ON OUTPUT FEEDBACK STABILITY AND PASSIVITY IN DISCRETE LINEAR SYSTEMS

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Abstract: Recent publications have shown that under some conditions continuous linear time-invariant systems become strictly positive real with constant feedback. This paper expands the applicability of this result to discrete linear systems. The paper shows the sufficient conditions that allow a discrete system to become stable and strictly passive via static (constant or nonstationary) output feedback.

Keywords: Control Systems, Discrete Systems, Stability, Passivity, Adaptive Control.

1. INTRODUCTION

Although the strict positive-real (SPR) property has played a crucial role in guaranteeing stability in systems with uncertainty (Steinberg and Coreless, 1985) and in adaptive control (Sobel et al., 1982; Barkana and Kaufman, 1985) of linear time-invariant (LTI) systems, most real-world systems are not inherently SPR. Therefore, it was important to find those systems that can become strictly positive real via constant or dynamic output feedback. In particular, such systems that need only a constant output feedback to become strictly positive real have been called "almost strictly positive real (ASPR)" (Barkana and Kaufman, 1985) because the ASPR property has been shown to be sufficient for stability with adaptive controllers. Many works (Fradkov, 1976; Owens et al., 1987; Teixeira, 1988; Gu, 1990; Huang et al., 1999) have contributed to define the continuous-time ASPR systems, namely, those special systems that not only can be stabilized, but also rendered SPR via constant output feedback. Simply summarized (Barkana, 2004b), any minimum-phase LTI systems {A, B, C} is ASPR if the matrical product CB is positive definite *symmetric*. Recently, Barkana (2004a) managed to eliminate the symmetry requirement from the ASPR conditions. It may be worth mentioning that only the transfer function should rigorously be called SPR, while the system should be called strictly passive. However, it is customary to use either name in LTI systems.

Although attempts at directly using the continuous systems results in discrete systems have failed, this paper nevertheless manages to extend the previous results, and thus to establish some useful relations related to the passivity of discrete systems with the realization

$$x_p(t+1) = A_p x_p(t) + B_p u_p(t) \qquad (1)$$

$$y_p(t) = C_p x_p(t) + D_p u_p(t) \qquad (2)$$

The main result of the paper is the proof that any proper minimum-phase discrete linear system with positive definite (and not necessarily symmetric) input-output gain matrix D_p can be stabilized and rendered strictly positive real via constant feedback and is therefore ASPR. In addition, the paper will show that systems that are not minimum-phase can be augmented and can become minimum-phase and thus, ASPR, via parallel feedforward. The paper will also show the applicability of the result in nonlinear and adaptive control.

2. ON ZERO DYNAMICS AND PASSIVITY IN DISCRETE LINEAR SYSTEMS

The stability properties of an LTI system are strongly related to the position of the "poles" of its transfer function, that are also the eigenvalues of the plant matrix A_p. The SPR properties of LTI systems are strongly related to both the pole and zero properties of the system. The paper will later show that the positive definite gain D_p is an integral part of the desired ASPR and SPR properties of the plant. One can use the output feedback gain K_e in the control signal

$$\begin{aligned} u_p(t) &= -K_e y_p(t) + v_p(t) = -K_e C_p x_p(t) \\ &\quad - K_e D_p u_p(t) + v_p(t) \end{aligned} \tag{3}$$

