NONLINEAR FLEXURE CONTROL USING SHAPE MEMORY ALLOY ACTUATORS [1]

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Abstract: This paper presents a nonlinear control scheme for deflection control of a flexible beam using Shape Memory Alloy (SMA) actuators. These actuators posses interesting properties in terms of force generation capacity, possibility of miniaturization, and power consumption. However, their use in precision applications is hampered by undesirable characteristics such as nonlinearities, hysteresis, extreme temperature dependencies, and slow response. By taking into account the nonlinear and thermal characteristics, a control scheme based on partial feedback linearization is developed in order to regulate the forces exerted by a differential SMA actuator pair attached to a flexible beam. A Lyapunov stability analysis is furnished and guidelines are provided for selecting controller gain parameters. Furthermore, performance of the developed control scheme is tested experimentally on a laboratory testbed.

Keywords: Flexible structures, shape memory alloy actuators, nonlinear control.

1. INTRODUCTION

Smart material systems offer great possibilities in terms of providing novel and economical solutions to engineering problems. Smart materials such as Piezo-electric Transducers (PZT), Shape Memory Alloys (SMA), Magneto- and Electro-Rheological Fluids (MRF and ERF), Magnetostrictive materials, and fiber-optic sensors have been used in such diverse areas as automotive vehicles, robots, orthodontic treatment, biotechnology, civil engineering structures, space structures, sports equipment, etc. (see e.g., (Janocha, 1999), (Otsuka and Wayman, 1998), (Srinivasan and McFarland, 2001)). SMA actuators have also been used for vibration control of flexible beams, see e.g., (Choi and Cheong, 1996). However, these works have not taken into account the nonlinear effects of SMA actuators and the effect of temperature on performance of the vibration control scheme. Noting that the force generated by an SMA actuator is highly nonlinear, the control objective is to regulate the magnitude of force exerted by the SMA actuator. The desired force corresponds to the desired position of the beam.

2. SYSTEM MODELING AND CONTROL

A simplified model of a flexible beam actuated by a differential two-string system is shown in Figure 1. Taking into account the non-linearity and temperature dependence of the SMA string, the dynamics of the system are given by

$$M\ddot{\delta} + K\delta = bn(\varepsilon_1, \varepsilon_2, T_1, T_2) \tag{1}$$

$$\dot{T}_1 + \beta(T_1 - T_a) = \alpha u_1 \tag{2}$$

$$\dot{T}_2 + \beta(T_2 - T_a) = \alpha u_2 \tag{3}$$

where δ is the $N \times 1$ vector of flexible modes, M and K are the $N \times N$ mass and stiffness matrices

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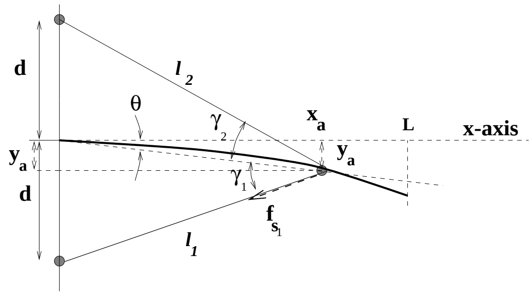

Fig. 1. Flexible beam driven by two SMA actuators.

of the flexible beam, respectively, obtained using the Lagrangian formulation (Meirovitch, 1975), b is the input-effect vector, $n(\varepsilon, T_1, T_2)$ is the nonlinear force generated by the SMA string, which depends on strain and temperature (Liang and Rogers, 1992). Moreover, ε_1, ε_2 are the strains induced in the SMA strings, T_1 is the temperature of SMA 1 (the lower wire), T_2 is the temperature of the SMA 2 (the upper wire), T_a is the ambient temperature, and α, β are positive constants. The inputs u_1 and u_2 correspond to the heat generated in the strings and are nonzero whenever the corresponding SMA is actuated. Otherwise, these inputs are zero.

Equations (2) and (3) are obtained from the heat transfer dynamics of a single wire expressed as (Madhill and Wang, 1998)

$$\rho c V \frac{dT}{dt} = Ri^2(t) - hA(T - T_a) \quad (4)$$

where ρ (kg/m^3) is the mass density of string material, c $(J/(kg^\circ C))$ is the specific heat, V (m^3) is the volume of string, i (A) is the electric current, R (Ω) is the string resistance, h $(W/(m^{2\circ}C))$ is the convection heat transfer coefficient, A (m^2) is the surface area of the string, and T_a is the ambient temperature. Note that the inputs u_1 and u_2 in (2) and (3) can only take positive values or zero corresponding to the current inputs applied to the two strings in Figure 1. Also, note from (4) that the values of α and β in (2) and (3) are dependent on geometric and physical properties of the SMA strings.

