FUZZY ADAPTIVE SLIDING MODE CONTROL FOR VEHICLE BRAKE SYSTEMS

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Abstract: This paper addresses an approach of continuously adjusting the brake torque to fulfil a comfortable and safe drive. A full-order model which consists of three subsystems is under consideration. The controllers are designed for two cases. In the first case considered the vehicle speed when the slip rate is small, and an adaptive control scheme is then developed. In the second case, the rapid of slip rate is deemed as the control target when the slip rate is large, and an adaptive sliding mode control is developed. Due to unknown changing condition, the fuzzy concept is adopted incorporated with the above-mentioned controllers. Asymptotic stability of the overall system is confirmed via Lyapunov stability theorem.

Keywords: Braking Control, Adaptive, Sliding Mode, Fuzzy, Lapunov Theorem.

1. Introduction

An anti-lock brake system (ABS), which is operated at urgent brake, has been developed as a standard product in automotive system. Limpert (1999) has shown that could control the wheel slip rate appropriately within the rang of 0.2 and 0.25, but it could not guarantee the stop of vehicle motion in safety distance. Thus, in this paper a continuous brake action is presented which considers both slip rate and safety distance. There are many technical literatures discussing the vehicle and wheel dynamics, e.g., Gissinger, *et al.* (1995) presented the vehicle brake dynamics obtained by system identification. Raza, *et al.* (1995) introduced a dynamics model that contains the hydraulic dynamics, but the effects of slip rate have not been considered. Kuang, *et al.* (1999) presented a dynamics model with consideration of hydraulic dynamics and slip rate dynamics, but the system's structure is too complicated. Dugoff, *et al.* (1970) proposed a full-order vehicle motion dynamics with simple and accurate form. In this paper, we have adopted the dynamics model proposed in Dugoff, *et al.* (1970) incorporated with a first-order hydraulic model which has been proposed in Thayer (1965) due to their simplicity, accuracy, and practicality. As for the previously developed the sliding mode techniques and fuzzy control have been applied to ABS, but accurate motion control can not be achieved. Gerdes, *et al.* (1997) developed a vehicle velocity and position control method by using the multiple sliding surfaces control methodology, but the comfort requirement has not been considered during brake. Huang, *et al.* (1999) presented a cruise control method by switching throttle and brake action without considering the slip rate. Yi, et al. (2001) planned a braking profile for deceleration, and the safety distance and comfortable performance can then be determined from this deceleration profile.

However, the system parameters as well as road condition should be known in priori which would very difficult for controller design. Hence, in this paper, to cope with imperfect knowledge of vehicle system and road conditions an adaptive fuzzy controller is developed which does not need information concerning the system parameters.

This paper is organized as follows: In **section 2**, the mathematical model of a vehicle motion system is derived. **Section 3** is presented to introduce the planning of "desired braking profile". In **section 4** the adaptive fuzzy controller which includes braking torque controller and hydraulic system controller is devised and the stability result is also shown to fulfil the requirements. In **section 5**, computer simulation results which can effectively verify the proposed controller are shown. Finally, some concluding remarks are discussed in **section 6**.

2. Model Formulation

In the section, the vehicle model with a servo brake is derived based on previous literatures.

2.1 Tire Force Dynamics

There are many literatures discussing the mathematical model of the tire force, among which the model proposed by Dugoff, *et al.* (1970), is the most accurate but not complicated as compared to the other proposed systems. In the above paper, the author proposed the longitudinal brake force F_x depends on the slip rate and can be mathematically represented as:

$$F_x = c_s \frac{s}{1-s} \qquad \text{if } s < \frac{\mu F_z}{2c_s + \mu F_z} \tag{1}$$

$$F_x = F_z \mu \left(1 - \frac{F_z \mu}{4c_s} \frac{1-s}{s}\right) \quad \text{if } s \geq \frac{\mu F_z}{2c_s + \mu F_z} \tag{2}$$

$$s = \frac{v_f - r\omega_f}{v_f} \tag{3}$$

where:

F_z : normal force (*N*)

v_f : vehicle velocity (*m/s*)