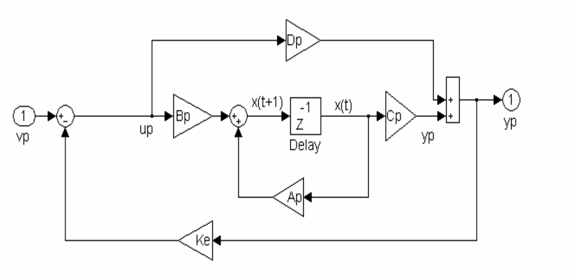

Figure 1. The original system closed via K_e

The algebraic loop in (3) results in

$$(I + K_e D_p) u_p(t) = -K_e C_p x_p(t) + v_p(t) \tag{4}$$

Define

$$K_{ec} = (I + K_e D_p)^{-1} K_e \tag{5}$$

to get

$$u_p(t) = -K_{ec} C_p x_p(t) + (I + K_e D_p)^{-1} v_p(t) \tag{6}$$

Substituting in (1)-(2) gives

$$\begin{aligned} x_p(t+1) &= \left(A_p - B_p K_{ec} C_p\right) x_p(t) \\ &\quad + B_p (I + K_e D_p)^{-1} v_p(t) \end{aligned} \tag{7}$$

$$\begin{aligned} y_p(t) &= C_p x_p(t) - D_p (I + K_e D_p)^{-1} K_e C_p x_p(t) \\ &\quad + D_p (I + K_e D_p)^{-1} v_p(t) \end{aligned} \tag{8}$$

Define

$$B_{pc} = B_p (I + K_e D_p)^{-1} \tag{9}$$

$$\begin{aligned} D_{pc} &= D_p (I + K_e D_p)^{-1} = (D_p^{-1} + K_e)^{-1} \\ &= (I + D_p K_e)^{-1} D_p \end{aligned} \tag{10}$$

$$\begin{aligned} C_{pc} &= C_p - D_p (I + K_e D_p)^{-1} K_e C_p \\ &= \left[I - D_p (I + K_e D_p)^{-1} K_e\right] C_p \\ &= \left(I + D_p K_e\right)^{-1} C_p \end{aligned} \tag{11}$$

The closed-loop system is

$$x_p(t+1) = A_{pc} x_p(t) + B_{pc} v_p(t) \tag{12}$$

$$y_p(t) = C_{pc} x_p(t) + D_{pc} v_p(t) \tag{13}$$

where the closed-loop system matrix A_{pc} can get any one of the forms

$$\begin{aligned} A_{pc} &= A_p - B_p K_{ec} C_p = A_p - B_{pc} K_e C_p \\ &= A_p - B_p K_e C_{pc} \end{aligned} \tag{14}$$

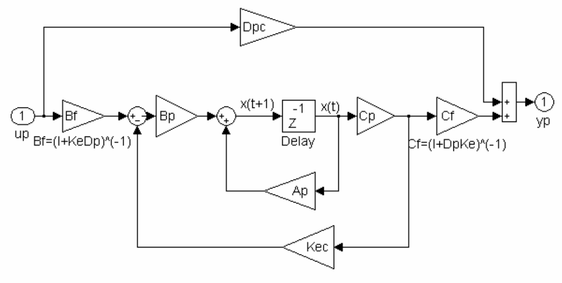

Figure 2. Equivalent closed-loop system

The closed-loop system is asymptotically stable if and only if there exist two positive definite symmetric (PDS) matrices, P and $\bar{Q}$ that satisfy the relation

$$A_{pc}^T P A_{pc} - P = -\bar{Q} \tag{15}$$

The zero dynamics is given by those trajectories that maintain the output at zero in spite of the presence of input commands. One gets from (2)

$$y_p(t) = C_p x_p(t) + D_p u_p(t) = 0 \tag{16}$$

that gives

$$u_p(t) = -D_p^{-1} C_p x_p(t) \tag{17}$$

Substituting in (1) gives the zero-dynamics equation

$$x\left(t+1\right) = A_z x_p(t) \tag{18}$$

where $A_z = A_p - B_p D_p^{-1} C_p$ is the system matrix for the zero dynamics. If the system is minimum-phase, there exist two positive definite matrices, P_z and Q_z, such that

$$A_z^T P_z A_z - P_z = -Q_z \tag{19}$$

Furthermore, in the system representation the closed-loop system is strictly passive (Hitz and Anderson, 1969) and its transfer function is SPR if there exist three positive definite symmetric (PDS) matrices of appropriate dimensions, P, Q and Q_0 that satisfy the relations

