ANALYSIS OF WINDOWING/LEAKAGE EFFECTS IN FREQUENCY RESPONSE FUNCTION MEASUREMENTS

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Abstract: This paper analyses the errors on the frequency response function measurement of a transfer function due to finite window effects (leakage). First an analysis of the rectangular and the Hanning window is made. It is shown that the leakage error consists of two components: a transient error due to initial and end conditions effects, and an interpolation error due to the combination of neighbouring spectral lines. Eventually, a new 'default' window is proposed with slightly better properties. This allows to reduce the measurement time with 25% if the leakage errors dominate.

Keywords**:** system identification, frequency response function, leakage, windowing.

1. INTRODUCTION

Frequency response function measurements of transfer functions (FRF) are intensively used in many engineering fields. For random excitations, these measurements are disturbed by leakage and noise. For these reasons we strongly advice to apply periodic excitation signals whenever it is possible (Pintelon and Schoukens, 2001). However in many applications the users prefer to apply random noise excitations and this for psychological or technological reasons. It is well known that under these conditions, the FRF measurements are disturbed by leakage (windowing) errors that are induced by the finite length of the measurement window. This paper gives a new insight in the nature of these errors. The kernel idea is based on the observation that 'leakage' errors in an FRF measurement are highly structured which allows to split them in two contributions. The first one is due to initial and end condition (transients) effects, the second are due to the interpolation of the transfer function over neighbouring frequency lines. Replacing the rectangular window by another one shifts the nature of the error from the first contribution to the second one. Explicit expressions are given to describe this behaviour. Bias and variance expressions are given, and the results are illustrated on some simulations.

In the literature a large number of windows is defined and their properties are intensively studied, keeping essentially spectral analysis applications in mind (Harris, 1978; Godfrey, 1993). The major contribution of this paper is to study these properties keeping FRF-measurements in mind which leads to new insights, and eventually to the definition of a new window. This allows to reduce the 'leakage errors' on the FRF measurements, while the noise sensitivity is not increased. On top of that, extremely simple expressions to calculate the leakage induced bias and variance errors are given.

2. THE GENERAL FRAMEWORK

2.1 Setup

Consider a stable, causal, discrete or continuous time, single-input-single-output linear time invariant system G with impulse response g_0, and transfer function G_0:

$$y(t) = g_0(t)*u_0(t) + v(t) = y_0(t) + v(t) \quad , \qquad (1)$$

with * the convolution, u_0, y_0 the exact input and output signal, and $v(t)$ disturbing noise. N samples of the input and output are measured at $kT_s = k/f_s$:

$$u_0(k), y(k) \text{ with } k = 0, \ldots, N-1 \ . \quad (2)$$

The results that are reported in this paper are only valid for stable systems excited with random inputs.

Assumption 1: *System*
The system G with impulse response $g_0(t)$ is assumed to be stable such that $|g_0(t)| \le \alpha_1 e^{-\beta_1 t}$ for $t, \alpha_1, \beta_1 > 0$; and causal such that $g_0(t < 0) = 0$.

Assumption 2: *Excitation*
The excitation is a filtered white noise sequence: $u_0(t) = f(t)*\rho(t)$, where $\rho(t)$ is an i.i.d. random signal with existing moments of any order, and $|f(t)| \le \alpha_2 e^{-\beta_2 t}$ for $t, \alpha_2, \beta_2 > 0$.
For continuous time systems, the power spectrum $S_{u_0u_0}(\omega)$ is band limited: $S_{u_0u_0}(|\omega| > \pi f_s) = 0$.
The band limited assumption is realized by applying anti-alias filters before sampling, to avoid aliasing.

Assumption 3: *Disturbing noise*
$v(t)$ is a stationary noise sequence with bounded second order moments.

