FAULT DETECTION FILTER FOR UNCERTAIN FUZZY SYSTEMS: AN LMI APPROACH [1]

Sing Kiong Nguang [*] **Peng Shi** [**] **Steven Ding** [***]

[*] *Dept. of ECE, The University of Auckland, Private Bag 92019 Auckland, New Zealand*
[**] *Division of Math.& Stat., SoT, University of Glamorgan, Pontypridd, CF37 1DL, Wales, United Kingdom*
[***] *Institute for AKS, FoE, University of Duisburg-Essen, Bismarckstrasse 81, 47057 Duisburg, Germany*

Abstract: The paper addresses the problem of designing a robust fault detection filter for an uncertain Takagi-Sugeno fuzzy models. The existence of a robust fault detection filter that guarantees i) the $\mathcal{L}_2$-gain from an exogenous input to a residual signal is less than a prescribed value and ii) the $\mathcal{L}_2$-gain from a fault signal to a residual signal is greater than a prescribed value is given in terms of the solvability of linear matrix inequalities. A numerical example is used to illustrate the effectiveness of the proposed design techniques.

Keywords: Fault detection; Fuzzy models; H-infinity optimization; Fault diagnosis; Estimation theory.

1. INTRODUCTION

In practices, structures of many systems are subject to random variations. These variations may result from component and interconnection failures, parameters shifting, tracking, sudden environmental disturbances, abrupt variations of the operating condition, etc. In order to avoid production deteriorations or damage to machines and humans, variations have to be detected as quickly as possible and decisions that stop the propagation of their effects have to be made.

Over the past two decades, the problem of fault detection (FD) in dynamic systems has attracted considerable attention of many researchers. Various model-based fault detection techniques have been proposed; see (Basseville, 1988; Frank, 1990; Isermann, 1984). In (Patton and Chen, 1991) and (Frank, 1994a), geometrical fault detection approaches have been developed to improve robustness against unknown disturbances. However, most of the above mentioned techniques rely on the system parameters to be known. We know that in reality the system parameters may either be uncertain or time-dependent. Though the problem of uncertain parameters is of crucial importance to the industrial implementation of FD methods, it has however received little attention with only a handful of works so far devoted to it. Recently, in (Ding et al., 1995) and (Frank and Ding, 1994b) frequency domain approaches have been developed based on the fact that faults and unknown input disturbances have different frequency characteristics. An LMI approach to H_-/H_∞ fault detection observers has been proposed in (Hou and Patton, 1996) and (Patton and Hou, 1997). In their papers, an H_- norm is defined as the smallest nonzero singular value of the transfer function matrix from faults to residuals at the

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zero frequency. Unfortunately, this H_- norm is not directly used in the analysis and synthesis of a fault detection observer. In (Rank, 1999) and (Ding et al., 2000), the worst-case fault sensitivity is measured by the smallest nonzero singular value of the transfer function matrix from faults to residuals over a finite frequency range. This proposed measure, however, can only be evaluated ineffective numerical optimizations such as nonlinear programming, frequency grid method, and genetic algorithm. Although many researchers have studied the problem of fault detection (FD) in linear systems with or without uncertainties for many years, the problem of fault detection (FD) in nonlinear systems remains as an open research area.

One of the main difficulties in designing a fault detection filter for nonlinear dynamical systems is that a rigorous mathematical model may be very difficult to obtain, if not impossible. However, many physical systems can be expressed either in some form of mathematical model locally or as an aggregation of a set of mathematical models. Fuzzy system theory enables us to utilise qualitative, linguistic information about a highly complex nonlinear system to construct a mathematical model for it. Recent studies (Tanaka and Sugeno, 1992; Tanaka et. al., 1996; Assawinchaichote and Nguang, 2002; Takagi and Sugeno, 1985; Nguang and Shi, 2003a,b; Nguang and Assawinchaichote, 2003c; Assawinchaichote and Nguang, 2004a,b) have shown that a fuzzy linear model can be used to approximate global behaviours of a highly complex nonlinear system. In this fuzzy linear model, local dynamics in different state space regions are represented by local linear systems. The overall model of the system is obtained by "blending" these linear models through nonlinear fuzzy membership functions. Unlike conventional modelling which uses a single model to describe the global behaviour of a system, fuzzy modelling is essentially a multi-model approach in which simple sub-models (linear models) are fuzzily combined to describe the global behaviour of the system.

