ROBUST $\mathcal{H}_\infty$ STATIC OUTPUT FEEDBACK CONTROL OF FUZZY SYSTEMS: AN ILMI APPROACH

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Abstract: This paper examines the problem of robust $\mathcal{H}_\infty$ static output feedback control of a Takagi-Sugeno (T-S) fuzzy system. The proposed robust $\mathcal{H}_\infty$ static output feedback controller guarantees the $\mathcal{L}_2$ gain of the mapping from the exogenous disturbances to the regulated output to be less than or equal to a prescribed level. The existence of a robust $\mathcal{H}_\infty$ static output feedback control is given in terms of the solvability of bilinear matrix inequalities. An iterative algorithm based on the linear matrix inequality is developed to compute robust $\mathcal{H}_\infty$ static output feedback gains. To reduce the conservatism of the design, the structural information of membership function characteristics is incorporated. A numerical example is used to illustrate the validity of the design methodologies.

Keywords: Fuzzy models; Static Controller; Lyapunov function; stabilization methods; H-infinity optimization

1. INTRODUCTION

In the past three decades, considerable attention has been devoted to the problem of nonlinear $\mathcal{H}_\infty$ control; see, (Ball and Helton, 1989; Basar and Olsder, 1982; van der Schaft, 1992; Isidori and Astolfi, 1992; Isidori, 1991; Basar, 1991; Hill and Moylan, 1980; Willems, 1972; Nguang, 1996; Nguang and Fu, 1996) for instance. This problem can be stated as follows. Given a dynamic system with the exogenous input and measured output, design a control law such that the $\mathcal{L}_2$ gain of the mapping from the exogenous input to the regulated output is minimized or no larger than some prescribed level. In general, there are two common methods to solve the nonlinear $\mathcal{H}_\infty$ control problems. One is based on the dissipativity theory and theory of differential games; see (Basar, 1991) and (Ball and Helton, 1989), and the other is based on the nonlinear version of the classical bounded real lemma as developed by (Willems, 1972) and (Hill and Moylan, 1980); see, e.g., (van der Schaft, 1991), (Isidori, 1991), and (Isidori and Astolfi, 1992). Both of these approaches convert the problem of nonlinear $\mathcal{H}_\infty$ control to the solvability of the so-called Hamilton-Jacobi equation (HJE). However, until now, it is still very difficult to find a global solution to the HJE.

The static output feedback problem has also attracted attentions of many researchers over the past two decades (Cao et al., 1998; Geromel et al., 1998; Kar, 1999; Iwasaki et al., 1994; Benton and Smith, 1998; Prempain and Postlethwaite, 2001; Syrmos et al., 1997) . The problem can be stated as follows: given a system, find a static output feedback so that the closed-loop system is stable. Normally, the existence of a full order output feedback control law is given in terms of to the solvability of two convex problems. However, the synthesis of a static output feedback gain or a fixed order controller is much more difficult. The main rationale is that the separation principle does not hold in such cases. A comprehensive survey on static output feedback can be found in (Syrmos et al., 1997).

A great amount of researches have been focused on describing a nonlinear system using a Takagi-Sugeno fuzzy model in recent years; see (Takagi and Sugeno, 1985; Wang et al., 1996; Tanaka et al., 1997; Kim and Lee, 2000; Cao et al., 1996; Ma et al., 1998; Lo and Lin, 2003; Assawinchaichote and Nguang, 2004; Nguang and Assawinchaichote, 2003; Nguang and Shi, 2003, 2001; Chen et al., 2000; Wang et al., 1992). In this fuzzy model, local dynamics in different state space regions are represented by local linear systems. The overall model of the nonlinear system is obtained by "blending" of these linear models through nonlinear fuzzy membership functions. Unlike conventional modelling techniques which use a single model to describe the global behavior of a nonlinear system, fuzzy modelling is essentially a multi-model approach in which simple sub-models (typically linear models) are fuzzily combined to described the global behavior of a nonlinear system. The T-S fuzzy model has been proved to be a very good representation for a certain class of nonlinear dynamic systems. Motivated by the fact that any smooth nonlinear dynamic system can be approximated by a T-S fuzzy model with linear models as fuzzy rule consequences (Takagi and Sugeno, 1985), recently, in (Lo and Lin, 2003), the problem of $\mathcal{H}_\infty$ static output feedback control of T-S fuzzy systems has been investigated.

