geq t_0$ is the initial instant for filtering.

In order to formulate conditions on the convergence of the FDP estimates (6) to the true FDP, the following functions are introduced:

$$\begin{aligned} \ell_k^\alpha(\tau) &= \frac{2}{\rho_k \tau} \int_{t_F}^{t_F+\tau} \bar{y}(t) \sin \omega_k (t - t_0)\, dt, \\ & \qquad\qquad\qquad\qquad\qquad\qquad k = \overline{1,n}, \\ \ell_k^\beta(\tau) &= \frac{2}{\rho_k \tau} \int_{t_F}^{t_F+\tau} \bar{y}(t) \cos \omega_k (t - t_0)\, dt, \end{aligned} \tag{7}$$

where $\bar{y}(t)$ is the "natural" output of the plant when the test signal (5) is absent $(u(t) = 0)$.

Definition 2.1. A disturbance $f(t)$ is called strongly FF-filterable if, for the given numbers δ^α and δ^β, there exists filtering time τ^* such that

$$\begin{gathered} \frac{|\ell_k^\alpha(\tau^*)|}{|\alpha_k(\tau^*)|} \leq \delta^\alpha, \quad \frac{\left|\ell_k^\beta(\tau^*)\right|}{|\beta_k(\tau^*)|} \leq \delta^\beta, \\ k = \overline{1,n}, \quad \tau \geq \tau^*. \end{gathered} \tag{8}$$

Conditions (8) can be examined by experiment.

If the disturbance $f(t)$ is strongly FF-filterable, then the filtering errors $\Delta\alpha_k(\tau) = \alpha_k - \alpha_k(\tau)$, $\Delta\beta_k(\tau) = \beta_k - \beta_k(\tau)$ $\left(k = \overline{1,n}\right)$ have the following properties: $\lim_{\tau\to\infty} \Delta\alpha_k(\tau) = \lim_{\tau\to\infty} \Delta\beta_k(\tau) = 0$ $\left(k = \overline{1,n}\right)$(Alexandrov,1999).

The estimates of the plant coefficients are found on the basis of the FDP estimates. In fact, the identity $w(s) = \dfrac{k(s)}{d(s)}$ and expressions (3) give the following system of the linear algebraic equations:

$$\hat{k}(s_k) - (\alpha_k + j\beta_k)\hat{\bar{d}}(s_k) = \alpha_k + j\beta_k, \quad k = \overline{1,n}, \tag{9}$$

where $\hat{\bar{d}}(s) = \hat{d}(s) - 1 = \hat{d}_n s^n + \cdots + \hat{d}_1 s$, $\hat{k}(s) = \hat{k}_\gamma s^\gamma + \cdots + \hat{k}_1 s + \hat{k}_0$, $s_k = j\omega_k$ $\left(k = \overline{1,n}\right)$.

Assertion 2.1. (Alexandrov,1994). If the plant (1) is completely controllable, then the system (9) has a unique solution d_i, k_j $\left(i = \overline{1,n},\ \ j = \overline{0,\gamma}\right)$ which does not depend on the choice of the frequencies ω_i $\left(\omega_i \neq \omega_j\ (i \neq j),\ \omega_i \neq 0\ \left(i = \overline{1,n}\right)\right)$.

Substituting the FDP by their estimates, the following *frequency equations* of identification

$$\hat{k}(s_k) - (\hat{\alpha}_k + j\hat{\beta}_k)\hat{\hat{d}}(s_k) = \hat{\alpha}_k + j\hat{\beta}_k, \quad k = \overline{1,n}, \tag{10}$$

are obtained.

In order to examine the requirements (2), the frequency techniques of model validation (Alexandrov, 1999) can be used.