Motivated by the lack of an efficient method to evaluate the worth-case fault sensitivity. The aim of this paper is to propose a new worst-case fault sensitivity measure that can be formulated in terms of LMIs which can be solved by an effective algorithm given in Boyd et. al. (1994). These LMIs can then be combined with other design objectives, such as robustness, pole constraints and input constraints to achieve a multi-objective fault detector. In this paper, the existence of a fault detection filter that not only guarantees the $\mathcal{L}_2$-gain from an exogenous input to a residual signal to be less than a prescribed value, but also guarantees that the $\mathcal{L}_2$-gain from a fault signal to a residual signal to be greater than a prescribed value is given in terms of the solvability of linear matrix inequalities.

This paper is organized as follows. In Section 2, system descriptions and definitions are presented. Based on an LMI approach, we develop a technique in Section 3 for designing a fuzzy H_∞ filter that guarantees the $\mathcal{L}_2$-gain of the mapping from the exogenous input noise to the residual signal is less than a prescribed value. The validity of this approach is demonstrated in Section 4. Finally, in Section 5, the conclusion is drawn.

2. SYSTEM DESCRIPTION AND DEFINITIONS

A fuzzy dynamic model has been proposed by Takagi and Sugeno (Takagi and Sugeno, 1985) to represent local linear input/output relations of nonlinear systems. This fuzzy linear model is described by IF-THEN rules and has been shown to be able to approximate a large class of nonlinear systems. Motivated by this, we generalise the TS fuzzy model to represent a fuzzy system with uncertainty and fault whose consequent parts are linear systems with uncertainty and fault.

As in (Tanaka and Sugeno, 1992) and (Tanaka et. al., 1996), we examine a TS fuzzy model with uncertain and fault, in which the ith rule is formulated as follows: Plant Rule i:
IF ν_1 is M_{i1} and $\cdots$ and ν_ϑ is $M_{i\vartheta}$ THEN

$$\begin{aligned} \dot{x} &= [A_i + \Delta A_i]x + [B_i + \Delta B_i]w \\ &\quad + [G_i + \Delta G_i]f \\ y &= [C_i + \Delta C_i]x + [D_i + \Delta D_i]w \\ &\quad + [J_i + \Delta J_i]f \end{aligned} \tag{1}$$

where $i = 1, 2, \cdots, r$, r is the number of IF-THEN rules, $M_{ij}(j = 1, 2, \cdots, \vartheta)$ are fuzzy sets, $x \in \Re^n$ is the state vector with $x(0) = 0$, $w \in \Re^p$ and $f \in \Re^q$ are, respectively, disturbances and faults which belong to $\mathcal{L}_2[0, \infty)$, $y \in \Re^\ell$ is the measurement. The matrices A_i, B_i, C_i, D_i, G_i and J_i are of appropriate dimensions. The matrix functions ΔA_i, ΔB_i, ΔC_i, ΔD_i, ΔG_i and ΔJ_i represent the time-varying uncertainties in the system and satisfy the following assumption.

Assumption 2.1.

$$\Delta A_i = E_{1_i} F(x,t) H_{1_i},\ \Delta B_i = E_{2_i} F(x,t) H_{2_i},$$
$$\Delta C_i = E_{3_i} F(x,t) H_{3_i},\ \Delta D_i = E_{4_i} F(x,t) H_{4_i},$$
$$\Delta G_i = E_{5_i} F(x,t) H_{5_i},\ \Delta J_i = E_{6_i} F(x(t),t) H_{6_i}$$

where H_{j_i} and E_{j_i} are known matrices which characterize the structure of the uncertainties. Furthermore, there exists a positive function ρ such that the following inequality holds:

$$\|F(x,t)\| \leq \rho. \tag{2}$$

Let $\varpi_i(\nu) = \prod_{k=1}^{\vartheta} M_{ik}(\nu_k)$ and $\mu_i(x) = \frac{\varpi_i(\nu)}{\sum_{i=1}^{r} \varpi_i(\nu)}$ where $M_{ik}(\nu_k)$ is the grade of membership of ν_k in M_{ik}. It is assumed in this paper that $\varpi_i(\nu) \geq 0$ for $i = 1, 2, ..., r$ which implies $\sum_{i=1}^{r} \varpi_i(\nu) > 0$ for all t. Therefore,$\mu_i(\nu) \geq 0$ for $i = 1, 2, ..., r$ and $\sum_{i=1}^{r} \mu_i(\nu) = 1$ for all t.

For the convenience of notations, we let $\varpi_i \triangleq \varpi_i(\nu)$, $\mu_i \triangleq \mu_i(\nu)$ and $S_\mu \triangleq \sum_{i=1}^{r} \mu_i$.

The resulting fuzzy system model is inferred as the weighted average of the local models of the form:

$$\begin{aligned} \dot{x} &= [A(\mu) + \Delta A(\mu)]x + [B(\mu) + \Delta B(\mu)]w \\ &\quad + [G(\mu) + \Delta G(\mu)]f \\ y &= [C(\mu) + \Delta C_2(\mu)]x + [D(\mu) + \Delta D(\mu)]w \\ &\quad + [J(\mu) + \Delta J(\mu)]f \end{aligned} \tag{3}$$

where $A(\mu) = \sum_\mu A_i, B_i(\mu) = S_\mu B_i, C_{1_i}(\mu) = S_\mu C_{1_i}, C_{2_i}(\mu) = S_\mu C_{2_i}$,

$\Delta A(\mu) = S_\mu E_{1_i} F(x,t) H_{1_i} := E_1(\mu) F(x,t) H_1(\mu)$,

$\Delta B(\mu) = S_\mu E_{2_i} F(x,t) H_{2_i} := E_2(\mu) F(x,t) H_2(\mu)$,

$\Delta C(\mu) = S_\mu E_{3_i} F(x,t) H_{3_i} := E_3(\mu) F(x,t) H_3(\mu)$,

$\Delta D(\mu) = S_\mu E_{4_i} F(x,t) H_{4_i} := E_4(\mu) F(x,t) H_4(\mu)$

$\Delta G(\mu) = S_\mu E_{5_i} F(x,t) H_{5_i} := E_5(\mu) F(x,t) H_5(\mu)$,

$\Delta J(\mu) = S_\mu E_{6_i} F(x,t) H_{6_i} := E_6(\mu) F(x,t) H_6(\mu)$.

In this paper, we seek for an n^{th}-order $\mathcal{H}_\infty$ fault detection fuzzy filter which is inferred as the weighted average of the local models of the form:

$$\begin{aligned} \dot{\hat{x}} &= \hat{A}(\mu)\hat{x} + \hat{B}(\mu)y \\ \hat{y} &= \hat{C}(\mu)\hat{x} \end{aligned} \tag{4}$$

where $\hat{x}(t)$ is the filter's state vector, $\hat{A}(\mu), \hat{B}(\mu)$ and $\hat{C}(\mu)$ are the matrix functions of appropriate dimensions. $\hat{y}$ is the estimate of $y(t)$. Let the residual signal be $e(t) = y(t) - \hat{y}(t)$, the aims of a robust fault detection filter are to satisfy the following inequalities:

$$\int_0^{T_d} e^T(t)e(t)\, dt < \gamma \int_0^{T_d} w^T(t)w(t)\, dt, \tag{5}$$

and

$$\int_0^{T_d} e^T(t)e(t)\, dt > \beta \int_0^{T_d} f^T(t)f(t)\, dt \tag{6}$$

where γ represents the performance criterion for the effect of disturbance on the residual signal $e(t)$ and it is useful for threshold selection. β stands for the performance criterion for the sensitivity of the residual signal $e(t)$ to the fault $f(t)$.