$$A_{pc}^T P A_{pc} - P = -Q - L^T L \tag{20}$$

$$A_{pc}^T P B_{pc} - C_{pc}^T = L^T W \tag{21}$$

$$D_{pc} + D_{pc}^T = W^T W + B_{pc}^T P B_{pc} + Q_0 \tag{22}$$

In this case, because the original open-loop plant is separated from strict positive realness only by a constant output feedback, it is called "almost strictly positive real (ASPR)" (Barkana and H. Kaufman, 1985; Barkana, 1987). Relations (20)-(22) can also be written in a more concise form. From (21) one gets

$$L^T = \left(A_{pc}^T P B_{pc} - C_{pc}^T\right) W^{-1} \qquad (23)$$

Substituting in (20) finally gives

$$A_{pc}^T P A_{pc} - P + \left(A_{pc}^T P B_{pc} - C_{pc}^T\right)\left(D_{pc} + D_{pc}^T - B_{pc}^T P B_{pc} - Q_0\right)^{-1} \bullet \left(B_{pc}^T P A_{pc} - C_{pc}\right) = -Q \qquad (24)$$

3. DISCRETE ASP THEOREM

This section represents the main objective of this paper, as it proves the following theorem for discrete linear systems:

Theorem 1: Consider the discrete linear system $\left\{A_p, B_p, C_p, D_p\right\}$, where $A_p \in R^{n,n}$, $B_p \in R^{n,m}$, $C_p \in R^{m,n}$, and $D_p \in R^{m,m}$. Assume that the system is strictly minimum-phase and that $D_p > 0$ (i.e., positive definite but not necessarily symmetric). Under these assumptions, the system is ASPR, namely, it can be stabilized and made strictly passive via static output feedback.

Proof: We showed that if a positive definite output feedback gain K_e is used, the resulting closed-loop system matrix is $A_{pc} = A_p - B_p K_{ec} C_p$, where K_{ec} is computed in (5) and satisfies

$$K_{ec} = (I + K_e D_p)^{-1} K_e = (K_e^{-1} + D_p)^{-1} \le D_p^{-1} \qquad (25)$$

Therefore

$$\begin{aligned} & A_{pc}^T P_z A_{pc} - P_z \\ &= \left(A_p - B_p K_{ec} C_p\right)^T P_z \left(A_p - B_p K_{ec} C_p\right) - P_z \\ &= \left(A_p - B_p D_p^{-1} C_p + B_p D_p^{-1} C_p - B_p K_{ec} C_p\right)^T \\ & P_z \left(A_p - B_p D_p^{-1} C_p + B_p D_p^{-1} C_p - B_p K_{ec} C_p\right) - P_z \end{aligned} \qquad (26)$$

After some algebra

$$\begin{aligned} A_{pc}^T P A_{pc} - P &= -Q_z + A_z^T P_z B_p \left(D_p^{-1} - K_{ec}\right) C_p \\ & + \left(B_p \left(D_p^{-1} - K_{ec}\right) C_p\right)^T P_z A_z \\ & + \left(B_p \left(D_p^{-1} - K_{ec}\right) C_p\right)^T P_z B_p \left(D_p^{-1} - K_{ec}\right) C_p \end{aligned} \qquad (27)$$

Define

$$K_{cc} = D_p^{-1} - K_{ec} > 0 \qquad (28)$$

$$\begin{aligned} Q_1 = Q_z - A_z^T P_z B_p K_{cc} C_p - C_p^T K_{cc}^T B_p^T P_z A_z \\ - C_p^T K_{cc}^T B_p^T P_z B_p K_{cc} C_p \end{aligned} \qquad (29)$$

It can be seen that $Q_1 > 0$ if the feedback gain K_e is sufficiently large, and thus K_{ec} is sufficiently large and K_{cc} sufficiently small. Therefore one first gets

$$\begin{aligned} A_{pc}^T P_z A_{pc} - P_z = \left(A_p - B_p K_{ec} C_p\right)^T P_z \\ \bullet \left(A_p - B_p K_{ec} C_p\right) - P_z = -Q_1 < 0 \end{aligned} \qquad (30)$$