3. PROBLEM STATEMENT

It has been assumed above that the amplitudes and frequencies of test signal (5) are given. In order to specify them a priori, a large volume of information about plant (1) is needed . In fact, first,the plant output and input are bounded by given numbers y^* and u^*:

$$|y(t)| \leq y^*, \quad |u(t)| \leq u^*, \quad t \geq t_0, \tag{11}$$

where y^* such that

$$|\bar{y}(t)| < y^*, \quad t \geq t_0. \tag{12}$$

The conditions (11) are provided by the choice of amplitudes ρ_k $\left(k = \overline{1,n}\right)$ for signal (5).

Second, it is intuitively clear that the test frequencies have to be chosen from a frequencies interval, where the corner frequencies of log magnitude of the plant (a natural frequencies band) are placed. At first glance, it contradicts the assertion 2.1, however, the assertion describes properties of the equations (9) but for identification the frequency equations (10), where the FDP's are substituted by their estimations, are used. In order to introduce a notion of a natural frequencies band, transfer function of plant (1) is represented as

$$w(s) = k\frac{\prod\limits_{i=1}^{p_1}(s+\omega_{1,i})\prod\limits_{i=1}^{p_2}\left(s^2+2\xi_{i,2}\omega_{i,2}+\omega_{i,2}^2\right)}{\prod\limits_{i=1}^{p_3}(s+\omega_{3,i})\prod\limits_{i=1}^{p_4}\left(s^2+2\xi_{i,4}\omega_{i,4}+\omega_{i,4}^2\right)}. \tag{13}$$

A set $L = \{\, |\omega_{1,1}|, |\omega_{1,2}|, \ldots, |\omega_{1,p_1}|; |\omega_{2,1}|, |\omega_{2,2}|, \ldots, |\omega_{2,p_2}|; \omega_{3,1}, \omega_{3,2}, \ldots, \omega_{3,p_3}; \omega_{4,1}, \ldots, \omega_{4,p_4}\}$ is called the *natural frequencies* of the plant (1).

The *lower* (ω_l) and *upper*(ω_u) boundaries of the natural frequencies are marked as

$$\omega_l = \min L \qquad \text{and} \qquad \omega_u = \max L.$$

Denote $\Omega_l = \{\omega : \omega \in (0, \omega_l)\}$,
$\Omega = \{\omega : \omega \in [\omega_l, \omega_u]\}$, $\Omega_u = \{\omega : \omega \in (\omega_u, \infty)\}$.

In the next section, it is shown that, if the test frequencies are taken from low-frequencies band $\left(\omega_k \in \Omega_l,\, k = \overline{1,n}\right)$ or upper-frequencies band $\left(\omega_k \in \Omega_u,\, k = \overline{1,n}\right)$, then small errors of the filtration may give large errors of identification and therefore a part of the test frequencies have to lie into the natural frequencies band $\left(\omega_k \in \Omega,\, k \in \overline{1,n}\right)$.

Problem 3.1. Find a way of the amplitudes and frequencies self-tuning of the test signal (5) such that the plant output and input satisfy the requirements (11) and a part of the test frequencies lie into the natural frequencies band $\left(\omega_k \in \Omega,\, k \in \overline{1,n}\right)$.

A solution of the problem is based on the following assertion (Alexandrov,2001).

Assertion 3.1. Let the plant (1) be exited by the test signal $u(t) = \rho_1 \sin \omega_1(t - t_0)$ and its output is fed to the Fourier's filter (6) $(n = 1)$. There exist a sufficiently large filtering time $\tau = \tau^*$ and a sufficiently small test frequency $\omega_1 \in \Omega_l$ such that a number

$$\bar{\omega}_l(\tau^*) = \left|\frac{\omega_1 \alpha_1(\tau^*)}{\beta_1(\tau^*)}\right| \tag{14}$$

is near to the lower (ω_l) boundary of the natural frequencies (the nearness depends on ω_1, τ^* and a difference of ω_l and a natural frequency that is nearest to ω_l).

A analogous assertion is proved for the estimate of the upper boundary ($\bar{\omega}_u$) of the natural frequencies band.