After designing a robust fault detection filter, the remaining task is to evaluate the residual signal. One of the widely used approaches is to choose a so-called threshold $J_{th} > 0$, i.e.,

$$\int_0^{T_d} e^T(t)e(t)\, dt > J_{th} \Rightarrow \text{ Faults } \Rightarrow \text{ alarm}$$
$$\int_0^{T_d} e^T(t)e(t)\, dt \leq J_{th} \Rightarrow \text{ No Fault}$$

In our system the threshold, $J_{th} = \gamma$ and the detectable fault $f(t)$ is

$$\int_0^{T_d} f^T(t)f(t)\, dt > \frac{\gamma}{\beta} \int_0^{T_d} w^T(t)w(t)\, dt \tag{7}$$

The ratio $\frac{\gamma}{\beta}$ is useful for evaluation of fault detection filters, i.e., FD1 is better than FD2 if $\frac{\gamma}{\beta}$ corresponding to FD1 is smaller than the one of FD2.

Before ending this section, we describe the problem under our study as follows.

Problem Formulation: Given prescribed $\gamma > 0$ and $\beta > 0$, design a robust fault detection filter of the form (4) such that (5) and (6) hold.

3. ROBUST FUZZY $\mathcal{H}_\infty$ FAULT DETECTION FILTER DESIGN

This section begins by formulating the fault sensitivity problem in terms of LMIs which can be solved by an effective algorithms given in Boyd et. al. (1994). Then, we combine these LMIs with a disturbance attenuation objective to yield a fault detection filter that not only guarantees the $\mathcal{L}_2$-gain from a fault signal to a residual signal to be greater than a prescribed value, but also the $\mathcal{L}_2$-gain from an exogenous input to a residual signal to be less than a prescribed value.

When the disturbance input, w is zero, the state-space form of the fuzzy system model (3) with the filter (4) is given by

$$\begin{aligned} \dot{\breve{x}} &= \begin{bmatrix} A(\mu) & 0 \\ \hat{B}(\mu)C(\mu) & \hat{A}(\mu) \end{bmatrix} \breve{x} + \begin{bmatrix} \Delta A(\mu) & 0 \\ \hat{B}(\mu)\Delta C(\mu) & 0 \end{bmatrix} \breve{x} \\ &\quad + \begin{bmatrix} G(\mu) + \Delta G(\mu) \\ \hat{B}(\hat{\mu})\Big\{ J(\mu) + \Delta J(\mu) \Big\} \end{bmatrix} f \end{aligned} \tag{8}$$

where $\breve{x} = [x^T \;\; \hat{x}^T]^T$. The closed-loop system (8) can be re-expressed as follows:

$$\dot{\breve{x}} = A_{cl}(\mu)\breve{x} + \tilde{B}_{cl}(\mu)\tilde{\mathcal{R}}^{-1}\tilde{v} \tag{9}$$

where $\tilde{v} = \tilde{\mathcal{R}} \begin{bmatrix} F(x,t)H_1(\mu)x \\ F(x,t)H_3(\mu)x \\ F(x,t)H_5(\mu)f \\ F(x,t)H_6(\mu)f \\ f \end{bmatrix}$, $\tilde{\mathcal{R}} = \text{diag}\Big\{ \delta I, \delta I, \beta\delta I, \beta I, \beta I \Big\}$, $A_{cl}(\mu) = \begin{bmatrix} A(\mu) & 0 \\ \hat{B}(\mu)C(\mu) & \hat{A}(\mu) \end{bmatrix}$ and $B_{cl}(\mu) = \begin{bmatrix} E_1(\mu) & 0 & E_5(\mu) & 0 \\ 0 & \hat{B}(\mu)E_3(\mu) & 0 & \hat{B}(\mu)E_6(\mu) \end{bmatrix}$

$\left.\begin{matrix} G(\mu) \\ \hat{B}(\mu)J(\mu) \end{matrix}\right]$. The following theorem provides sufficient conditions for the closed-loop system (9) to have (6).