Equation (30) implies that a discrete minimum-phase system can be stabilized via positive definite constant output feedback. Furthermore, if K_{cc} is sufficiently small -- possibly smaller than the value needed for (30) -- one can also write

$$\begin{aligned} Q = Q_1 - \left(A_{pc}^T P_z B_{pc} - C_{pc}^T\right)^T \left(K_{cc}^T + K_{cc}\right) \\ \bullet \left(A_{pc}^T P_z B_{pc} - C_{pc}^T\right) > 0 \end{aligned} \qquad (31)$$

$$\begin{aligned} A_{pc}^T P_z A_{pc} - P_z + \left(A_{pc}^T P_z B_{pc} - C_{pc}^T\right)^T \left(K_{cc}^T + K_{cc}\right) \\ \bullet \left(A_{pc}^T P_z B_{pc} - C_{pc}^T\right) = -Q < 0 \end{aligned} \qquad (32)$$

This is equivalent with the SPR condition (24) if one replaces $K_{cc}^T + K_{cc}$ by $D_{pc}^T + D_{pc} - B_{pc}^T P B_{pc} - Q_0$. Therefore, under the assumptions of the theorem, the closed loop system is SPR and the original open-loop system is ASPR.

4. PARALLEL FEEDFORWARD OR DUALITY OF STABILIZABILITY AND "PASSIVABILITY"

The previous sections showed that a minimum-phase plant $\left\{A_p, B_p, C_p, D_p\right\}$ with $D_p > 0$ is ASPR, or it can be rendered SPR via some constant output feedback. Yet, this result may not look too encouraging, as practical plant may be strictly proper (i. e., $D_p > 0$) and not necessarily minimum-phase. How can such a plant be made ASPR? The results of this paper allow a solid theoretical basis to the use of parallel feedforward in discrete systems (Barkana, 1983; Barkana, 1989). This section will show that stabilizability and "passivability" are dual, as formulated in the following theorem.

Theorem 2: Consider the strictly proper discrete linear plant $G:\left\{A_p, B_p, C_p, 0\right\}$, where $A_p \in R^{n,n}$, $B_p \in R^{n,m}$, and $C_p \in R^{m,n}$. Assume that there exists a proper, static or dynamic, stabilizing feedback configuration $H:\left\{A_f, B_f, C_f, D_f\right\}$ such that the closed-loop system is asymptotically stable. In this case the augmented system $G_a = G + H^{-1}$ is proper and strictly minimum-phase and is therefore ASPR.

A simple example could be useful to illustrate the parallel feedforward idea. Assume that the SISO transfer function $G(z)=B(z)/A(z)$ can be stabilized by the constant feedback k. Therefore, the closed-loop plant with the transfer function $G(z)=B(z)/\left(A(z)+kB(z)\right)$ is asymptotically stable. It is then easy to see that the augmented system $G_a=G+H^{-1}=B(z)/A(z)+1/k$, namely, $G_a=\left(A(z)+kB(z)\right)/\left(kA(z)\right)$ is strictly minimum-phase.

In the general SISO case, let the stabilizing configuration be $H(z)=N(z)/M(z)$. Assume that the closed-loop plant with the transfer function $G_c(z)=G(z)/\left(1+H(z)G(z)\right)$, namely, $G_c(z)=B(z)/\left(A(z)M(z)+B(z)N(z)\right)$ is asymptotically stable. It is then easy to see that the augmented system $G_a=G+H^{-1}=B(z)/A(z)+M(z)/N(z)$ or $G_a=\left(A(z)M(z)+B(z)N(z)\right)/\left(A(z)N(z)\right)$ is strictly minimum-phase.