ω_f : wheel angular velocity (*rad/s*)

s : wheel slip rate

c_s : longitudinal tire stiffness (*N*)

μ :friction coefficient

r : wheel radius (*m*)

Furthermore, from Henry, *et al.* (1980), the friction coefficient can be defined as:

$$\mu = \mu_0 e^{-A_s \cdot s \cdot v_f} \tag{4}$$

where:

μ_0: friction coefficient when *s=0*

A_s : adhesion reduction coefficient

Here the effect of suspension system is not taken into account, and all the road surface characteristic coefficients F_z , c_s , A_s and μ_0 are assumed to be constant but unknown.

2.2 Tire Dynamics Model

The wheel is assumed to be a homogeneous rigid-like body, then the rotational dynamics can be represented as:

$$I\dot{\omega}_f = F_x r - T_B \tag{5}$$

where I is the wheel inertial and T_B is the brake torque.

2.3 Servo-Hydraulic System Model

Chao (1965) and Thayer (1997) presented the servo-hydraulic dynamics that can be approximately modelled as a first-order system. Thus, the transfer function can be written as:

$$\frac{T_B}{U} = \frac{c}{s+\tau} \tag{6}$$

where:

T_B: Braking Torque of the Servo Hydraulic System

U : Control Input Voltage

c, τ : Parameters of the Hydraulic System

2.4 Complete Brake Model

Based on above derivations, the complete full-order dynamic model for the hydraulic brake system is divided into two cases and is described as:

Case 1: If $s < \frac{\mu F_z}{2c_s + \mu F_z}$,

$$\dot{v}_f = \frac{c_s}{M}\left(1 - \frac{v_f}{r\omega_f}\right) \tag{7}$$

$$\dot{\omega}_f = \frac{1}{I}\left[c_s\left(\frac{v_f}{\omega_f} - r\right) - T_B\right] \tag{8}$$

$$\dot{T}_B = -\frac{1}{\tau} T_B + cu \tag{9}$$

Case 2: If $s \geq \frac{\mu F_z}{2c_s + \mu F_z}$,

$$\dot{v}_f = \frac{-F_z \mu}{M} + \frac{F_z^2 \mu^2}{4Mc_s} \frac{1-s}{s} \tag{10}$$

$$\dot{s} = \frac{r(\dot{v}_f \omega_f - v_f \dot{\omega}_f)}{v_f^2} \tag{11}$$

$$\dot{T}_B = -\frac{1}{\tau} T_B + cu \tag{12}$$

Where the wheel dynamics in case 2 has been replaced by slip rate dynamics for control purpose.

3. Planning of Desired Braking Profile

Both drive comfort and safety are general requirements in contemporary automotive industry. Yi, *et al.* (2001) presented the desired braking profiles that can be precisely defined according to comfort and safety conditions. Generally, the jerk for deceleration can be mathematically represented as:

$$j(t) = \begin{cases} \frac{a_d \pi}{2\Delta t_a} \sin\left[\frac{\pi}{\Delta t_a}(t - t_1)\right] & t_1 \leq t < t_2 \\ 0 & t_2 \leq t < t_3 \\ -\frac{a_d \pi}{2\Delta t_c} \sin\left[\frac{\pi}{\Delta t_c}(t - t_3)\right] & t_3 \leq t \leq t_4 \end{cases} \tag{13}$$

where:

a_d : desired deceleration constant (*m/s*^{2})

$\Delta t_a = t_2 - t_1$, $\Delta t_b = t_3 - t_2$, $\Delta t_c = t_4 - t_3$ are time parameters designed to satisfy the comfort and safety demands. The Δt_a and Δt_c have been selected to limit the following jerk condition for comfort:

$$-10\ m/s^2 < j(t) < 10\ m/s^2$$

By integrating $j(t)$ twice and third times, the desired velocity and distance functions are respectively written as:

$$v_{des}(t) = \begin{cases} v_0 + \frac{a_d}{2} - \frac{a_d \Delta t_a}{2\pi} \sin\left[\frac{\pi}{\Delta t_a}(t - t_1)\right] & t_1 \leq t < t_2 \\ v_{1f} + a_d t & t_2 \leq t < t_3 \\ v_{2f} + \frac{a_d t}{2} + \frac{a_d \Delta t_c}{2\pi} \sin\left[\frac{\pi}{\Delta t_c}(t - t_3)\right] & t_3 \leq t \leq t_4 \end{cases} \tag{14}$$

$$x_{des}(t)=\begin{cases} x_0+v_0t+\frac{a_dt^2}{4}+\frac{a_d\Delta t_a^2}{2\pi^2}\cos\left[\frac{\pi}{\Delta t_a}(t-t_1)\right] & t_1\le t<t_2 \\ x_{1f}+v_{1f}t+\frac{a_dt^2}{2} & t_2\le t<t_3 \\ x_{2f}+v_{2f}t+\frac{a_dt^2}{4}-\frac{a_d\Delta t_c^2}{2\pi^2}\cos\left[\frac{\pi}{\Delta t_c}(t-t_3)\right] & t_3\le t\le t_4 \end{cases} \quad (15)$$

If it is desirable to stop the vehicle at $t=t_4$, we can set:

$$v_{des}(t_4)=0\Rightarrow v_{2f}+\frac{a_d\Delta t_a}{2}=0 \quad (16)$$

Similarly, we can let:

$$x_{des}(t_4)<d+\varepsilon$$

$$\Rightarrow x_{2f}+v_{2f}\Delta t_c+\frac{a_d\Delta t_c^2}{4}+\frac{a_d\Delta t_c^2}{2\pi^2}<d+\varepsilon \quad (17)$$

where d is the distance measured from the preceding vehicle as the brake is initially applied and ε is some allowable safety distance.

4. Adaptive Fuzzy Controller Design

In this section, an adaptive fuzzy controller is devised to achieve brake trajectory tracking as mentioned previously. Therefore, the controller design will be accomplished in two loops, namely, outer loop and inner loop which are designed to control the vehicle motion dynamics and servo hydraulic dynamics, respectively.

4.1 Vehicle Motion Controller Design

Case 1:

In this case, the control objective is to force the vehicle motion to follow the desired deceleration profiles smoothly. Thus, we consider the equation (7) and (8), and by taking the time derivative of equation (7), we have:

$$\ddot{v}_f=c_1\frac{v_f^2}{\omega_f^3}+c_2\frac{v_f}{\omega_f^2}+c_3\frac{1}{\omega_f}+c_4\frac{v_f}{\omega_f^2}T_B \quad (18)$$

where $c_1=\frac{c_s^2}{IMr}$, $c_2=\frac{c_s^2}{M}\left(\frac{1}{Mr^2}-\frac{1}{I}\right)$, $c_3=-\frac{c_s^2}{M^2r}$, and $c_4=-\frac{c_s}{IMr}$. Let the error trajectory be defined as:

$$e(t)=v_f-v_{fd} \quad (19)$$

Then, by taking twice time derivative of (18), the following equality holds:

$$\ddot{e}(t)=c_1\frac{v_f^2}{\omega_f^3}+c_2\frac{v_f}{\omega_f^2}+c_3\frac{1}{\omega_f}+c_4\frac{v_f}{\omega_f^2}T_B-\ddot{v}_{fd} \quad (20)$$

If the control input T_B is designed as:

$$T_{Bd}\equiv-\frac{1}{\hat{c}_4}\frac{\omega_f^2}{v_f}q_1\equiv T_{Bd1} \quad (21)$$

where:

$$q_1=\delta_1\dot{e}(t)+\delta_2e(t)+\delta_3\int e(t)dt+\hat{c}_1\frac{v_f^2}{\omega_f^3}+\hat{c}_2\frac{v_f}{\omega_f^2}+\hat{c}_3\frac{1}{\omega_f}-\ddot{v}_{fd}$$