4. SENSITIVITY ANALYSIS OF IDENTIFICATION ERRORS

Denote the maximal relative errors of filtration and identification as

$$\eta_{\alpha,\beta} = \max_{1\leq k\leq n}\left\{\hat{\alpha}_k \div \alpha_k\,,\, \hat{\beta}_k \div \beta_k\right\}, \tag{15}$$

$$\eta_{d,k} = \max_{1\leq k\leq n}\left\{\hat{d}_k \div d_k\,,\, \hat{k}_k \div k_k\right\} \tag{16}$$

respectively.

Definition 4.1. Number $C = \dfrac{\eta_{dk}}{\eta_{\alpha\beta}}$ is called a sensitivity coefficient of identification errors with respect to filtration errors.

Assertion 4.1. There exists a set of the test frequencies $\omega_k \in \Omega_l$ $\left(k = \overline{1,n}\right)$ and the strongly FF-filterable disturbance $f(t)$ such that the sensitivity coefficient C is larger than any given positive number C^* $(C > C^*)$.

The proof of this assertion is based on two assertions (properties) and a lemma that are formulated below.

Using transfer function (4) the following expression for the FDP is obtained

$$\alpha_k = \frac{\sum\limits_{q=0}^{[\frac{n+\gamma}{2}]} l_{2q}\omega_k^{2q}}{\sum\limits_{q=0}^{n} m_q\omega_k^{2q}}, \quad \beta_k = \frac{\sum\limits_{q=0}^{\{\frac{n+\gamma}{2}\}-1} l_{2q+1}\omega_k^{2q+1}}{\sum\limits_{q=0}^{n} m_q\omega_k^{2q}}, \quad (17)$$

$\left(k = \overline{1,n}\right)$, where $\ell_0 = k_0$, $\ell_1 = k_1 - k_0 d_1$, ..., $[\cdot]$ and $\{\cdot\}$ are nearest to $\cdot$ integer number such that $[\cdot] \leq \cdot \leq \{\cdot\}$.

The FDP (17) can be approximated by

$$\alpha_k^l = l_0, \quad \beta_k^l = \sum_{q=0}^{\{\frac{n}{2}\}-1} l_{2q+1}\omega_k^{2q+1}, \quad k = \overline{1,n}. (18)$$

The following assertion is almost obvious.

Property 4.1. For any small number $\delta_l > 0$ there exists a set of the test frequencies $\omega_k \in \Omega_\ell$ $\left(k = \overline{1,n}\right)$ such that

$$\alpha_k^l \div \alpha_k < \delta_l, \quad \beta_k^l \div \beta_k < \delta_l, \quad k = \overline{1,n}. \quad (19)$$

■

The FDP estimates can be represented as

$$\begin{aligned} \hat{\alpha}_k &= \alpha_k^l + \varepsilon_\alpha^l(\omega_k) + \Delta\alpha_k(\tau), \\ \hat{\beta}_k &= \beta_k^l + \varepsilon_\beta^l(\omega_k) + \Delta\beta_k(\tau), \end{aligned} \quad (20)$$

where $\varepsilon_\alpha^l(\omega_k) = \alpha_k - \alpha_k^l$ and $\varepsilon_\beta^l(\omega_k) = \beta_k - \beta_k^l$ $\left(k=\overline{1,n}\right)$.

Lemma 4.1. For the specified test frequencies ω_k $\left(k = \overline{1,n}\right)$ there exists a strongly FF-filterable disturbance $f(t)$ and filtration time τ^* such that the following equalities

$$\begin{aligned} \varepsilon_\alpha^l(\omega_k) &= -\Delta\alpha_k(\tau^*), \\ \varepsilon_\beta^l(\omega_k) &= -\Delta\beta_k(\tau^*), \end{aligned} \quad k = \overline{1,n} \quad (21)$$

hold. ■

The lemma proof is given in Appendix.

The equalities

$$\hat{\alpha}_k = \alpha_k^l, \quad \hat{\beta}_k = \beta_k^l, \quad k = \overline{1,n}, \quad (22)$$

follow (20) with $\tau = \tau^*$ and conditions (21).