Theorem 3.1. Consider the system (9) satisfies Assumption 2.1 and $A_{cl}(\mu)$ is stable. Suppose there exist a scalar $\delta > 0$, $\beta > 0$, matrices $X > 0$, $Y > 0$, $\mathcal{A}_{ij}$ and $\mathcal{B}_i$ satisfying the following inequality:

$$X - Y > 0 \tag{10}$$

$$\begin{bmatrix} \tilde{\Psi}_{1_{ii}} & (*)^T \\ \tilde{\Psi}_{2_{ii}} & \tilde{\Psi}_{3_{ii}} \end{bmatrix} < 0 \;\; \forall\, i = 1, 2, \cdots, r \tag{11}$$

$$\begin{bmatrix} \tilde{\Psi}_{1_{ij}} + \tilde{\Psi}_{1_{ji}} & (*)^T \\ \tilde{\Psi}_{2_{ij}} + \Psi_{2_{ji}} & \tilde{\Psi}_{3_{ij}} + \tilde{\Psi}_{3_{ji}} \end{bmatrix} < 0 \; \forall\, i < j \leq r \tag{12}$$

$$\tilde{\Psi}_{1_{ij}} = \begin{bmatrix} YA_i + A_i^T Y + \delta\rho[H_{1_i}^T H_{1_i} + H_{3_i}^T H_{3_i}] & (*)^T \\ \mathcal{A}_{ij} & \begin{pmatrix} A_i^T X + XA_i + \mathcal{B}_i C_j \\ + C_i^T \mathcal{B}_j^T - C_i^T C_j \\ + \delta\rho[H_{1_i}^T H_{1_j} + H_{3_i}^T H_{3_j}] \end{pmatrix} \end{bmatrix} \tag{13}$$

$$\tilde{\Psi}_{2_{ij}} = \begin{bmatrix} E_{1_i} & 0 & E_{5_i} & 0 & G_i \\ XE_{1_i} & \mathcal{B}_i E_{3_j} - C_i^T & XE_{5_i} & \mathcal{B}_i E_{6_j} - C_i^T & XG_i + \mathcal{B}_i J_j - C_i^T J_j \end{bmatrix}^T \tag{14}$$

$$\tilde{\Psi}_{3_{ij}} = -\begin{bmatrix} \delta I & (*)^T & (*)T & (*)^T & (*)^T \\ 0 & \delta I + E_{3_i}^T E_{3_j} & (*)^T & (*)^T & (*)^T \\ 0 & 0 & \beta I & (*)^T & (*)^T \\ 0 & E_{6_i}^T E_{3_j} & 0 & E_{6_i}^T E_{6_j} + \beta I & (*)^T \\ 0 & J_i^T E_{3_j} & 0 & J_i^T E_{6_j} & J_i^T J_j - \beta(1 + \tilde{\aleph})I \end{bmatrix} \tag{15}$$

where $\tilde{\aleph} = \rho^2 \sum_{i=1}^{r} \sum_{j=1}^{r} \left\| H_{5_i}^T H_{5_j} + H_{6_i}^T H_{6_j} \right\|$. Then the inequality (6) is guaranteed. Moreover, a suitable fault detection filter is given as follows:

$$\hat{B}_i = (Y - X)^{-1} \mathcal{B}_i \tag{16}$$

$$\hat{C}_i = C_i \tag{17}$$

$$\hat{A}_{ij} = (Y - X)^{-1} \Big\{ - XA_i Y - \mathcal{B}_i C_j Y + \mathcal{A}_{ij} - A_i^T Y - \delta\rho[H_{1_i}^T H_{1_j} + H_{3_i}^T H_{3_j}] \Big\} \tag{18}$$

Proof: The detail of the proof has been omitted due to the page limit. ∇∇∇

Remark 3.1. In (Hou and Patton, 1996)-(Ding et al., 2000), the worst-case fault sensitivity measure can only be evaluated by algorithms such as nonlinear programming, frequency grid methods, and also genetic algorithms which are very ineffective. However, in Theorem 3.1, the worst-case fault sensitivity of the system (9) is formulated in terms of LMIs can be solved by an effective algorithm given in Boyd et. al. (1994).