$\delta_i, i=1,2$ are some positive constants, and $\hat{c}_i, i=1,2,3,4$ are the estimates of $c_i, i=1,2,3,4$, respectively. we get, after substituting (21) into (20):

$$\ddot{e}(t)=-\delta_1\dot{e}(t)-\delta_2e(t)-\delta_3\int e(t)dt+\tilde{c}_1\frac{v_f^2}{\omega_f^3}+\tilde{c}_2\frac{v_f}{\omega_f^2}+\tilde{c}_3\frac{1}{\omega_f}-\tilde{c}_4\frac{q_1}{\hat{c}_4} \quad (22)$$

where $\tilde{c}_i=c_i-\hat{c}_i$, $i=1,2,\ldots,4$. The above equation can also be rewritten as the following matrix form:

$$\dot{X}=AX+BW\tilde{C} \quad (23)$$

where:

$$X=\left[\int edt \quad e \quad \dot{e}\right]^T$$

$$A=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -\delta_1 & -\delta_2 & -\delta_3\end{bmatrix} \qquad B=\begin{bmatrix}0 & 0 & 1\end{bmatrix}^T$$

$$W=\begin{bmatrix}\frac{v_f^2}{\omega_f^3} & \frac{v_f}{\omega_f^2} & \frac{1}{\omega_f} & -\frac{q_1}{\hat{c}_4}\end{bmatrix} \quad \tilde{C}=\begin{bmatrix}\tilde{c}_1 & \tilde{c}_2 & \tilde{c}_3 & \tilde{c}_4\end{bmatrix}^T$$

To prove the stability of the closed-loop system (23), we define the Lyapunov function candidate as:

$$V_1=\frac{1}{2}X^TPX+\frac{1}{2}\Gamma^{-1}\tilde{C}^T\tilde{C} \quad (24)$$

where P and Γ are some diagonal positive definite matrices. Then by differentiating V_1 with respect to time, we get:

$$\dot{V}_1=\frac{1}{2}X^T(A^TP+PA)X+\tilde{C}^TW^TB^TPX+\Gamma^{-1}\tilde{C}^T\dot{\tilde{C}}$$

If we further let the adaptive laws be:

$$\dot{\tilde{C}}=-\Gamma W^TB^TPX \quad (25)$$

then the following result is achieved:

$$\dot{V}_1=-\frac{1}{2}X^TQX$$

where Q satisfies the Lyapunov equation $A^TP+PA=-Q$. From the Lyapunov stability theory, we can conclude that $\dot{e}(t)$, $e(e)$ and $\int e(t)dt\to 0$ asymptotically.

Case 2:

In this case, an adaptive sliding mode control scheme will be advocated to decrease the slip rate rapidly. Therefore, we consider equation (10) and (11).

$$\dot{v}_f=-\frac{F_z}{M}\mu_0e^{-A_s\cdot s\cdot v_f}\left(1-\frac{F_z}{4c_s}\mu_0e^{-A_s\cdot s\cdot v_f}\frac{1-s}{s}\right) \quad (26)$$

$$\dot{s}=\frac{1}{v_f}\left[s-\left(1+\frac{Mr^2}{I}\right)\right]\frac{F_z}{M}\mu_0e^{-A_s\cdot s\cdot v_f}\cdot\left(1-\frac{F_z}{4c_s}\mu_0e^{-A_s\cdot s\cdot v_f}\frac{1-s}{s}\right)+\frac{r}{I}\frac{1}{v_f}T_B \quad (27)$$

The wheel dynamics has been replaced by the slip rate dynamics according to equation (3). We merely consider equation (27) for controller design. Define:

$$\frac{I}{r}\dot{s}=\frac{I}{r}f+\frac{1}{v_f}T_B \quad (28)$$

And from the fact that:

$$\mu_0 e^{-A_s s v_f} \le \alpha_a$$

we have:

$$\left|\frac{I}{r} f\right| \le p_1\left|\frac{s}{v_f}\right| + p_2\left|\frac{1}{s v_f}\right| + p_3\left|\frac{1}{v_f}\right| \quad (29)$$

where $p_i, i = 1,2,3$ are some positive constants. To rapidly decrease the slip rate *s*, we design the controller as:

$$T_{Bd} = -sign(s) v_f \left(ks + \hat{p}_1\left|\frac{s}{v_f}\right| + \hat{p}_2\left|\frac{1}{s v_f}\right| + \hat{p}_3\left|\frac{1}{v_f}\right| \right) \quad (30)$$
$$\equiv T_{Bd2}$$

where k is a positive constant, and $\hat{p}_i, i = 1,2,3$ are the estimates for $p_i, i = 1,2,3$. For the stability proof, we define the Lyapunov function as follows:

$$V_2 = \frac{1}{2}\left(\frac{I}{r} s^2 + \sum_{l=1}^{3} \Psi_l^{-1} \tilde{p}_l^2 \right) \quad (31)$$

where Ψ is some diagonal positive definite matrices. Then, by differentiating V_2 with respect to time and invoking (28), (29), (30), the following equation satisfies:

$$\dot{V}_2 \le -ks^2 + \tilde{p}_1\left|\frac{s^2}{v_f}\right| + \tilde{p}_2\left|\frac{1}{v_f}\right| + \tilde{p}_3\left|\frac{s}{v_f}\right| - \sum_{l=1}^{3} \Psi_l^{-1} \tilde{p}_l \dot{\hat{p}}_l \quad (32)$$

If we further let the adaptive laws be:

$$\dot{\hat{p}}_1 = \Psi_1\left|\frac{s^2}{v_f}\right|, \dot{\hat{p}}_2 = \Psi_2\left|\frac{1}{v_f}\right|, \dot{\hat{p}}_3 = \Psi_3\left|\frac{s}{v_f}\right|$$

we get:

$$\dot{V}_2 \le -ks^2$$

From Barbalat's lemma, we can infer $s \to 0$ as $t \to \infty$.

4.2 *Servo Hydraulic System Control*

In the servo hydraulic system, the true control input is designed so that the output brake torque T_B can follow the desired brake torque T_{Bd}. For this sake, the model reference adaptive control technique will be solicited in the controller design. Let the reference model be:

$$\frac{T_{Bm}}{T_{Bd}} = \frac{\tau_m}{s + \tau_m} \quad (33)$$

where τ_m is some large positive constant. In state space representation, the hydraulic model and the reference model can be rewritten as:

$$\dot{T}_B = -\tau T_B + cu \quad (34)$$

$$\dot{T}_{Bm} = -\tau_m T_{Bm} + \tau_m T_{Bd} \quad (35)$$

Let the control input u be designed in the following form:

$$u = \hat{k}_1 T_{Bd} - \hat{k}_2 T_B \quad (36)$$

and be substituted into (34) to obtain:

$$\dot{T}_B = -\tau_m T_B - c\tilde{k}_2 T_B + \tau_m T_{Bd} + c\tilde{k}_1 T_{Bd} \quad (37$$

where $\tilde{k}_1 = \hat{k}_1 - k_1^*, \tilde{k}_2 = \hat{k}_2 - k_2^*$, and $k_r^*, r = 1,2$ satisfy:

$$\tau_m = \begin{cases} ck_1^* \\ \tau + ck_2^* \end{cases} \quad (38)$$

Define $e_T = T_B - T_{Bm}$, then the error dynamics:

$$\dot{e}_T = -\tau_m e_T - c\tilde{k}_2 T_B + c\tilde{k}_1 T_{Bd} \quad (39)$$

To guarantee the closed-loop system stability, the Lyapunov function is defined in the following form:

$$V_T = \frac{1}{2}\left(e_T^2 + \sum_{r=1}^{2} \Gamma_r^{-1} c\tilde{k}_r^2 \right) \quad (40)$$

where $\Gamma_i, i = 1,2$ are positive constants and *c*, which is known to be positive. Then, by substituting (39) into the time derivative of V_T, we have:

$$\dot{V}_T = -\tau_m e_T^2 - c\tilde{k}_2 T_B e_T + c\tilde{k}_1 T_{Bd} e_T + \sum_{r=1}^{2} \Gamma_r^{-1} c\tilde{k}_r \dot{\hat{k}}_r \quad (41)$$

Similarly, the following adaptive laws:

$$\dot{\hat{k}}_1 = \Gamma_1 T_{Bd} e_T, \dot{\hat{k}}_2 = \Gamma_2 T_B e_T$$

are applied to (41) to cancel the coupling terms and result in:

$$\dot{V}_T = -\tau_m e_T^2 \quad (42)$$

Again, by Barbalat's lemma, asymptotic convergence of e_T is confirmed.