2.2. The hidden nature of the leakage errors

From the measurements (2), the FRF $G_0(\Omega_l)$ is retrieved at $f_l = lf_s/N$, $l = 0, \ldots, N/2$, with $G_0(\Omega)$ the Fourier transform of the impulse response $g_0(t)$, and $\Omega_l = j2\pi f_l$ for continuous time systems, and $\Omega_l = e^{j2\pi f_l/f_s}$ for discrete time systems. The discrete Fourier transform $U_0(l), Y(l)$ of the input/output signal (Brigham, 1974) is :

$$X(l) = \frac{1}{N}\sum_{k=0}^{N-1} x(k)e^{-j\frac{2\pi}{N}kl} \ . \quad (3)$$

The following remarkably simple relation holds (Pintelon *et al.*, 1997; Pintelon and Schoukens, 2001):

$$Y_0(l) = G_0(\Omega_l)U_0(l) + T_0(\Omega_l) \ , \quad (4)$$

with G_0 and T_0 smooth rational functions of the frequency Ω . T_0 can be interpreted as a generalized ‘transient’ term. Some of these ideas were already reported before (Rabiner and Allen, 1979; Douce and Balmer, 1985; Antoni, 2004). With the DFT definition (3), $U_0(l), Y_0(l), V(l)$ are an $O(N^{-1/2})$, and the transient $T_0(\Omega_l)$ is an $O(N^{-1})$ (Pintelon and Schoukens, 2001).

In absence of disturbing noise $v(t) = 0$ the FRF estimate is given by:

$$\hat{G}(\Omega_l) = \frac{Y_0(l)}{U_0(l)} = G_0(\Omega_l) + \frac{T_0(\Omega_l)}{U_0(l)} \ . \quad (5)$$

It is the last term in (5) that causes the leakage errors in the FRF measurements. These look like noise in FRF measurements because $U_0(l)$ is random. However, this hides a highly structured nature that can be described by a smooth function T_0 in Ω . Windowing methods exploit this smooth behaviour of T_0 to reduce the leakage errors. Note that the leakage errors in (5) disappear as an $O(N^{-1/2})$.

It is common practice to average $\hat{G}(\Omega_l)$ over multiple measurements (Bendat and Piersol, 1980):

$$\hat{G}^M(\Omega_l) = \frac{\sum_{m=1}^{M} Y^{[m]}(l)\bar{U}_0^{[m]}(l)}{\sum_{m=1}^{M} U_0^{[m]}(l)\bar{U}_0^{[m]}(l)} \ , \quad (6)$$

where $X^{[m]}(l)$ is the spectrum of the signal in the m^{th} realization of the experiment. This estimate converges for $M \to \infty$ to the noise free solution ($v(t) = 0$) if the output noise $v(t)$ is not correlated with the input $u_0(t)$. At a given frequency Ω_l :

$$\lim_{M\to\infty} \hat{G}^M = \frac{\lim_{M\to\infty}\sum_{m=1}^{M} Y_0^{[m]}\bar{U}_0^{[m]}}{\lim_{M\to\infty}\sum_{m=1}^{M} U_0^{[m]}\bar{U}_0^{[m]}} \ne G_0 \quad (7)$$

Due to the leakage effects, this limit is still biased.

2.3 Windows

The Fourier transform of a discrete time signal $x(t)$ is an infinite sum $\sum_{t=-\infty}^{\infty} x(t)e^{-j\omega t}$. This infinite sum is restricted to a finite one in the DFT by considering only a finite number of samples: it is calculated on the ‘windowed’ signal:

$$x_w(t) = w(t)x(t) \ , \quad (8)$$

with $w(t) = 0$ if t is outside the interval $[0, N-1]$. A large number of different windows is proposed in the literature (Harris, 1978), here we focus on the rectangular and the Hanning window:

Rectangular (Dirichlet) window:

$$w(t) = 1 \text{ for } t = 0, 1, \ldots, N-1 \ . \quad (9)$$

Hanning window:

$$w(t) = 0.5 - 0.5\cos(2\pi t/N) \ . \quad (10)$$

There exist a simple relation between the DFT spectra obtained with the Hanning window (X_{Hann}) and the rectangular window (X_{Rect}):

$$X_{\text{Hann}}(l) = \frac{2X_{\text{Rect}}(l) - X_{\text{Rect}}(l-1) - X_{\text{Rect}}(l+1)}{4} \quad (11)$$

which is proportional the 2^{nd} order difference.