2.1 Kinematic Relationships

The input effect vector b in (1) depends on the place where the SMA strings are attached. In order to obtain this vector refer to Figure 1, which is comprised of a cantilevered flexible beam attached to SMA strings. Let f_{s_1} be the force exerted by SMA 1, which is placed at x_a along the link. We assume that this force is measurable, for example by using a force sensor mounted on the rigid base, where the other end of the string is attached. Neglecting curvature of the beam, the tangential and normal components of f_{s_1}, denoted by $f_{s_1 T}$ and $f_{s_1 N}$, are obtained as

$$f_{s_1 T} = f_{s_1} cos(\gamma_1), \quad f_{s_1 N} = f_{s_1} sin(\gamma_1). \quad (5)$$

The angles γ_1 and γ_2 can be obtained from

$$\gamma_1 = tan^{-1}\left(\frac{d - y_a}{x_a}\right) + \theta$$
$$\gamma_2 = tan^{-1}\left(\frac{d + y_a}{x_a}\right) - \theta \quad (6)$$

with y_a given by

$$y_a = \sum_{i=1}^{N} \phi_i(x_a)\delta_i \quad (7)$$

where $\phi_i(x_a)$ is the i-th mode shape function. Using the method of virtual work and Lagrangian formulation (Meirovitch, 1975), one can obtain the input effect vector b given by

$$b_i = sin(\gamma)\phi_i(x_a) \quad (8)$$

Assuming that $\theta \approx y_a/x_a$ is small enough and $d \gg y_a$ we have

$$b_i = \frac{d}{\sqrt{d^2 + x_a^2}}\phi_i(x_a), \quad i = 1,\ 2,\ \cdots, N\cdot \quad (9)$$

The ε term in (1) is a function of the modal vector δ as described next. Referring to Figure 1, the length l_1 can be obtained from the trigonometric relationship

$$l_1^2 = d^2 + x_a^2 - 2dx_a cos(\pi/2 - \theta). \quad (10)$$

Now defining $l_{10} = \sqrt{d^2 + x_a^2}$, where l_{10} is the value of l_1, the strain ε_1 is given by

$$\varepsilon_1 = \frac{\sqrt{l_{10}^2 - 2dy_a} - l_{10}}{l_{10}}$$
$$= (1 - 2\frac{d}{l_{10}^2}y_a)^{1/2} - 1\cdot \quad (11)$$

Noting that lengths of the two strings are equal when there is no deflection, i.e., $l_{20} = l_{10}$, ε_2 can be obtained in a similar way as follows

$$\varepsilon_2 = \frac{\sqrt{l_{10}^2 + 2dy_a} - l_{10}}{l_{10}}$$
$$= (1 + 2\frac{d}{l_{10}^2}y_a)^{1/2} - 1\cdot \quad (12)$$

Expanding the square root function in (11) and neglecting higher order terms, the strain relationships for the two strings can be obtained as

$$\varepsilon_1 = -\varepsilon_2 = -\frac{d}{l_{10}^2}y_a$$
$$= -\frac{d}{l_{10}^2}\sum_{i=1}^{N}\phi_i(x_a)\delta_i := \varepsilon \quad (13)$$

where ε is the strain term given in (1). Equation (13) may also be written in the following form

$$\varepsilon = -\Phi_a \delta \tag{14}$$

where

$$\Phi_a = \frac{d}{l_{10}^2}[\phi_1(x_a)\ \phi_2(x_a)\ \cdots\ \phi_N(x_a)]. \tag{15}$$