With $f(t) = 0$ and $\check{x} = [x^T \;\; \hat{x}^T]^T$, the state-space form of the fuzzy system model (3) with the filter (4) can be expressed as follows:

$$\dot{\check{x}} = A_{cl}(\mu)\check{x}(t) + B_{cl}(\mu)\mathcal{R}^{-1} v \tag{19}$$

where $v = \mathcal{R} \begin{bmatrix} F(x,t)H_1(\mu)x \\ F(x,t)H_3(\mu)x \\ F(x,t)H_2(\mu)w \\ F(x,t)H_4(\mu)w \\ w \end{bmatrix}$, $\mathcal{R} = \mathrm{diag}\Big\{\delta I, \delta I, \gamma I, \gamma I, \gamma I\Big\}$, $A_{cl}(\mu) = \begin{bmatrix} A(\mu) & 0 \\ \hat{B}(\mu)C(\mu) & \hat{A}(\mu) \end{bmatrix}$ and $B_{cl}(\mu) = \begin{bmatrix} E_1(\mu) & 0 & E_2(\mu) & 0 & B(\mu) \\ 0 & \hat{B}(\mu)E_3(\mu) & 0 & \hat{B}(\mu)E_4(\mu) & \hat{B}(\mu)D(\mu) \end{bmatrix}$.

Theorem 3.2. Consider the system (19) satisfies Assumption 2.1. Suppose there exist a scalar $\delta > 0$, $\gamma > 0$, matrices $X > 0$, $Y > 0$, $\mathcal{A}_{ij}$ and $\mathcal{B}_i$ satisfying the following inequality:

$$X - Y > 0 \tag{20}$$

$$\begin{bmatrix} \Psi_{1_{ii}} & (*)^T \\ \Psi_{2_{ii}} & \Psi_{3_{ii}} \end{bmatrix} < 0 \forall i = 1, 2, \cdots, r \tag{21}$$

$$\begin{bmatrix} \Psi_{1_{ij}} + \Psi_{1_{ji}} & (*)^T \\ \Psi_{2_{ij}} + \Psi_{2_{ji}} & \Psi_{3_{ij}} + \Psi_{3_{ji}} \end{bmatrix} < 0 \; \forall\, i < j \leq r \tag{22}$$

$$\Psi_{1_{ij}} = \begin{bmatrix} YA_i + A_i^T Y + \delta\rho[H_{1_i}^T H_{1_i} + H_{3_i}^T H_{3_i}] & (*)^T \\ \mathcal{A}_{ij} & \begin{pmatrix} A_i^T X + XA_i + \mathcal{B}_i C_j \\ + C_i^T \mathcal{B}_j^T + \aleph C_i^T C_j \\ + \delta\rho[H_{1_i}^T H_{1_j} + H_{3_i}^T H_{3_j}] \end{pmatrix} \end{bmatrix} \tag{23}$$

$$\Psi_{2_{ij}} = \begin{bmatrix} YE_{1_i} & 0 & YE_{3_i} & 0 & YB_i \\ XE_{1_i} & \mathcal{B}_i E_{3_j} + \aleph C_i^T & XE_{2_i} & \mathcal{B}_i E_{4_i} + \aleph C_i^T & XB_i + \mathcal{B}_i D_i + \aleph C_i^T D_i \end{bmatrix}^T$$

$$\Psi_{3_{ij}} = -\begin{bmatrix} \delta I & (*)^T & (*)^T \\ 0 & (\delta I - \aleph E_{3_i}^T E_{3_j}) & (*)^T \\ 0 & 0 & \gamma I \\ 0 & \aleph E_{4_i}^T E_{3_j} & 0 \\ 0 & D_i^T E_{3_j} & 0 \end{bmatrix}$$

$$\begin{matrix} (*)^T & (*)^T \\ (*)^T & (*)^T \\ (*)^T & (*)^T \\ (\gamma I - \aleph E_{4_i}^T E_{4_j}) & (*)^T \\ D_i^T E_{4_j} & (\gamma I - \aleph D_i^T D_j) \end{matrix} \Bigg] \quad (24)$$

where $\aleph = 1 + \rho^2 \sum_{i=1}^r \sum_{j=1}^r \left\| H_{2_i}^T H_{2_j} + H_{4_i}^T H_{4_j} \right\|$.