3. ANALYSIS OF THE LEAKAGE ERROR

From (4) it is seen that the leakage errors in an FRF measurement are due to the presence of the transient term $T_0(\Omega)$. Since this is a smooth function it can be reduced by differencing the spectra $U_0(l), Y_0(l)$ over neigbouring lines before taking the division (this is the frequency domain interpretation of windowing in the time domain). However, a new interpolation error occurs because also G_0 is frequency dependent. The error due to the transient term (the leakage generating mechanism) will be called e_1, the 'interpolation' error of G_0 will be called e_2.

In the subsections below the rectangular (no difference) and the Hanning window (2^{nd} order difference) are analysed. Next, a new window is proposed that applies only a 1^{st} order difference. It will have slightly better characteristics than the Hanning window and can replace it as the default choice in practice. During these discussions it is assumed that the disturbing noise $v(t) = 0$. At the end the impact of the disturbing noise is analysed for the three proposed windows.

3.1 Rectangular window

For a rectangular window(Bendat and Piersol, 1980; Brigham, 1974), it is found immediately that in the noiseless case:

$$\begin{aligned}\hat{G}_{\text{Rect}}(\Omega_l) &= \frac{G_0(\Omega_l)U_0(l)}{U_0(l)} + \frac{T_0(l)}{U_0(l)} \\ &= (G_0(\Omega_l) + O(N^{-1/2})) \\ &= (G_0(\Omega_l) + e_{1\,\text{Rect}}(l))\end{aligned} \quad (12)$$

Since there is no averaging over neighbouring lines $e_{2\,\text{Rect}}(l) = 0$. Hence the leakage errors disappear as an $O(N^{-1/2})$ for stationary random excitations. The averaged estimate is

$$\hat{G}^M_{\text{Rect}}(\Omega_l) = G_0(\Omega_l) + \frac{\frac{1}{M}\sum_{m=1}^{M}\bar{U}_0^{[m]}(l)T_0^{[m]}(\Omega_l)}{\frac{1}{M}\sum_{m=1}^{M}U_0^{[m]}(l)\bar{U}_0^{[m]}(l)} \quad (13)$$

Systematic contributions

In this section we analyse the systematic error that remains if $M \to \infty$. $T_0^{[m]}$ is the sum of two transient contributions at the beginning and the end of the window and the alias term. Each of these contributions depend on the input signal ($u(t), t < 0$ for the begin transient; $u(N-t), t > 0$ for the end transient; $u(t), t = 0, 1, \ldots, N-1$ for the alias term). Hence, a weak correlation between $T_0^{[m]}(\Omega_l)$ and $U_0^{[m]}$ will exist. It is shown (Schoukens *et al.*, 2004) that this results eventually in a systematic error contribution that can be bounded at a given frequency by:

$$\lim_{M\to\infty} \hat{G}^M_{\text{Rect}} = \frac{E\{\bar{U}_0^{[m]}T_0^{[m]}\}}{E\{U_0^{[m]}\bar{U}_0^{[m]}\}} = O(N^{-1}), \quad (14)$$

at all excited frequencies ($E\{U_0^{[m]}(l)\bar{U}_0^{[m]}(l)\} \neq 0$).

$$\lim_{M\to\infty} \hat{G}^M_{\text{Rect}}(\Omega_l) = G_0(\Omega_l) + O(N^{-1}). \quad (15)$$

Variance

In the absence of disturbing noise, the variance of $\hat{G}^M_{\text{Rect}}(\Omega_l)$ is completely set by the variance of $e_{1\,\text{Rect}}(l)$ which is bounded by an $O(M^{-1}N^{-1})$ under Assumptions 1 and 2 (Schoukens *et al.*, 2004).