Property 4.2. If the FDP estimates are represented by (22), then the solution of frequency equations (10) is unique and it has the following view

$$\hat{d}_i = 0, \quad i = \overline{1,n}. \quad (23)$$

Proof of this property is bulky (for shortness it is omitted), but its idea may be explained by an example when $n = 2$. In this case the frequency equations (10) are rewritten as

$$\begin{aligned} \hat{k}_0 + \jmath\omega_k\hat{k}_1 - (\hat{\alpha}_k + \jmath\hat{\beta}_k)(\jmath\omega_k\hat{d}_1 - \omega_k^2\hat{d}_2) &= \\ = \hat{\alpha}_k + \jmath\hat{\beta}_k, \quad k = \overline{1,2}. \end{aligned} \quad (24)$$

In accordance with expression (18) the FDP estimates $\hat{\alpha}_k = l_0$ and $\hat{\beta}_k = l_1\omega_k$ $\left(k = \overline{1,2}\right)$ and then the system has the obvious solution $\hat{d}_1 = \hat{d}_2 = 0$, $\hat{k}_0 = l_0$, $\hat{k}_1 = l_1$. This solution is unique since the determinant of the system is equal to $[l_1(\omega_1 - \omega_2)(\omega_1 + \omega_2)]^2$ and it is not zero.

Now using properties 4.1 and 4.2 and lemma 4.1 the assertion 4.1 can be easily proved. In fact, let any large number C^* be specified. Take $\delta_l = 1/C^*$ and find the frequencies $\omega_k \in \Omega_l$ $\left(k = \overline{1,n}\right)$ for which the inequalities (19) are fulfilled.

The equalities (23) give $\eta_{dk} \geq 1$. On the other hand, the expressions(19) and (22) give $\eta_{\alpha\beta} < \delta_l$ and therefore the assertion 4.1 is proved.

5. PROCEDURE OF SELFTUNING

During the selftuning process of the test signal, the plant (1) is exited by the following signal

$$\begin{aligned} u(t) &= \rho_{[i]}^{[j]} \sin\omega_{[i]}(t - t_0), \quad (25) \\ t_{[i]}^{[j-1]} &\leq t < t_{[i]}^{[j]}, \quad t_{[i+1]}^{[0]} = t_{[i]}^{[n_{[i-1]}]}, \\ i &= \overline{1,n_\omega} \quad j = \overline{1,n_{[i]}}, \end{aligned}$$

where i $\left(i = \overline{1,n_\omega}\right)$ is a number of an interval of the frequencies tuning, j $\left(j = \overline{1,n_{[i]}}\right)$ is a number of a subinterval of the amplitude tuning.

Durations of all subintervals are equal

$$T_{[i]} = t_{[i]}^{[j]} - t_{[i]}^{[j-1]} = \frac{2\pi}{\omega_{[i]}} p_{[i]}, \quad i = \overline{1,n_\omega},$$

where $p_{[i]}$ $\left(i = \overline{1,n_\omega}\right)$ are given numbers.

Procedure 5.1

(1) Feed to the plant (1) signal (25) with a given sufficiently small frequency $\omega_{[1]} = \omega^*$ and an amplitude $\rho_{[1]}^{[0]} = u^*$; examine the first condition (11). If it is satisfied,the searched amplitude is found. Otherwise, put $\rho_{[1]}^{[1]} = u^*/\delta$, where $\delta > 1$ is a given number, and so on until the condition (11) is satisfied for $\rho_{[1]}^{[n_{[1]}]} = \rho^*$.

Measure the outputs $\alpha_1(\tau^*)$ and $\beta_1(\tau^*)$ of the Fourier's filter (6), where $n = 1$, $\rho_1 = \rho^*$, $\omega_1 = \omega_{[1]}$ (a way of determination of filtering time (τ^*) from the conditions (8) of the strongly FF-filterability is given after the procedure).