Considering the right hand side of (1), the term $n(\varepsilon_1, \varepsilon_2, T_1, T_2)$ is the difference between two external forces acting on the flexible link by the SMA strings. Referring to (13), and changing the dependency of $n(\cdot)$ to ε we have

$$n(\varepsilon, T_1, T_2) = A(\sigma(-\varepsilon, T_1) - \sigma(\varepsilon, T_2)) \tag{16}$$

where A is the cross sectional area of the SMA string and the $\sigma(\cdot)$ terms represent stresses induced by the two SMA strings. The appearance of ε in (16) and its dependence on δ as given by (13) affects the magnitudes of flexural modes given by (1). In this regard, the open-loop flexural modes of the system are not the same as the eigenvalues of the flexible beam given by $eig(M^{-1}K)$. To illustrate this, a Taylor series expansion of (16) around $\delta = 0$ yields

$$\begin{aligned} n(\varepsilon, T_1, T_2) = A((&\sigma(0, T_1) - \sigma(0, T_2)) \\ &+ \bigtriangledown_\delta(\sigma(-\varepsilon, T_1) \\ &- \sigma(\varepsilon, T_2))|_{\delta=0} + \cdots) \end{aligned} \tag{17}$$

where $\bigtriangledown_\delta$ denotes the gradient operator with respect to δ. Neglecting higher order terms, equation (1) can be approximated as

$$M\ddot{\delta} + K_e\delta = Ab(\sigma(0, T_1) - \sigma(0, T_2)) \tag{18}$$

where K_e is the equivalent stiffness matrix given by

$$\begin{aligned} K_e = K - Ab \bigtriangledown_\delta (&\sigma(\Phi_a\delta, T_1) \\ &- \sigma(-\Phi_a\delta, T_2))|_{\delta=0}) \end{aligned} \tag{19}$$

The above relationship indicates that the open-loop vibration modes of the flexible beam are temperature dependent and are given by the eigenvalues of $M^{-1}K_e$, rather than those of $M^{-1}K$.

2.2 *Development of the Control Scheme*

The force generated by an SMA actuator is a nonlinear function of the string temperature and strain. Besides, the actuator nonlinearity is not well known and varies with the operating point (Liang and Rogers, 1992). In order to deal with the nonlinear and uncertain characteristics of SMA actuators, the control strategy in this paper is targeted towards regulating the actuation force to a desired value that corresponds to the desired position of the flexible beam. Towards this end, let us consider the system dynamics given by (1)–(3) and define the output as

$$y_o = \Psi\delta \tag{20}$$

where Ψ is a constant matrix that relates modal variables to the output of interest y_o. The numerical value of Ψ depends on the appropriate modal shape functions determined by geometric boundary conditions of the partial differential equation describing the behavior of the beam–*clamped-free* for the beam in this study (see e.g., (Meirovitch, 1975)). The output under control can be the displacement of the tip position of the beam or a location close to the tip. Taking the time derivative of y_o twice and using (1) we have

$$\ddot{y}_o = \Psi M^{-1}(bn_o(\varepsilon, T_1, T_2) - K\delta) \tag{21}$$

where $n_o(\varepsilon, T_1, T_2)$ is the desired $n(\varepsilon, T_1, T_2)$ corresponding to y_o. Note that $n(\cdot)$ can be measured by measuring the tensional forces acting on the SMA strings. Let us further take $n_o(\cdot)$ according to

$$n_o(\varepsilon, T_1, T_2) = (\Psi M^{-1}b)^{-1}(v + \Psi M^{-1}K\delta) \tag{22}$$

where v is a new control input. Substituting the above equation in (21) yields

$$\ddot{y}_o = v \tag{23}$$

Denoting the reference position by y_r and the error by $e = y_r - y_o$, let us choose v as

$$v = -K_d\dot{y}_o + K_pe \tag{24}$$

where K_d and K_p are feedback gains. It is also assumed that $\dot{y}_o$ is known by measurement or estimation, e.g., by numerical differentiation of modal variables or using a state observer. The resulting closed-loop system will then be given by

$$\ddot{e} + K_d\dot{e} + K_pe = 0\cdot \tag{25}$$

Now substituting (24) in (22) yields

$$\begin{aligned} n_o(\varepsilon, T_1, T_2) = \frac{1}{\Psi M^{-1}b}(&K_pe + \\ &+ K_d\dot{e} + \Psi M^{-1}K\delta)\cdot \end{aligned} \tag{26}$$