3.2 Hanning window

The errors for the rectangular window are completely due to the leakage term $T_0(l)/U_0(l)$. Differencing twice the input/output spectra (applying a Hanning window) results eventually in:

$$\hat{G}_{\text{Hann}}(l) = \frac{2Y_0(l) - Y_0(l+1) - Y_0(l-1)}{2U_0(l) - U_0(l+1) - U_0(l-1)}. \quad (16)$$

Define $\Delta = f_s/N$. Using the smoothness of G_0 and T_0 we have that:

$$\begin{aligned}G_0(\Omega_{l\pm1}) &= G_0 \pm G_0^{(1)}\Delta + G_0^{(2)}\frac{\Delta^2}{2} + O(N^{-3}) \\ T_0(\Omega_{l\pm1}) &= T_0 \pm T_0^{(1)}\Delta + T_0^{(2)}\frac{\Delta^2}{2} + O(N^{-3})O(N^{-1})\end{aligned} \quad (17)$$

and $X^{(n)}$ the n^{th} derivative of $X(\Omega_l)$. The last $O(N^{-1})$ is because $T_0^{(3)}$ is an $O(N^{-1})$. Substituting (17) in (16) results in

$$\hat{G}_{\text{Hann}}(l) = G_0(\Omega_l) + e_{1\,\text{Hann}}(l) + e_{2\,\text{Hann}}(l) + O(N^{-3}) \quad (18)$$

with the leakage (e_1) and interpolation (e_2) error:

$$e_{1\,\text{Hann}}(l) = \frac{-T_0^{(2)}(\Omega_l)\Delta^2}{2U_0(l) - U_0(l+1) - U_0(l-1)} = O(N^{-5/2}) \quad (19)$$

$$\begin{aligned}e_{2\,\text{Hann}}(l) &= -G_0^{(1)}\Delta\frac{U_0(l+1) - U_0(l-1)}{2U_0(l) - U_0(l+1) - U_0(l-1)} \\ &\quad -G_0^{(2)}\frac{\Delta^2}{2}\frac{U_0(l+1) + U_0(l-1)}{2U_0(l) - U_0(l+1) - U_0(l-1)} \\ &= O(N^{-1}) + O(N^{-2})\end{aligned} \quad (20)$$

In this case the leakage error $e_{1\,\text{Hann}}$ is reduced to an $O(N^{-5/2})$, but compared with the rectangular window a new 'interpolation' term $e_{2\,\text{Hann}}$ appears which is $O(N^{-1})$. Hence the Hann window reduces the error from an $O(N^{-1/2})$ to an $O(N^{-1})$, and it switches the nature of the dominant error from 'leakage' errors to 'interpolation' errors.

Again, an averaging procedure is needed:

$$\hat{G}^M_{\text{Hann}}(\Omega_l) = \frac{\sum_{m=1}^{M} Y^{[m]}_{0\text{Hann}}(l)\bar{U}^{[m]}_{0\text{Hann}}(l)}{\sum_{m=1}^{M} U^{[m]}_{0\text{Hann}}(l)\bar{U}^{[m]}_{0\text{Hann}}(l)} \quad (21)$$

Systematic contributions

It is shown (Schoukens *et al.*, 2004) that

$$\lim_{M\to\infty} \hat{G}^M_{\text{Hann}}(\Omega_l) = G_0(\Omega_l) + 2G_0^{(1)}\frac{P^{(1)}_{u_0u_0}}{6P_{u_0u_0}}\Delta^2 - G_0^{(2)}\frac{\Delta^2}{6}$$

$$= G_0(\Omega_l) + O(N^{-2}) \quad (22)$$

with $P_{u_0u_0} = E\{|U_0(l)|^2\}$.

Compared to the rectangular window, the systematic errors are reduced from $O(N^{-1})$ to an $O(N^{-2})$.

Remark: for a white noise excitation, $P^{(1)}_{u_0u_0}(\Omega_l) = 0$.