Then the inequality (5) is guaranteed. Moreover, a suitable robust filter is given as follows:

$$\hat{B}_i = (Y - X)^{-1} \mathcal{B}_i \quad (25)$$

$$\hat{C}_i = C_i \quad (26)$$

$$\hat{A}_{ij} = (Y - X)^{-1} \Big\{ - X A_i Y - \mathcal{B}_i C_j Y + \mathcal{A}_{ij} - A_i^T Y - \delta \rho [H_{1_i}^T H_{1_j} + H_{3_i}^T H_{3_j}] \Big\} \quad (27)$$

Proof: The detail of the proof has been omitted due to the page limit. ∇∇∇

Remark 3.2. When ΔA_i, ΔB_i, ΔC_i and ΔD_i are all zero(i.e., not uncertainty in the system), Theorem 3.2 will reduce to the result given in Nguang and Assawinchaichote (2003c).

Combining Theorems 3.2 and 3.1, we have the following theorem which provides sufficient conditions for the existence of a robust fault detection filter that guarantees both disturbance rejection and fault detection.

Theorem 3.3. Consider the system (3) satisfies Assumption 2.1. For given $\gamma > 0$ and $\beta > 0$, suppose there exist a scalar $\delta > 0$, matrices $X > 0$, $Y > 0$, $\mathcal{A}_{ij}$ and $\mathcal{B}_i$ such that the inequalities (20)-(22) and (10)-(12) hold. Then the conditions given in (5)-(6) hold. Moreover, a suitable robust fault detection filter is given as follows:

$$\hat{B}_i = (Y - X)^{-1} \mathcal{B}_i \quad (28)$$

$$\hat{C}_i = C_i \quad (29)$$

$$\hat{A}_{ij} = (Y - X)^{-1} \Big\{ - X A_i Y - \mathcal{B}_i C_j Y + \mathcal{A}_{ij} - A_i^T Y - \delta \rho [H_{1_i}^T H_{1_j} + H_{3_i}^T H_{3_j}] \Big\} \quad (30)$$

Proof: This is the consequence of Theorems 3.1 and 3.2. ∇∇∇

4. ILLUSTRATIVE EXAMPLES

Consider an uncertain nonlinear system which is governed by the following state equations:

$$\begin{aligned} \dot{x}_1 &= -0.2x_1 - 0.67x_1^3 + (0.75 + \Delta R)x_2 \\ &\quad +0.1w - 0.5f \\ \dot{x}_2 &= x_1 + 0.1w - 0.5f \\ y &= x_1 + x_2 + 0.1w + 0.3f \end{aligned}$$

where x_1 and x_2 are the state vectors, f is the fault input, w is the disturbance input, y is the measured output and ΔR is the uncertain term with $\Delta R \in [0 \quad 0.02]$. It is assumed that $x_1 \in [-1.5 \quad 1.5]$. The nonlinear system (31) can be exactly represented by two TS fuzzy rules with the membership functions$M_1(x_1) = 1 - \frac{x_1^2}{2.25}$ and $M_2(x_1) = 1 - M_1(x_1) = \frac{x_1^2}{2.25}$. Using these membership functions, the uncertain nonlinear system can be written by the following TS fuzzy model:

Plant Rule 1: IF x_1 is $M_1(x_1)$ THEN

$$\dot{x} = [A_1 + \Delta A_1]x + Bw + Gf,$$
$$y = Cx + Dw + Jf,$$

Plant Rule 2: IF x_1 is $M_2(x_1)$ THEN

$$\dot{x} = [A_2 + \Delta A_2]x + Bw + Gf, ,$$
$$y = Cx + Dw + Jf$$

where

$$A_1 = \begin{bmatrix} -0.2 & -0.75 \\ 1 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} -1.7275 & -0.75 \\ 1 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \ C = \begin{bmatrix} 1 & 1 \end{bmatrix}, \ D = 0.1, \ G = \begin{bmatrix} -0.5 \\ -0.5 \end{bmatrix},$$