A main goal of the controller is to make $n(\cdot)$ approach $n_o(\cdot)$. Towards this end, let us define the output to be controlled as

$$y = n(\varepsilon, T_1, T_2) - n_o(\varepsilon, T_1, T_2)\cdot \tag{27}$$

Following the (Hirschorn, 1979), (Byrnes and Isidori, 1985), the output variable is differentiated once for the input to appear in the input-output dynamics. Thus, starting from

$$\dot{y} = \frac{\partial n}{\partial\varepsilon}\dot{\varepsilon} + \frac{\partial n}{\partial T_1}\dot{T}_1 + \frac{\partial n}{\partial T_2}\dot{T}_2 - \dot{n}_o \tag{28}$$

and utilizing (14), (2), (3), (22), (24), and (26), $\dot{y}$ can be written after some algebraic manipulations, in the following form

$$\dot{y} = h_1 u + h_2 + h_3 \tag{29}$$

where u and h_1 are defined as follows

$$u = \begin{cases} u_1 & u_1 \geq 0,\ u_2 = 0 \\ -u_2 & u_1 = 0,\ u_2 \geq 0 \end{cases}$$
$$h_1 = \begin{cases} \alpha \dfrac{\partial n}{\partial T_1} & u \geq 0 \\ \alpha \dfrac{\partial n}{\partial T_2} & u < 0 \end{cases} \tag{30}$$

and h_2, h_3 are given by

$$\begin{aligned} h_2 &= K_d n(\varepsilon, T_1, T_2) - \frac{K_d \Psi M^{-1} K}{\Psi M^{-1} b} \delta \\ &\quad + \frac{K_p \Psi - \Psi M^{-1} K}{\Psi M^{-1} b} \dot{\delta} \\ h_3 &= -\beta \frac{\partial n}{\partial T_1}(T_1 - T_a) - \beta \frac{\partial n}{\partial T_2}(T_2 - T_a) \\ &\quad - \frac{\partial n}{\partial \varepsilon} \Phi_a \dot{\delta} \end{aligned} \tag{31}$$

with Φ_a defined as

$$\Phi_a = \frac{d}{l_{10}}[\phi_1(x_a)\ \phi_2(x_a)\ \cdots \phi_n(x_a)]. \tag{32}$$

Referring to (30) and (31), h_1 is of known sign and approximate value, h_2 is completely known, and the value of h_3 is not available, since it is assumed that T_1 and T_2 are not measured. Let us choose the control law as

$$u = \hat{h}_1^{-1}(-k_y y - h_2) \tag{33}$$

where $\hat{h}_1$ is an estimate of h_1 and k_y is a feedback gain whose approximate value will be determined later in this paper based on a Lyapunov stability analysis.

The above control scheme acts on part of the system dynamics that is related to the input-output behavior. The other part of the dynamics, often referred to as *internal dynamics* (Slotine and Li, 1991), can be obtained by solving for $n(\cdot)$ in (27), i.e.,

$$\begin{aligned} n(\varepsilon, T_1, T_2) &= y + \frac{1}{\Psi M^{-1} b}(K_p e + K_d \dot{e} \\ &= \Psi M^{-1} K \delta) \end{aligned} \tag{34}$$

and substituting it in (1), which results in

$$\begin{aligned} \ddot{\delta} &= -M^{-1}((I - \frac{b\Psi M^{-1}}{\Psi M^{-1} b})K + K_p \frac{b\Psi}{\Psi M^{-1} b})\delta \\ &\quad - K_d \frac{M^{-1} b \Psi}{\Psi M^{-1} b}\dot{\delta} + M^{-1} b(y + K_p \frac{y_r}{\Psi M^{-1} b})\cdot \end{aligned} \tag{35}$$

It can be concluded that if the flexure dynamics given by (35) are controllable then they can be stabilized by a proper choice of feedback gains K_p, K_d. It should also be noted that the output location vector Ψ, and the input vector b are at our disposal and can be chosen to satisfy controllability properties. This is usually the case as can be verified for a model with a single mode (scalar δ), which is the most important mode to control.