The major drawback of the above mentioned papers (Takagi and Sugeno, 1985; Wang et al., 1996; Tanaka et al., 1997; Kim and Lee, 2000; Cao et al., 1996; Ma et al., 1998; Lo and Lin, 2003; Assawinchaichote and Nguang, 2004; Nguang and Assawinchaichote, 2003; Nguang and Shi, 2003, 2001; Chen et al., 2000; Wang et al., 1992) is that their design methodologies do not incorporate membership function characteristics, which may lead to conservative design methodologies. Motivated by this drawback, in this paper, $\mathcal{S}$-procedure has been used to incorporate input membership function characteristics into our design. We show that the existence of a robust $\mathcal{H}_\infty$ static output feedback control can be expressed in terms of the solvability of bilinear matrix inequalities. To compute a solution to these BMIs, an iterative algorithm (Cao et al., 1998) based on the linear matrix inequality has been developed.

The rest of this paper is organized as follows. In Section 2, system description and problem formulation are given. Main results are presented in Section 3. The validity of our approach is demonstrated by an example from literatures in Section 4. Finally, conclusions are given in Section 5.

2. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

Consider a nonlinear dynamic plant whose operation space can be partitioned into several regimes according to premise variables. The i-th plant local linear model in the T-S fuzzy model is,

Plant Rule i:

IF ν_1 is M_{i1} and $\cdots$ and ν_p is M_{ip}, THEN

$$\begin{aligned} \dot{x} &= [A_i + \Delta A_i]x + B_{1_i}w + [B_{2_i} + \Delta B_{2_i}]u \\ z &= [C_{1_i} + \Delta C_{1_i}]x + [D_{12_i} + \Delta D_{12_i}]u \\ y &= C_2 x \end{aligned} \tag{1}$$

where $i = 1, \cdots, r,\ r$ is the number of fuzzy rules; ν_k are premise variables, M_{ik} are fuzzy sets, $k = 1, \cdots, p,\ p$ is the number of premise variables; $x \in \Re^n$ is the state vectors, $u \in \Re^m$ is the input, $z \in \Re^l$ and $y \in \Re^p$ are controlled and measured output, respectively. $w \in \Re^q$ is the disturbance which belongs to $\mathcal{L}_2[0, \infty)$. $A_i,\ B_{1_i},\ B_{2_i},\ C_{1_i},\ D_{12_i},\ C_2$ are of appropriate dimensions. $\Delta A_i,\ \Delta B_{2_i},\ \Delta C_{1_i}$ and ΔD_{12_i} represent the uncertainties in the system and satisfy the following assumption.

Assumption 2.1. The parameter uncertainties considered here are norm-bounded, in the form

$$\begin{bmatrix} \Delta A_i & \Delta B_{2_i} \\ \Delta C_{1_i} & \Delta D_{12_i} \end{bmatrix} = \begin{bmatrix} H_{1i} \\ H_{2i} \end{bmatrix} F_i(t) \begin{bmatrix} E_{1i} & E_{2i} \end{bmatrix} \tag{2}$$

where $H_{1i},\ H_{2i},\ E_{1i}$ and E_{2i} are known real constant matrices of appropriate dimensions, and $F_i(t)$ is an unknown matrix function with Lebesgue-measurable elements and satisfies $F_i^T(t)F_i(t) \le I$, in which I is the identity matrix of an appropriate dimension.