(2) Calculate the lower boundary estimate $\bar{\omega}_l(\tau^*)$ by formulae (14).

(3) Repeat the operations 1-2, putting in the signal (25): $\omega_{[2]} = \omega_{[1]}/\delta_\omega$, where $\delta\omega > 1$ is a given number and find new lower boundary estimate – $\omega_l(\tau^{**})$; examine the condition

$$\hat{\bar{\omega}}_l(\tau^*) \div \hat{\bar{\omega}}_l(\tau^{**}) \le \varepsilon_\omega, \qquad (26)$$

where ε_ω is a given sufficiently small number.

If the inequality (26) is satisfied then the searched $\hat{\bar{\omega}}_l = \bar{\omega}_l(\tau^{**})$. Otherwise, put $\omega_{[3]} = \omega_{[2]}/\delta_\omega$ and so on until this inequality is satisfied.

(4) Repeat the operations 1-3 for a sufficiently large frequency $\omega_{[1]}$ and find the upper boundary estimate $\hat{\bar{\omega}}_u$ of the natural frequencies of the plant.

(5) Choice the n frequencies of the set $\hat{\bar{\Omega}} = \{\omega : \omega \in [\hat{\bar{\omega}}_l, \hat{\bar{\omega}}_u]\}$ (for example, calculate of them as $\omega_1 = \hat{\bar{\omega}}_l$, $\omega_k = \hat{\bar{\omega}}_l + \dfrac{\hat{\bar{\omega}}_u - \hat{\bar{\omega}}_l}{n-1}(k-1)$ $\left(k = \overline{2,n}\right)$); repeat operation 1 for each frequency ω_k $\left(k = \overline{1,n}\right)$; find the FDP estimates $\hat{\alpha}_k$, $\hat{\beta}_k$ $\left(k = \overline{1,n}\right)$; solve the frequency equations (10), that gives the plant coefficients estimations.

(6) Form the vector L of the natural frequencies of the identified plant; find the set of these frequencies Ω^{id}. If $\omega_k \in \Omega^{id}$ $\left(k = \overline{1,n}\right)$, then the selftuning is finished. Otherwise, repeat the procedure 5.1, decreasing the numbers δ^α, δ^β and ε_ω, until the requirement $\omega_k \in \Omega^{id}$ $\left(k = \overline{1,n}\right)$ is satisfied.

In order to find the filtering time τ^* for operation 1, this operation is formed from the pause-intervals, where $\rho_{[i]}^{[j]} = 0$, and the test-intervals, where $\rho_{[i]}^{[j+1]} = \rho^*$.

Let $\tau^* = T_{[1]}$. Using the outputs of filters (6) and (7), the inequalities (8), in which δ^α and δ^β are given numbers, are examined. If they are satisfied than $\tau^* = T_{[1]}$. Otherwise, the operation 1 is repeated for $\tau^* = T_{[1]}\delta_T$, where $\delta_T > 1$ is a given number, and so on until the conditions (8) are satisfied.

Remark 5.1 The described way of finding of the filtration time τ^* serves for an experimental test of the a priori assumption about the strongly FF-filtrability of the disturbance $f(t)$. If the filtering time τ^*, for which the conditions (8) satisfy, does not exist, then the frequency $\omega_{[1]}$ is changed until the conditions (8) are satisfied.

Let us introduce a class of disturbances $f(t)$ for which the convergence of procedure 5.1 is readily proved.

Definition 5.1. The disturbance $f(t)$ is contiguously stationary if

$$\max_{t_{[i]}^{[j-1]} + t_F \le t \le t_{[i]}^{[j]}} |\bar{y}(t)| \div \max_{t_{[i]}^{[j]} + t_F \le t \le t_{[i]}^{[j+1]}} |\bar{y}(t)| \le \varepsilon_y, \qquad (27)$$

where ε_y is a given sufficiently small number.

The following assertion is almost obvious.