Variance

The variance of $\hat{G}^M_{\text{Hann}}(\Omega_l)$ is dominated by the first term in (20) and equals (Schoukens *et al.*, 2004):

$$\text{var}(\hat{G}^M_{\text{Hann}}(\Omega_l)) = \frac{|G_0^{(1)}(\Omega_l)\Delta|^2}{3M} = O(M^{-1}N^{-2}) . \quad (23)$$

Estimation of the bias and variance

From equations (22), (23) the level of the systematic errors (for white noise excitation only) and the variance can be estimated as (Schoukens *et al.*, 2004)

$$\lim_{M\to\infty} \hat{G}^M_{\text{Hann}}(\Omega_l) - G_0(\Omega_l) \approx \frac{\text{diff}(\text{diff}(\hat{G}^M_{\text{Hann}}(\Omega_{l-1})))}{6},$$

$$\text{with } \text{diff}(X(l)) = X(l+1) - X(l) \quad (24)$$

for white noise excitation ($P^{(1)}_{u_0u_0}(\Omega_l) = 0$), and

$$\text{std}(\hat{G}^M_{\text{Hann}}(\Omega_l)) \approx \frac{\text{diff}(\hat{G}^M_{\text{Hann}}(\Omega_{l-1}))}{\sqrt{3M}} . \quad (25)$$

Hence it is possible to quantify very easily the impact of the windowing effects.

3.3 The Diff window

The Hanning window reduces the leakage effects on the FRF-measurements from an $O(N^{-1})$ to an $O(N^{-2})$ (systematic error and variance). This error reduction is obtained due to a shift of the nature of the errors from 'leakage' (e_1) errors to 'interpolation' (e_2) errors. The latter one grow with the width of the interpolation interval which is 2 bins (3 lines) for the Hanning window. An alternative window with a smaller width should allow for a better balancing between the leakage and interpolation errors. This idea is elaborated below.

A new window

An alternative for the 3-lines 2nd order difference of the Hanning window is to make only a 1st order difference of the spectra that combines only 2 lines:

$$\hat{G}_{\text{Diff}}(\Omega_{l+\frac{1}{2}}) = \frac{Y_0(l+1) - Y_0(l)}{U_0(l+1) - U_0(l)} = \frac{Y_{0\text{Diff}}(l)}{U_{0\text{Diff}}(l)},$$

$$\text{and } \hat{G}^M_{\text{Diff}}(\Omega_l) = \frac{\sum_{m=1}^{M} Y^{[m]}_{0\text{Diff}}(l)\bar{U}^{[m]}_{0\text{Diff}}(l)}{\sum_{m=1}^{M} U^{[m]}_{0\text{Diff}}(l)\bar{U}^{[m]}_{0\text{Diff}}(l)} . \quad (26)$$

Applying again the Taylor series representation (17), but this time around $\Omega_{l+1/2}$ results in:

$$\hat{G}_{\text{Diff}}(\Omega_{l+\frac{1}{2}}) = G_0(\Omega_{l+\frac{1}{2}}) + e_{1\text{Diff}}(l) + e_{2\text{Diff}}(l) \quad (27)$$

with leakage error:

$$e_{1\text{Diff}}(l) = T_0^{(1)}(\Omega_{l+\frac{1}{2}})\Delta\frac{1}{U_0(l+1) - U_0(l)} = O(N^{-3/2}) , \quad (28)$$

and interpolation error $e_{2\text{Diff}}(l)$ equals

$$G_0^{(1)}(\Omega_{l+\frac{1}{2}})\frac{\Delta}{2}\frac{U_0(l+1) + U_0(l)}{U_0(l+1) - U_0(l)} + G_0^{(2)}(\Omega_{l+\frac{1}{2}})\frac{\Delta^2}{8}$$

$$= O(N^{-1}) + O(N^{-2}) , \quad (29)$$

$e_{2\text{Diff}}$ is reduced w.r.t. $e_{2\text{Hann}}$ by working around the middle frequency $\Omega_{l+1/2}$. In that case an approximation is made over only half a bin to the left and to the right instead of a full bin for the Hann window. The leakage error increased to $O(N^{-3/2})$, but this is not important because it is not the dominating error. More detailed results are given in the next section, but first a time domain interpretation is made.