Now let us write (35) in the form

$$\dot{\Delta} = A_\Delta \Delta + b_\Delta (y + \frac{K_p y_r}{\Psi M^{-1} b}) \tag{36}$$

where

$$A_\Delta = \begin{bmatrix} 0 & I \\ A_{21} & -K_d \dfrac{M^{-1} b \Psi}{\Psi M^{-1} b} \end{bmatrix}$$
$$\Delta^T = [\delta^T\ \ \dot{\delta}^T],\ b_\Delta = \begin{bmatrix} 0 \\ M^{-1} b \end{bmatrix} \tag{37}$$

with

$$\begin{aligned} A_{21} &= -M^{-1}((I - \frac{b\Psi M^{-1}}{\Psi M^{-1} b})K \\ &\quad + K_p \frac{b\Psi}{\Psi M^{-1} b})\cdot \end{aligned} \tag{38}$$

It is assumed that A_Δ can be made Hurwitz by a proper choice of K_p, K_d, Ψ, and b. Thus, the equilibrium value of Δ, i.e., when y and $\dot{\Delta}$ are set to zero, is given by

$$\bar{\Delta} = -A_\Delta^{-1} b_\Delta \frac{K_p y_r}{\Psi M^{-1} b}\cdot \tag{39}$$

Then defining $\tilde{\Delta} = \Delta - \bar{\Delta}$ and rewriting (36) in terms of $\tilde{\Delta}$ yields

$$\dot{\tilde{\Delta}} = A_\Delta \tilde{\Delta} + b_\Delta y \tag{40}$$

Now let us consider the closed loop system dynamics described by (29) and (40) and choose the Lyapunov function candidate

$$V = \frac{1}{2} y^2 + \epsilon_\Delta^2 \tilde{\Delta}^T P_\Delta \tilde{\Delta} \tag{41}$$

where ϵ_Δ^2 is a nonzero constant and P_Δ is a positive-definite matrix. A proof of the closed-loop system stability can be established based on the above Lyapunov function as described in section 2.3. The result of this analysis can be used to provide qualitative guidelines for choosing control parameters K_p, K_d, and k_y as described later.

2.3 Stability Analysis

Since A_Δ in (37) is Hurwitz, for any positive-definite matrix Q_Δ there exists a positive-definite matrix P_Δ satisfying the following Lyapunov equation

$$A_\Delta^T P_\Delta + P_\Delta A_\Delta = -Q_\Delta\cdot \tag{42}$$

Considering the dynamics of the system given by (29) and (40) and the Lyapunov function (41), we have

$$\dot{V} = -k_y h_1 \hat{h}_1^{-1} y^2 - \epsilon_\Delta \tilde{\Delta}^T Q_\Delta \tilde{\Delta} + y((1 - h_1 \hat{h}_1^{-1}) h_2 + h_3) + 2\epsilon_\Delta y b_\Delta^T P_\Delta \tilde{\Delta} \cdot \quad (43)$$

Furthermore, utilizing (26), (31) can be written in the form

$$\begin{aligned} h_2 &= K_d y + c_1 y_r + \gamma_2^T \tilde{\Delta} \\ h_3 &= \gamma_3 + \gamma_4^T \dot{\tilde{\Delta}} \end{aligned} \quad (44)$$

where

$$\begin{aligned} \gamma_2^T &= \left[-\frac{K_d K_p \Psi}{\Psi M^{-1} b} \quad \frac{-K_d^2 \Psi + K_p \Psi - \Psi M^{-1} K}{\Psi M^{-1} b} \right] \\ \gamma_3 &= -\beta\left(\frac{\partial n}{\partial T_1}(T_1 - T_a) + \frac{\partial n}{\partial T_2}(T_2 - T_a)\right) \\ \gamma_4^T &= \left[0 \; 0 \; \cdots \; 0 \; - \frac{\partial n}{\partial \varepsilon} \Phi_a \right]_{1 \times 2n} \\ c_1 &= \frac{K_p K_d - \gamma_2^T A_\Delta^{-1} b_\Delta K_p}{\Psi M^{-1} b} \cdot \end{aligned} \quad (45)$$

Considering an arbitrary bounded region $\Omega \subset R^{2n+2}$ around the origin of the state space of $(y, \tilde{\Delta}, T_1 - T_a, T_2 - T_a)$ the following inequalities can be written

$$\|\gamma_2 \tilde{\Delta}\| \leq c_2 \|\tilde{\Delta}\|, \;\; |h_3| \leq c_3 + c_4 \|\tilde{\Delta}\| \quad (46)$$

where $\| \cdot \|$ denotes the *2*-norm, c_2 is the norm of γ_2, and c_3, c_4 are upper bounds of γ_3 and γ_4, respectively.