By using a center-average defuzzifer, product inference and singleton fuzzifer, the local models can be integrated into a global nonlinear model:

$$\begin{aligned} \dot{x} &= \sum_{i=1}^{r} \mu_i[(A_i + \Delta A_i)x + B_{1_i}w + (B_{2_i} + \Delta B_{2_i})u] \\ z &= \sum_{i=1}^{r} \mu_i[(C_{1_i} + \Delta C_{1_i})x + (D_{12_i} + \Delta D_{12_i})u] \\ y &= C_2 x \end{aligned} \tag{3}$$

where $\nu = [\nu_1, \nu_2, \cdots, \nu_p]^T$, $\omega_i(\nu) = \prod_{k=1}^{p} M_{ik}(\nu_k)$, $\omega_i(\nu) \ge 0$, $\sum_{i=1}^{r} \omega_i(\nu) > 0$, $\mu_i = \frac{\omega_i(\nu)}{\sum_{i=1}^{r} \omega_i(\nu)}$, $\mu_i \ge 0$, $\sum_{i=1}^{r} \mu_i = 1$. Here, $M_{ik}(\nu_k)$ denote the grade of membership of $\nu_k(t)$ in M_{ik}.

For the nonlinear plant represented by (3), the fuzzy static output feedback controller is inferred as follows:

$$u(t) = \sum_{i=1}^{r} \mu_i(\nu(t)) K_i y(t). \tag{4}$$

where K_i is the local controller gain for each plant rule.

Before proceeding with our controller design, we recall the following matrix inequality lemma which will be used throughout the proof.

Lemma 2.1. (Wang et al., 1992) Let $G,\ H,\ E$ and $F(t)$ be real matrices of appropriate dimensions with $F(t)$ being a matrix function. Then we have

(a) for any $\varepsilon > 0$ and $F^T(t)F(t) \le I$,

$$HF(t)E + E^T F^T(t) H^T \le \varepsilon H H^T + \frac{1}{\varepsilon} E^T E, \tag{5}$$

(b) for any $\varepsilon > 0$ such that $\varepsilon E^T E < I$ and $F(t)F^T(t) \le I$,

$$\begin{aligned} &[G + HF(t)E]^T [G + HF(t)E] \le \\ &G^T (I - \varepsilon H H^T)^{-1} G + \frac{1}{\varepsilon} E^T E. \end{aligned} \tag{6}$$

Problem Formulation : Given a prescribed $\mathcal{H}_\infty$ performance $\gamma > 0$, design a fuzzy controller (4) such that

$$\int_0^\infty z^T(t) z(t) dt \le \gamma^2 \int_0^\infty w^T(t) w(t) dt \tag{7}$$

For the convenience of the notation $(*)$ as an ellipsis for terms that are induced by symmetry.

3. MAIN RESULTS

In this section, we shall present our procedure for designing a robust $\mathcal{H}_\infty$ static output feedback control gain for the system (3). In particular, we are interested in finding a controller of the form (4) that ensures (7).

The closed-loop system of (3) with (4) can be written as follows:

$$\begin{aligned} \dot{x} = &\sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i \mu_j \Big\{ [A_i + B_{2_i} K_j C_2 + \\ &H_{1i} F_i(t)(E_{1i} + E_{2i} K_j C_2)] x + B_{1i} w(t) \Big\} \end{aligned} \tag{8}$$

$$\begin{aligned} z = &\sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i \mu_j \Big\{ C_{1_i} + D_{12_i} K_j C_2 + \\ &H_{2i} F_i(t)(E_{1i} + E_{2i} K_j C_2) \Big\} x(t) \end{aligned} \tag{9}$$

Before presenting the main result, the following theorem is needed.