4.3 *Composite Controller Design with Fuzzy Concept*

The vehicle motion controller is designed separately according to different conditions. These two types of controllers are switched based on the slip condition. However, since the road conditions are unknown, the switching value for slip rate may be not valid. To circumvent this problem, the fuzzy concept is incorporated to form a composite control action. Based on fuzzy concepts, the switching condition can be written in the following linguistic rules:

IF s is large **THEN** $T_{Bd} = T_{Bd1}$

IF s is small **THEN** $T_{Bd} = T_{Bd2}$

where the membership functions of the linguistic terms "large" and "small" are defined as:

$$\mu_l(s) = \begin{cases} 1 & \forall s > \alpha \\ \dfrac{s-\alpha}{\beta-\alpha} & \forall \alpha \le s \le \beta \\ 0 & \forall s < \alpha \end{cases} \quad (43)$$

$$\mu_s(s) = \begin{cases} 1 & \forall s < \alpha \\ \dfrac{-s+\beta}{\beta-\alpha} & \forall \alpha \le s \le \beta \\ 0 & \forall s > \beta \end{cases} \quad (44)$$

, respectively, where α and β are some positive constant. The plots of $\mu_l(s)$ and $\mu_s(s)$ are shown in figure_1.

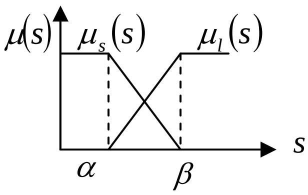

Fig. 1. Membership Functions of μ_s and μ_l

Therefore, after defuzzification process, the composite control law is proposed as:

$$T_{Bd} = \frac{\mu_s T_{Bd1} + \mu_l T_{Bd2}}{\mu_s + \mu_l} \qquad (45)$$

Remark: The choice of parameters α and β in the membership function should be carefully made. Theoretically, the actual switching value s_d must lie in the region $\alpha \le s_d \le \beta$, i.e., the range of s_d must be roughly estimated form some experimental tests.

5. Computer Simulation

In this section, computer simulations are performed to show the effectiveness of the proposed fuzzy control scheme. A very small initial slip rate is reasonably assumed. In the simulation cases, the initial velocity of vehicle as the controller brake torque is applied is set to be 108 *km/hr*. From the simulation results shown below, the velocity of vehicle will track the desired stopping trajectory rapidly, and the vehicle can accurately follow the stopping trajectory so as to guarantee the drive safety. The parameters are selected and listed in the following: M=1240 kg, F=Mg/4 N, c_s=10F N, I=2.1 kg-m^2, R=0.3 m, d=200 m, δ_1=1, δ_2=10, δ_2=700, k=1, Γ=0.2, Ψ=2, Φ=1. The values of α and β are defined as $\alpha = 0.04$ and $\beta = 0.06$. As the initial values of slip rate is small (for *s(0)=0.0001*), the simulation results are shown in figure_2-figure_6. In this case, the tracking performances of vehicle displacement, velocity, and deceleration are demonstrated in figure_2-figure_4. Figure_5 shows the slip rate, whereas figure_6 shows the exerting brake torque profile.