$J = 0.3, \Delta A_1 = E_{1_1} F(x) H_{1_1}, \Delta A_2 = E_{1_2} F(x) H_{1_2}$ and $x = [x_1^T \ x_2^T]^T$. Assuming that $\|F(x,t)\| \leq \rho = 1$, we have $E_{1_1} = E_{1_2} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$ and $H_{1_1} = H_{1_2} = \begin{bmatrix} 0 & 0.2 \\ 0 & 0 \end{bmatrix}$. Using the LMI optimization algorithm and Theorem 3.3 with $\gamma = 1$ and $\beta = 50$, we obtain $\delta = 0.9$

$$X = \begin{bmatrix} 6.1981 & -1.9765 \\ -1.9765 & 3.8902 \end{bmatrix}, Y = \begin{bmatrix} 2.8156 & 0.3580 \\ 0.3580 & 1.9811 \end{bmatrix}$$

$$\hat{A}_{11} = \begin{bmatrix} -119.5101 & -63.8225 \\ -169.8249 & -92.0664 \end{bmatrix}, \hat{B}_1 = \begin{bmatrix} 35.0705 \\ 48.1303 \end{bmatrix}$$

$$\hat{A}_{12} = \begin{bmatrix} -121.7980 & -70.6690 \\ -172.5591 & -101.8545 \end{bmatrix}, \hat{B}_2 = \begin{bmatrix} 30.4406 \\ 42.8851 \end{bmatrix}$$

$$\hat{A}_{21} = \begin{bmatrix} -79.6283 & -43.2809 \\ -126.7547 & -66.9750 \end{bmatrix}, \hat{C}_1 = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$\hat{A}_{22} = \begin{bmatrix} -82.2281 & -50.3366 \\ -129.9071 & -77.0323 \end{bmatrix}, \hat{C}_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

Remark 4.1. A square wave with amplitude =0.1 and frequency=0.1Hz was used to simulate the input disturbance. The fault input signal was simulated by a step change at 50 seconds. The ratio of the filter error energy to the disturbance input noise energy is depicted in Figure 1. From Figure 1, one can see that when $f(t) = 0$ the ratio of the residual signal energy to the disturbance input noise noise is less than 1, but when $f(t) = 1$, $t \geq 50$ sec, this ratio becomes greater than the threshold, $J_{th} = 1$ which indicates that the fault has occurred. This confirms that the proposed robust fuzzy fault detection filter guarantees both disturbance rejection and fault detection.

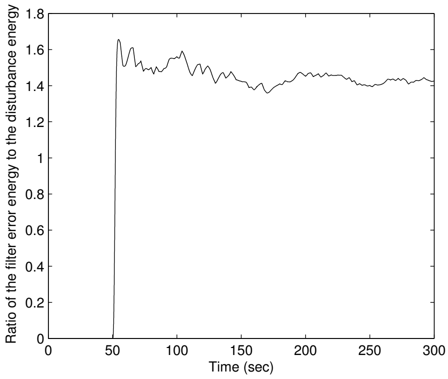

Fig. 1. The ratio of the filter error energy to the disturbance noise energy, $\left(\int_0^{T_d} e^T(t)e(t)dt / \int_0^{T_d} w^T(t)w(t)dt\right)$.

5. CONCLUSION

The problem of designing a robust fault detection filter for an uncertain Takagi-Sugeno fuzzy models has been addressed in the paper. The worst case fault sensitivity has been formulated in terms of LMIs which can be effectively solved by an algorithm proposed in Boyd et. al. (1994). Sufficient conditions for the existence of a robust fault detection filter have been derived. The proposed fault detection filter not only guarantees the $\mathcal{L}_2$-gain from an exogenous input to a residual signal is less than a prescribed value, but also ensures the $\mathcal{L}_2$-gain from a fault signal to a residual signal is greater than a prescribed value. The effectiveness of the proposed design techniques has been demonstrated on a numerical example.

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