Theorem 3.1. Given a prescribed $\mathcal{H}_\infty$ performance $\gamma > 0$ and positive constants ε_1 and ε_2, if there exist symmetric matrices P and Y_{ij} and matrices K_i satisfying the following conditions:

$$P > 0 \tag{10}$$

$$\Omega_{ii} + \begin{bmatrix} Y_{ii_{n\times n}} & 0 \\ 0 & 0 \end{bmatrix} < 0,\ i = 1, 2, \cdots, r \tag{11}$$

$$\Omega_{ij} + \Omega_{ji} + \begin{bmatrix} 2Y_{ij_{n\times n}} & 0 \\ 0 & 0 \end{bmatrix} < 0,\ i < j \le r \tag{12}$$

$$\begin{pmatrix} Y_{11} & Y_{12} & \cdots & Y_{1r} \\ Y_{12} & Y_{22} & \cdots & Y_{2r} \\ \vdots & & \ddots & \vdots \\ Y_{1r} & Y_{2r} & \cdots & Y_{rr} \end{pmatrix} > 0 \tag{13}$$

where

$$\Omega_{ij} = \begin{bmatrix} A_i^T P + P A_i - P B_{2_i} B_{2_i}^T P & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T \\ B_{2_i}^T P + K_j C_2 & -I & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T \\ H_{1i}^T P & 0 & -\frac{1}{\varepsilon_1} I & (*)^T & (*)^T & (*)^T & (*)^T \\ E_{1i} + E_{2i} K_j C_2 & 0 & 0 & -\varepsilon_1 I & (*)^T & (*)^T & (*)^T \\ C_{1_i} + D_{12_i} K_j C_2 & 0 & 0 & 0 & \varepsilon_2 H_{2i} H_{2i}^T - I & (*)^T & (*)^T \\ E_{1i} + E_{2i} K_j C_2 & 0 & 0 & 0 & 0 & -\varepsilon_2 I & (*)^T \\ B_{1_i}^T P & 0 & 0 & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix} \tag{14}$$

then inequality (7) holds.

Proof: This theorem can be proved by following the same approach as in (Nguang and Shi, 2001) and (Nguang and Shi, 2003). The detail of the proof has been omitted due to the page limit. ■

Note that the matrix inequalities (11) and (12) are bilinear matrix inequalities (BMI) and cannot be solved by a convex optimization algorithm. An iterative algorithm (ILMI) based on the linear matrix inequality (LMI) has been employed to solve this BMI problem in (Cao et al., 1998) and (Kim and Lee, 2000). To apply the ILMI method on (11) and (12), the negative quadratic term $-PB_{2_i}B_{2_i}^T P$ has to be replaced by $-X^T B_{2_i} B_{2_i}^T P - P B_{2_i} B_{2_i}^T X + X^T B_{2_i} B_{2_i}^T X$, where X is the additional design matrix variable. This replacement is due to the fact that for any X and P of the same dimension, we have

$$\begin{aligned} -PB_{2_i}B_{2_i}^T P \le &-X^T B_{2_i} B_{2_i}^T P - P B_{2_i} B_{2_i}^T X \\ &+ X^T B_{2_i} B_{2_i}^T X \end{aligned} \tag{15}$$

Note that in Lemma 3.1, the membership function characteristics of the fuzzy system have been ignored. These membership function characteristics are crucial in many cases and may render less conservative results. Based on the so-called (outer) ellipsoidal approximation algorithm, a new result is derived in the following text by incorporating the membership function characteristics. Before we proceed with the development, the following definition is needed.

Definition 3.1. Let I_ξ be the set of indices for the fuzzy rules that contains the origin $x = 0$:

$$I_\xi \equiv \{\xi | h_\xi(0) \neq 0\}.$$

We also define $\mathcal{R}_{ij}$ as the region where the fuzzy rule i and fuzzy rule j are activated:

$$\mathcal{R}_{ij} \equiv \{x | \mu_i(x)\mu_j(x) > 0\}$$

We assume that each region $\mathcal{R}_{ij}$ $(i, j \notin I_\xi)$ can be outer approximated by a union of ellipsoids $\mathcal{E}_{ijk}$ for $k = 1, \cdots, m$, where m is the number of ellipsoids. That is, matrices T_{ijk} and f_{ijk} exist such that $\mathcal{R}_{ij} \subseteq \bigcup_{k=1}^{m} \mathcal{E}_{ijk}$ where $\mathcal{E}_{ijk} = \{x| \; \|T_{ijk}x + f_{ijk}\| \leq 1\}$. Note that the ellipsoids $\mathcal{E}_{ijk}$ can also be represented as the following LMI form:

$$\begin{bmatrix} x \\ 1 \end{bmatrix}^T \begin{bmatrix} T_{ijk}^T T_{ijk} & (*)^T \\ f_{ijk}^T T_{ijk} & -(1 - f_{ijk}^T f_{ijk}) \end{bmatrix} \begin{bmatrix} x \\ 1 \end{bmatrix} \leq 0 \quad (16)$$

Remark 3.1. Suppose that $\mathcal{R}_{ij} = \{x| \; d_1 \leq c^T x \leq d_2\}$, then it is easy to see that we can take $T_{ij1} = 2c^T/(d_2 - d_1)$ and $f_{ij1} = -(d_2 + d_1)/(d_2 - d_1)$.

Using (15) and the $\mathcal{S}$-procedure, we have the following theorem which incorporates the membership function characteristics.

Theorem 3.2. Given a prescribed $\mathcal{H}_\infty$ performance $\gamma > 0$ and positive constants ε_1 and ε_2, if for $i = 1, \cdots, r$, $i < j$ and $k = 1, \cdots, m$ there exist symmetric matrices P and Y_{ij}, matrices K_i, scalars λ_{iik} and λ_{ijk}, and an auxiliary matrix variable X satisfying the following conditions:

For region $\mathcal{R}_{ij}$ $(i, \; j \in I_\xi)$.

$$P > 0 \quad (17)$$

$$\Phi_{ii} + \begin{bmatrix} Y_{ii} & 0 \\ 0 & 0 \end{bmatrix} < 0 \quad (18)$$

$$\Phi_{ij} + \Phi_{ji} + \begin{bmatrix} 2Y_{ij} & 0 \\ 0 & 0 \end{bmatrix} < 0 \quad (19)$$

For the other regions.

$$\lambda_{iik} > 0, \lambda_{ijk} > 0 \quad (20)$$

$$\Psi_{iik} + \begin{bmatrix} Y_{ii} & 0 \\ 0 & 0 \end{bmatrix} < 0 \quad (21)$$

$$\Psi_{ijk} + \Psi_{jik} + \begin{bmatrix} 2Y_{ij} & 0 \\ 0 & 0 \end{bmatrix} < 0 \quad (22)$$

$$\begin{pmatrix} Y_{11} & Y_{12} & \cdots & Y_{1r} \\ Y_{12} & Y_{22} & \cdots & Y_{2r} \\ \vdots & & \ddots & \vdots \\ Y_{1r} & Y_{2r} & \cdots & Y_{rr} \end{pmatrix} > 0 \quad (23)$$

where

$$\Phi_{ij} = \begin{bmatrix} \begin{pmatrix} A_i^T P + P A_i \\ -X^T B_{2_i} B_{2_i}^T P \\ -P B_{2_i} B_{2_i}^T X \\ +X^T B_{2_i} B_{2_i}^T X \end{pmatrix} & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T \\ H_{1i}^T P & 0 & -\frac{1}{\varepsilon_1} I & (*)^T & (*)^T & (*)^T & (*)^T \\ E_{1i} + E_{2i} K_j C_2 & 0 & 0 & (*)^T & (*)^T & (*)^T & (*)^T \\ C_{1_i} + D_{12_i} K_j C_2 & 0 & 0 & -\varepsilon_1 I & (*)^T & (*)^T & (*)^T \\ E_{1i} + E_{2i} K_j C_2 & 0 & 0 & 0 & \varepsilon_2 H_{2i} H_{2i}^T - I & (*)^T & (*)^T \\ B_{1_i}^T P & 0 & 0 & 0 & 0 & -\varepsilon_2 I & (*)^T \\ & & & & 0 & 0 & -\gamma^2 I \end{bmatrix} \quad (24)$$