Time domain interpretation

Making the difference over two neighbouring frequencies can be interpreted as applying a complex window in the time domain (Figure 1):

$$w(k) = 1 - e^{j\frac{2\pi}{N}k} , \quad (30)$$

Systematic contributions

It is shown that (Schoukens *et al.*, 2004):

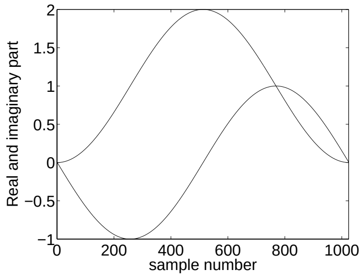

Fig. 1Real and imaginary part of the complex window corresponding to the 1st order difference operation.

$$\lim_{M \to \infty} \hat{G}^{M}_{\mathrm{Diff}}(\Omega_{l+\frac{1}{2}}) = G_0(\Omega_{l+\frac{1}{2}}) + O(N^{-2}) \qquad (31)$$

which is of the same order as the Hanning window.

Variance

The variance becomes (Schoukens *et al.*, 2004):

$$\mathrm{var}(\hat{G}^{M}_{\mathrm{Diff}}(\Omega_{l+\frac{1}{2}})) = \frac{\left|G_0^{(1)}(\Omega_{l+\frac{1}{2}})\Delta\right|^2}{4M} = O(M^{-1}N^{-2}) \qquad (32)$$

So the variance is slightly reduced (-1.25 dB) compared to the Hanning window. This allows to reduce the measurement time with 25% for the same level of variance of the leakage error on the measured FRF.

Estimation of the bias and variance

Again a simple estimate for the standard deviation is obtained (Schoukens *et al.*, 2004):

$$\mathrm{std}(\hat{G}^{M}_{\mathrm{Diff}}(\Omega_{l+\frac{1}{2}})) \approx \left|\mathrm{diff}(\hat{G}^{M}_{\mathrm{Diff}}(\Omega_{l-\frac{1}{2}}))\right| / \sqrt{4M}. \qquad (33)$$

No expression is given for the limit error, because the systematic transient contributions are not simply described.

3.4 Conclusion

In Table 1 all the results of the previous discussions are collected. It is seen that for FRF measurements, the Hanning window is superior to the rectangular window, while the diff window even does a little bit better on all aspects studied. So the diff window can replace the Hanning window as default choice in FRF measurements.

4. NOISE ANALYSIS

The analysis in Section 3 was made assuming that the disturbing noise equals zero. The three windows resulted eventually in the same type of estimates:

$$\hat{G} = \frac{Z}{X_0} = \frac{Z_0 + N_Z}{X_0}, \qquad (34)$$

where Z and X are defined in eq. (12), (16), and (26). For multiple measurements $Z^{[l]}, X_0^{[l]}$, $l = 1, \ldots, M$ are available, and the H_1 averaging technique is used (Bendat and Piersol, 1980):

$$\hat{G}^{M} = \sum_{l=1}^{M} Z^l \bar{X}_0^l \Big/ \sum_{l=1}^{M} X_0^l \bar{X}_0^l. \qquad (35)$$

The variance for $\hat{G}^{M}_{\mathrm{Rect}}, \hat{G}^{M}_{\mathrm{Diff}}, \hat{G}^{M}_{\mathrm{Hann}}$ is approximately given by

$$\sigma_G^2 = \frac{\sigma_{N_Z}^2}{ME\{|X|^2\}}, \qquad (36)$$

This shows that under Assumptions 2 and 3, the noise sensitivity of all these estimators is the same and the variance due to the disturbing noise is

$$\sigma_G^2 = \frac{\sigma_V^2}{ME\{|U_0|^2\}}. \qquad (37)$$

with $E\{\ \}$ the expected value taken over the successive realizations of the input signal.

For small M, $\frac{1}{M}\sum_{l=1}^{M} |U_0^{[l]}(k)|^2$ can be significantly different from $E\{|U_0|^2\}$. At some frequencies large drops in the realized power spectrum appear, jeopardizing the FRF measurement completely. Therefore, it is advised to choose M large enough to avoid these dips (Pintelon and Schoukens, 2001). .