Utilizing the above inequalities, and defining the following constants

$$c_5 = |1 - h_1 \hat{h}_1|, \;\; c_6 = 2\|b_\Delta^T P_\Delta\|, \;\; c_7 = h_1 \hat{h}_1^{-1} \quad (47)$$

$\dot{V}$ can be further written in the form

$$\dot{V} \leq -\eta^T \Lambda \eta + 2\alpha^T \eta \quad (48)$$

where

$$\begin{aligned} \eta &= \begin{bmatrix} |y| \\ \|\tilde{\Delta}\| \end{bmatrix} \\ \Lambda &= \begin{bmatrix} k_y c_7 - c_5 K_d & \frac{1}{2}(c_2 c_5 + c_4 + \epsilon_\Delta c_6) \\ \frac{1}{2}(c_2 c_5 + c_4 + \epsilon_\Delta c_6) & \lambda_{min}(Q_\Delta) \end{bmatrix} \\ \alpha^T &= \frac{1}{2}[c_3 + c_1 c_5 y_r \;\; 0] \cdot \end{aligned} \quad (49)$$

From (48), it follows that if Λ is positive-definite, the system trajectories would converge to a residual set whose size is determined by the ellipse $\eta^T \Lambda \eta = \alpha^T \eta$. The diagonals of the ellipse representing the order of errors are given by $\sqrt{\alpha^T \Lambda^{-1} \alpha / \lambda_{max}(\Lambda)}$ and $\sqrt{\alpha^T \Lambda^{-1} \alpha / \lambda_{min}(\Lambda)}$, where $\lambda_{max}(\Lambda)$ and $\lambda_{min}(\Lambda)$ are the maximum and minimum eigenvalues of Λ, respectively.

In order to guarantee stability of the closed loop system, let us study the effects of controller gains k_y, K_p, and K_d on the closed-loop system stability and the magnitudes of errors. Towards this end, Λ would be positive-definite if the following conditions are satisfied

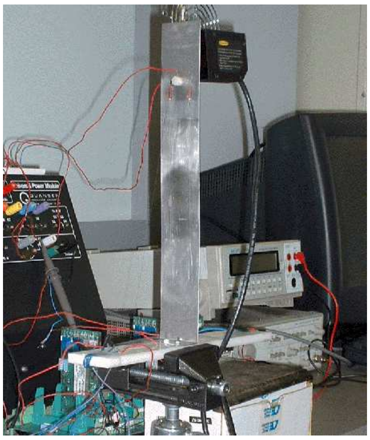

Fig. 2. Flexible beam setup.

$$\begin{aligned} k_y c_7 - c_5 K_d &> 0 \\ (k_y c_7 - c_5 K_d)\lambda_{min}(Q_\Delta) &> \frac{1}{4}(c_2 c_5 + c_4 + \epsilon_\Delta c_6)^2 \cdot \end{aligned} \quad (50)$$

The first condition in (50) can be re-written as

$$k_y > \left| \frac{1}{h_1 \hat{h}_1^{-1}} - 1 \right| K_d \quad (51)$$

which implies that k_y should be sufficiently larger than K_d. As for the second condition in (50), assume that $K_p = \omega_N^2$, and $K_d = 2\xi\omega_N$, where ω_N is the closed-loop undamped natural frequency and ξ is the damping ratio. Thus K_p and K_d are of orders $O(\omega_N^2)$ and $O(\omega_N)$, respectively. Similarly, it can be verified that c_2, c_4, and c_6 are of orders $O(\omega_N^3)$, $O(\omega_N)$, and $O(\omega_N)$, respectively, which when substituted in the second inequality in (50) yield $k_y > O(\omega_N^6)$. Thus, these relationships provide qualitative measures for selection of the order of k_y in relation to K_p and K_d to guarantee stability. In order to study the effects of controller gains on the magnitudes of errors, consider the following term

$$\begin{aligned} \alpha^T \Lambda^{-1} \alpha = \lambda_{min}(Q_\Delta)(c_3 + c_1 c_5 y_r)^2 / \\ (4(\lambda_{min}(Q_\Delta)(k_y c_7 - c_5 K_d) \\ - \frac{1}{4}(c_2 c_5 + c_4 + \epsilon_\Delta c_6)^2) \end{aligned} \quad (52)$$

which can be verified to be of order $O(\omega_N^{-1})$. Furthermore, it can be verified that the larger diagonal of the ellipse given by $\sqrt{\alpha^T \Lambda^{-1} \alpha / \lambda_{max}(\Lambda)}$, is of order $O(\omega_N^{-4})$.