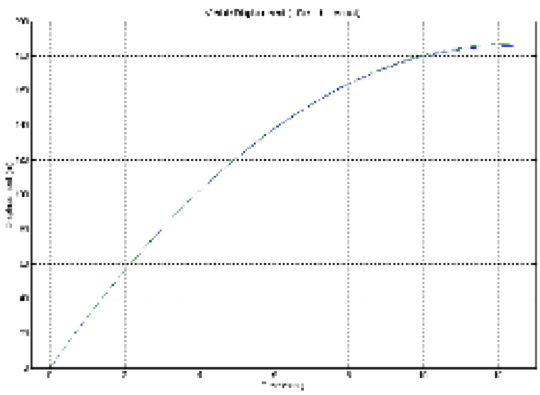

Fig. 2. Vehicle displacement trajectory

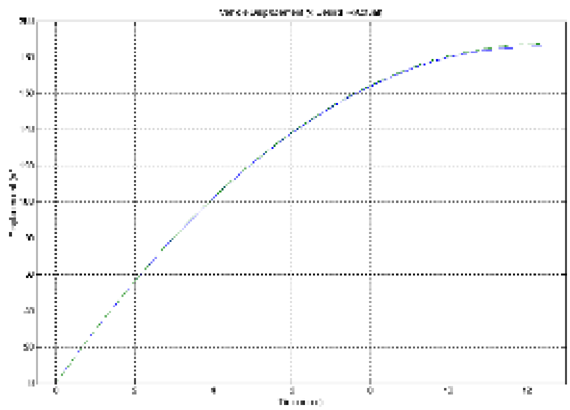

Fig. 3. Vehicle speed tracking trajectory

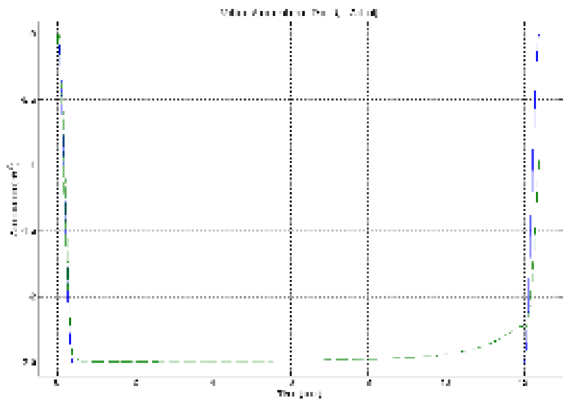

Fig. 4. Vehicle acceleration tracking trajectory

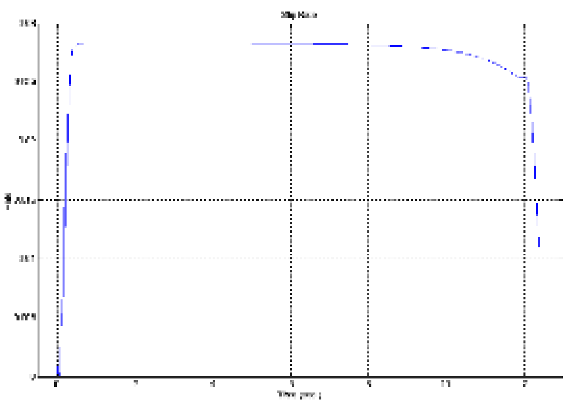

Fig. 5. Wheel Slip Rate

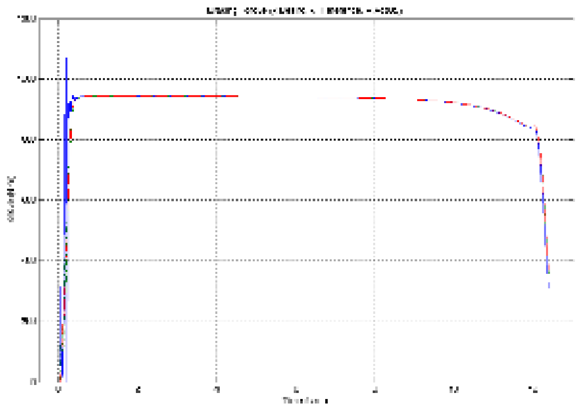

Fig. 6. Braking Torque

6. Conclusion

In this paper, an adaptive sliding mode control scheme with fuzzy composition that does not need the information regarding the system parameters or road conditions is proposed to achieve brake trajectory tracking of vehicle motion. Under such control approach, the slip rate and the vehicle motion will be properly adjusted to reach the optimal condition. The servo hydraulic system which is approximated by a first-order dynamic system is also taken into account in the controller design for completeness. In addition, with careful design of stopping trajectories, both the driver's safety and comfort can be guaranteed. Asymptotic stability is guaranteed via Lyapunov stability theory and demonstrated by computer simulations.

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