$$\Psi_{ijk} = \begin{bmatrix} \begin{pmatrix} A_i^T P + P A_i \\ -X^T B_{2_i} B_{2_i}^T P \\ -P B_{2_i} B_{2_i}^T X \\ +X^T B_{2_i} B_{2_i}^T X \\ -\lambda_{ijk} T_{ijk}^T T_{ijk} \end{pmatrix} & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T \\ H_{1i}^T P & 0 & -\frac{1}{\varepsilon_1} I & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T \\ E_{1i} + E_{2i} K_j C_2 & 0 & 0 & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T \\ C_{1_i} + D_{12_i} K_j C_2 & 0 & 0 & -\varepsilon_1 I & (*)^T & (*)^T & (*)^T & (*)^T \\ E_{1i} + E_{2i} K_j C_2 & 0 & 0 & 0 & \varepsilon_2 H_{2i} H_{2i}^T - I & (*)^T & (*)^T & (*)^T \\ B_{1_i}^T P & 0 & 0 & 0 & 0 & -\varepsilon_2 I & (*)^T & (*)^T \\ -\lambda_{ijk} f_{ijk}^T T_{ijk} & 0 & 0 & 0 & 0 & 0 & -\gamma^2 I & (*)^T \\ & & & & 0 & 0 & 0 & \lambda_{ijk}(1 - f_{ijk}^T f_{ijk}) \end{bmatrix} \quad (25)$$

then inequality (7) holds.

Proof: (18) and (19) can be obtained straight from Theorem 3.1 and (15). (21) and (22) can be obtained by combining (18) and (19) with (16) through the $\mathcal{S}$-procedure. ■

4. NUMERICAL EXAMPLE

To illustrate the validation of the results obtained previously, we consider the following problem of balancing an inverted pendulum on a cart. The equations of motion of the pendulum (Wang et al., 1996) are

$$\dot{x_1} = x_2$$
$$\dot{x_2} = \frac{g\sin(x_1) - amlx_2^2\sin(2x_1)/2 - a\cos(x_1)u}{4l/3 - aml\cos^2(x_1)} + w \quad (26)$$

where x_1 denotes the angle of the pendulum from the vertical position, and x_2 is the angular velocity. $g = 9.8m/s^2$ is the gravity constant, m is the mass of the pendulum, $a = 1/(m+M)$, M is the mass of the cart, $2l$ is the length of the pendulum, and u is the force applied to the cart. In the simulation, the pendulum parameters are chosen as $m = 2kg$, $M = 8kg$, and $2l = 1.0m$.

We approximate the system (26) by the T-S fuzzy model given in (Lo and Lin, 2003):

$$\begin{aligned}
&Rule\ 1: IF x_1\ \ is\ M_1,\ THEN\\
&\quad \dot{x} = (A_1 + \Delta A_1)x + B_1 w + (B_{2_1} + \Delta B_{2_1})u\\
&\quad z = C_1 x + D_{12}u\\
&\quad y = C_2 x\\
&Rule\ 2: IF x_1\ \ is\ M_2,\ THEN\\
&\quad \dot{x} = (A_2 + \Delta A_2)x + B_1 w + (B_{2_2} + \Delta B_{2_2})u\\
&\quad z = C_1 x + D_{12}u\\
&\quad y = C_2 x
\end{aligned}$$

where $A_1 = \begin{bmatrix} 0 & 1 \\ 17.2941 & 0 \end{bmatrix}$, $B_{2_1} = \begin{bmatrix} 0 \\ -0.1765 \end{bmatrix}$, $A_2 = \begin{bmatrix} 0 & 1 \\ 9.3600 & 0 \end{bmatrix}$, $B_{2_2} = \begin{bmatrix} 0 \\ -0.0052 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C_1 = \begin{bmatrix} 1 & 0.3 \end{bmatrix}$, $D_{12} = 0.01$ and $C_2 = \begin{bmatrix} 9 & 0.1 \end{bmatrix}$. The disturbance attenuation level γ is set to be equal to 1 in this example and $\varepsilon_1 = \varepsilon_2 = 1$. The membership functions for Rule 1 and Rule 2 are shown in Figure 1. Let the uncertain terms