Assertion 5.1. If the disturbance $f(t)$ is contiguously stationary and the strongly FF-filterable then procedure 5.1 converges to the frequencies $\omega_k \in \Omega^{id}$ $\left(k = \overline{1,n}\right)$ and the requirements (11) to the input and output plant are satisfied.

Remark 5.2. The number ε_y may be essentially enhanced, if the amplitude of signal (25) is tuned, when the filtration time τ^* is searched.

MATLAB-function "Finite-frequency identification" was created (Alexandrov and Orlov, 2005) on the base of the procedure 5.1. Applications of this function show its effectiveness.

6. CONCLUSION

In this paper it is shown that, if the frequencies of the test signal (5) are chosen outside of the natural frequencies band of plant (1), then the sensitivity of identification results to filtration errors can be very high (assertion 4.1).

In connection with it, the finite-frequency method is added by the procedure 5.1. This procedure (by selftuning of the test frequencies) gives the part of the test frequencies into the natural frequencies band. In addition, in order to carry out the requirements (11) to boundaries of the input and output plant, the selftuning of the test signal amplitudes is proposed.

This development of the finite-frequency method gives new possibilities for identification of the real plants.

REFERENCES

Alexandrov, A.G. (1994). Finite-frequency method of identification. *10-th IFAC Symposium on System Identification.* Preprints, Vol. 2, pp. 523-527.

Alexandrov, A.G. (1999). Finite - Frequency Identification and model validation of stable plant. 14 th World Congress of IFAC, Preprints, Beijing, China, vol. H, pp.295 - 301.

Alexandrov, A.G. (2001). Finite-frequency identification: boundaries of frequencies of a test signal. *Automation and Remote Control*, Vol. 62, No 11.

Alexandrov, A.G. (2004). Finite-frequency identification: test signal selftuning. *The collection of proceedings: "Robust control and frequency identification"*. Electrostal: EPI MISA. pp. 67-97 (in Russian).

Alexandrov, A.G. and Yu.F. Orlov. (2005). ADAPLAB-M: directive for identification with test signal selftuning. *Proceedings of IV International conference "System identification and control problems "*. SICPRO'2005. Institute of Control Sciences, Moscow, ISBN 5-201-14975-8.

Bunich, A.L. and N.N. Bakhtadze. (2003). Synthesis and application of discrete control systems with the identifier. *Nauka*, Moscow (in Russian).

Granichin, O.N. and B.T. Polyak. (2003). Randomized algorithms and optimization at almost any noises. *Nauka*, Moscow (in Russian).

Ljung, L. (1999). System Identification – Theory for the User. *Prentice-Hall, Englewood Cliffs*, New-Jersey.

Tsypkin, Ya.Z. and A.S. Poznyak. (1989). The generalized of instrumental variable method in problems of identification of linear plants. *Reports of an academy of sciences of the USSR*. Vol. 306. No 5. pp. 1068-1072 (in Russian).

Wong, K.Y. and E. Polak. (1967). Identification of linear discrete time systems using instrumental variable method. *IEEE Trans. Automat. Control*. Vol. AC-12. pp. 707-718.

APPENDIX

A.1 PROOF OF LEMMA 4.1

Let disturbance $f(t)$ have the form

$$f(t) = \sum_{k=1}^{2n} \rho_k^f \sin \omega_k^f (t - t_0), \quad t \geq t_0, \tag{A.1}$$

where ρ_k^f and ω_k^f $\left(k = \overline{1, 2n}\right)$ are unknown numbers that should be determined from conditions (21) and inequalities

$$\omega_i^f \neq \omega_j, \quad i = \overline{1, 2n} \quad j = \overline{1, n}, \tag{A.2}$$

which signify that disturbance (A.1) is strong FF-filterable, amplitudes ρ_k^f $\left(k = \overline{1, 2n}\right)$ satisfy the following inequalities

$$\sum_{k=1}^{2n} |\rho_k^f| \leq f^*, \tag{A.3}$$

where f^* is given number The filtration errors have the following structure

$$\begin{aligned} \Delta\alpha_k(\tau) &= e_k^\alpha(\tau) + \ell_k^\alpha(\tau), \\ \Delta\beta_k(\tau) &= e_k^\beta(\tau) + \ell_k^\beta(\tau), \end{aligned} \quad k = \overline{1, n}, \tag{A.4}$$

where $e_k^\alpha(\tau)$ and $e_k^\beta(\tau)$ are vanished functions.