3. EXPERIMENTAL EVALUATION

An experimental testbed was constructed to evaluate performance of the controller as shown in Figure 2. A schematic diagram of this setup is shown in

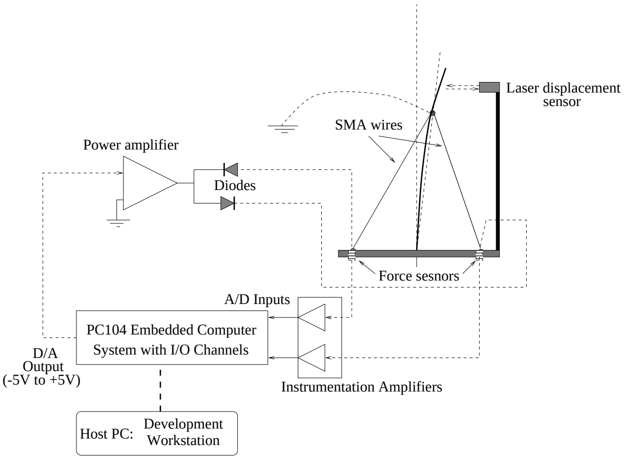

Fig. 3. Schematic diagram of the flexible beam setup.

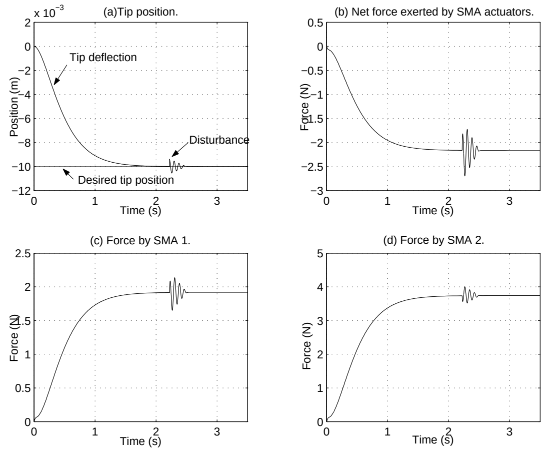

Fig. 4. Experimental results for a constant reference input: SMA 1 and 2 refer to the left, and right SMA strings in Figure 3, respectively.

Figure 3 consisting of a laser displacement sensor, two piezo-resistive force sensors, Flexinol SMA string, and instrumentation and power amplifiers.

Figure 4 shows experimental results when the control law given by (33) is applied to the system. Figure 4(a) shows the tip deflection measured by the laser displacement sensor and the reference desired position. At some point during its motion, a disturbance is applied to the flexible beam by hitting it with a finger. It is observed that the tip position approaches the desired position while damping out vibrations due to the disturbance. Figure 5 shows the results when a square wave reference input is used to drive the flexible beam by $\pm 0.5\ cm$. The values of controller gains were set at $K_d = 8,\ K_p = 16,\ k_y = 80,000$.

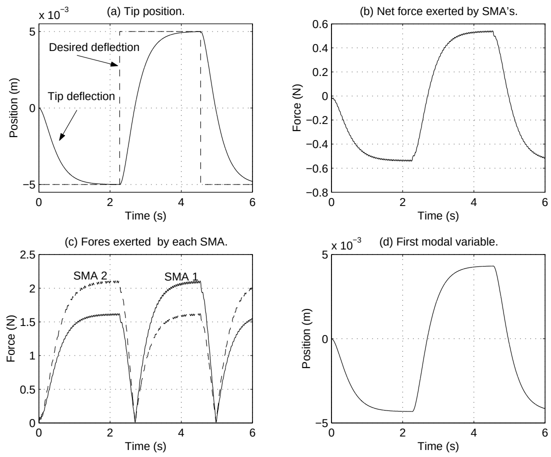

Fig. 5. Experimental results for a square wave reference input.

4. CONCLUSION

In this paper, a nonlinear control scheme was developed for position control of a flexible beam actuated by Shape Memory Alloy strings. SMA actuators exhibit highly nonlinear effects with characteristics that are highly temperature dependent. Since actuator temperature is not readily measurable, it is treated as a disturbance term that has to be compensated by the control scheme. The force is regulated to a value corresponding to a desirable position of the beam endpoint. Stability analysis of the closed-loop system is established along with experimental results that illustrate the performance of the proposed controller.

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