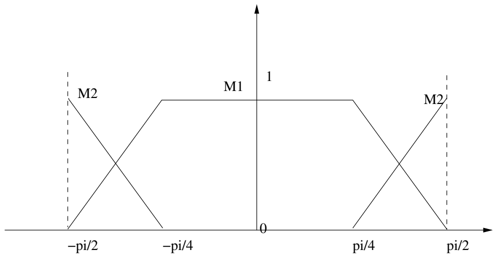

Fig. 1. Membership functions

be given as$H_{1_1} = H_{1_2} = \begin{bmatrix} 0 & 0 \\ 0.15 & 0 \end{bmatrix}$,, $E_{1_1} = E_{1_2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $E_{2_1} = E_{2_2} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $H_{2_1} = H_{2_2} = 0, \|F(t)\| \leq I$. From the membership functions given in Figure 1, we have $I_\xi = \{1\}$

For $\mathcal{R}_{12}$,

$$T_{121} = T_{122} = \frac{8}{\pi}\begin{bmatrix} 1 & 0 \end{bmatrix}, f_{121} = 3, f_{122} = 3.$$

For $\mathcal{R}_{22}$,

$$T_{221} = T_{222} = \frac{8}{\pi}\begin{bmatrix} 1 & 0 \end{bmatrix},\ f_{221} = 3, f_{222} = 3$$

Applying the ILMI method given in (Cao et al., 1998), we have

$$P = \begin{bmatrix} 36.83 & -6.78 \\ -6.78 & 2.26 \end{bmatrix}, Y_{11} = \begin{bmatrix} 201.71 & -49.60 \\ -49.60 & 12.20 \end{bmatrix}$$
$$Y_{12} = \begin{bmatrix} -22.34 & 5.49 \\ 5.49 & -1.35 \end{bmatrix}, Y_{22} = \begin{bmatrix} 96.28 & -31.20 \\ -31.20 & 12.1 \end{bmatrix}$$

and the static output feedback gains

$$K_1 = 400.49,\ K_2 = 47.47. \quad (27)$$

A square wave with amplitude =1 and frequency=0.1Hz was used to simulate the disturbance input noise $w(t)$. The ratio of the regulated output energy to the disturbance input noise energy ($\frac{\int_0^t z^T(t)z(t)dt}{\int_0^t w^T(t)w(t)dt}$) obtained using the fuzzy controller gain (27) is given in Figure 2. We can see that after 5 seconds, the ratio of the regulated output energy to the disturbance input noise energy tends to be a constant value which is about 0.1. Hence, the disturbance attenuation level γ is about 0.3, which is less than the prescribed level 1.

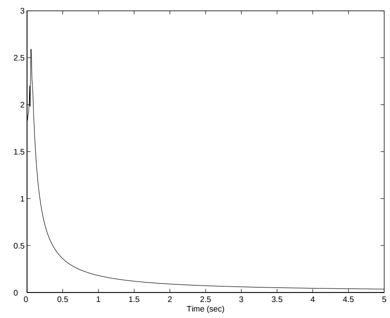

Fig. 2. The ratio of the regulated output energy to the disturbance input noise energy

5. CONCLUSION

Design of a robust $\mathcal{H}_\infty$ static output feedback controller for a T-S fuzzy system has been provided in this paper. The existence of a static output feedback control law has been expressed in terms of the solvability of bilinear matrix inequalities. To compute a solution to the BMIs, an iterative algorithm based on the linear matrix inequality has been proposed. The conservatism of the design has been reduced by incorporating the input membership structural information. A numerical example has been given to illustrate the validity of our design.

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