Table 1 Comparison of the rectangular, Hanning, and diff window

window	leakage error e_1	interpolation error e_2	systematic error ($M \to \infty$)	variance
w_{Rect}	$O(N^{-1/2})$	0	$O(N^{-1})$	$O(M^{-1}N^{-1})$
w_{Hann}	$O(N^{-5/2})$	$O(N^{-1})$	$O(N^{-2})$	$O(M^{-1}N^{-2})$: $\left\|G_0^{(1)}(\Omega_l)\Delta\right\|^2/(3M)$
w_{Diff}	$O(N^{-3/2})$	$O(N^{-1})$	$O(N^{-2})$	$O(M^{-1}N^{-2})$: $\left\|G_0^{(1)}(\Omega_{l+\frac{1}{2}})\Delta\right\|^2/(4M)$

5. SIMULATIONS

A discrete time system is excited with white Gaussian noise. $M = 64$ experiments of 8192 points are processed, such that 1024 frequency points in the frequency band of interest are available. This simulation is repeated 1000 times. No disturbing noise is added ($v(t) = 0$) in order to be able to emphasize the effects that are described in this paper. The mean and the standard deviation for the three FRF-estimators are calculated and the results are shown in Figure 2. For

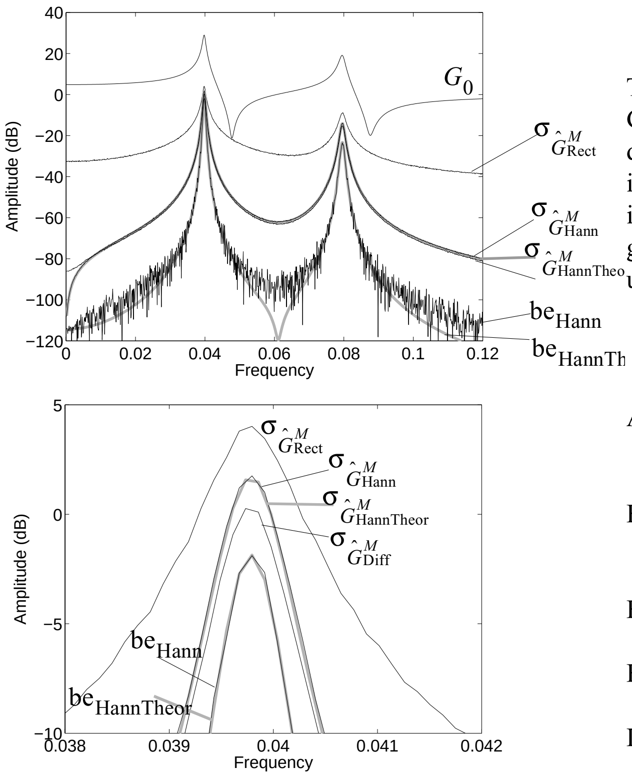

Fig. 2 Comparison of 3 FRF-estimators: G_{Hann}, G_{Diff}, G_{Rect}, together with the exact value G_0 of the FRF. Top: global view; Bottom: zoom around the first resonance freq. The experimental and theoretical standard deviations (σ) and the bias errors (be) are shown.

the Hanning window, the theoretically predicted and experimentally observed standard deviations are compared and a good agreement is found. This is also true for the systematic error. Note also that the new window does slightly better than the Hanning window as was expected from the theory.

6. CONCLUSIONS

In this paper, an analysis of the windowing/leakage effects on FRF-measurements is made. It turns out that the leakage errors in FRF-measurements have hiddenly a highly structured nature that can be used to reduce their impact. The arguments used in window analysis for spectral analysis applications can not be unaltered transferred to FRF-measurements. Replacing the rectangular window by a Hanning shifts the nature of the error from leakage to interpolation. It turns out that an alternative ‘diff’ window can be proposed with slightly better properties. It allows to reduce the measurement time with 25% if leakage errors dominate. If the output noise is the dominating error source, both windows have the same disturbing noise sensitivity. Eventually, simple but accurate expressions to estimate the variance that is induced by the leakage effect are given.

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