In order to find functions $\ell_k^\alpha(\tau)$ and $\ell_k^\beta(\tau)$ $\left(k = \overline{1, n}\right)$, function $\bar{y}(t)$ is necessary. To this effect the equation (1) rewritten in state space as

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + \boldsymbol{\psi} f, \quad \bar{y} = \boldsymbol{c}^T \boldsymbol{x}, \tag{A.5}$$

where A is a matrix, $\boldsymbol{\psi}$, $\boldsymbol{c}$ are vectors.

The solution of these equations under condition (A.1) is

$$\begin{aligned} \bar{y}(t) = \sum_{k=1}^{2n} \rho_k^f \Big[&\alpha_k^f \sin \omega_k^f (t - t_0) + \\ + \beta_k^f \cos \omega_k^f (t - t_0) &+ æ\left(t, \omega_k^f\right)\Big] + æ^0(t), \end{aligned} \tag{A.6}$$

where $\alpha_k^f = \operatorname{Re} w^f(j\omega_k^f)$, $\beta_k^f = \operatorname{Im} w^f(j\omega_k^f)$ $\left(k = \overline{1, 2n}\right)$, $w^f(s) = \boldsymbol{c}^T (Es - A)^{-1} \boldsymbol{\psi}$,

$$\begin{aligned} æ\left(t, \omega_k^f\right) &= \boldsymbol{c}^T e^{A(t - t_0)} \operatorname{Im}(j\omega_k^f - A)^{-1} \boldsymbol{\psi}, \\ æ^0(t) &= \boldsymbol{c}^T e^{A(t - t_0)} \boldsymbol{x}(t_0). \end{aligned} \tag{A.7}$$

Substituting function (A.6) into the expression (7) and taking into account (A.4) and equalities (21) the following system of the linear algebraic equations for determination of the amplitudes ρ_k^f $\left(k = \overline{1, 2n}\right)$ is derived

$$\begin{aligned} \sum_{i=1}^{2n} q_{ki}^\alpha(\tau^*) \rho_i^f &= -\varepsilon_\alpha^l(\omega_k) - e_k^\alpha(\tau^*) - q_{k0}^\alpha(\tau^*), \\ & \qquad k = \overline{1, n}, \\ \sum_{i=1}^{2n} q_{ki}^\beta(\tau^*) \rho_i^f &= -\varepsilon_\beta^l(\omega_k) - e_k^\beta(\tau^*) - q_{k0}^\beta(\tau^*), \end{aligned} \tag{A.8}$$

where $q_{k0}^\alpha(\tau^*)$ and $q_{k0}^\beta(\tau^*)$ are vanished functions.

Choosing a filtration time τ^* and the disturbance frequencies ω_i^f $\left(i = \overline{1, 2n}\right)$ from interval $(0, \infty)$, when the test frequencies ω_k and amplitudes ρ_k $\left(k = \overline{1, n}\right)$ are specified, a unique solution of system (A.8) can be always obtained. For the condition (A.3) to be fulfilled the numbers $\varepsilon_\alpha^l(\omega_k)$ and $\varepsilon_\beta^l(\omega_k)$ in the right hand side of equations (A.8) must be decreased by decreasing of the frequencies ω_k $\left(i = \overline{1, n}\right)$. It is easily shown that the coefficients of the left hand side of this equations are almost independent on the frequencies ω_k $\left(i = \overline{1, n}\right)$ if frequencies ω_i^f $\left(i = \overline{1, 2n}\right)$ are taken to be sufficiently large.