Now, one can proceed with the general proof:

Proof*:* Given the system G with representation (1)-(2) stabilized by the feedback controller H with the representation

$$x_{fb}(t+1)=A_{fb}x_{fb}(t)+B_{fb}y_p(t) \tag{33}$$

$$y_{fb}(t)=C_{fb}x_{fb}(t)+D_{fb}y_p(t) \tag{34}$$

The control signal is

$$u_p(t)=-y_{fb}(t) \tag{35}$$

The stabilized closed loop system is

$$x(t+1)=Ax(t) \tag{36}$$

where

$$x(t)=\begin{bmatrix} x_p(t) \\ x_{fb}(t) \end{bmatrix} \tag{37}$$

$$A=\begin{bmatrix} A_p-B_pD_{fb}C_p & -B_pC_{fb} \\ B_{fb}C_p & A_{fb} \end{bmatrix} \tag{38}$$

The matrix A is Hurwitz, as it represents the asymptotically stable closed loop system. Now one inverts the proper feedback controller to get

$$x_{ff}(t+1)=A_{ff}x_{ff}(t)+B_{ff}y_{ff}(t) \tag{39}$$

$$y_{ff}(t)=C_{ff}x_{ff}(t)+D_{ff}u_{ff}(t) \tag{40}$$

Here

$$A_{ff}=A_{fb}-B_{fb}D_{fb}^{-1}C_{fb} \tag{41}$$

$$B_{ff}=-B_{fb}D_{fb}^{-1} \tag{42}$$

$$C_{ff}=-D_{fb}^{-1}C_{fb} \tag{43}$$

$$D_{ff}=D_{fb}^{-1} \tag{44}$$

The augmented system is

$$x_a(t+1)=A_ax_a(t)+B_ay_a(t) \tag{45}$$

$$y_a(t)=C_ax_a(t)+D_au_a(t) \tag{46}$$

Here

$$x_a(t)=\begin{bmatrix} x_p(t) \\ x_{ff}(t) \end{bmatrix} \tag{47}$$

$$A_a=\begin{bmatrix} A_p & 0 \\ 0 & A_{fb}-B_{fb}D_{fb}^{-1}C_{fb} \end{bmatrix} \tag{48}$$

$$B_a=\begin{bmatrix} B_p \\ B_{fb}D_{fb}^{-1} \end{bmatrix} \tag{49}$$

$$C_a=\begin{bmatrix} C_p & -D_{fb}^{-1}C_{fb} \end{bmatrix} \tag{50}$$

$$D_a=D_{ff}=D_{fb}^{-1} \tag{51}$$

The zero dynamics system matrix of the augmented system is given by $A_{az}=A_a-B_aD_a^{-1}C_a$. Substituting the appropriate matrices, finally gives

$$A_{az}=A=\begin{bmatrix} A_p-B_pD_{fb}C_p & -B_pC_{fb} \\ B_{fb}C_p & A_{fb} \end{bmatrix} \tag{52}$$

In a recent application of a non-minimum phase UAV, Barkana (2004b) showed how the use of usually available basic knowledge on plant stabilizability can not only guarantee stability of the adaptive control system in uncertain environments, but also achieve high performance with nonlinear adaptive controller in some of the most difficult real-world plants.

5. NONLINEAR CONTROL OF ASPR SYSTEMS:

While strictly proper continuous systems (with $D_p=0$) can be SPR, in discrete systems the direct input-output gain D_{pc} appears as an integral part of the SPR relations (20)-(22). To illustrate its significance, assume that the plant

$$x_p(t+1)=A_{pc}x_p(t)+B_{pc}u_p(t) \tag{53}$$

$$y_{p1}(t)=C_{pc}x_p(t) \tag{54}$$

$$y_p(t)=y_{p1}+D_{pc}u_p(t) \tag{55}$$

is SPR. The SPR system is asymptotically stable and we expect it to remain stable for any positive-definite gain, either constant or nonstationary. First, attempt to ignore the direct input output component of $y_p(t)$ and use the nonlinear control signal

$$u_p(t)=-K(x_p,t)y_{p1}(t)=-K(x_p,t)C_{pc}x_p(t) \tag{56}$$

Here, $K(x,t)$ is uniformly positive definite. The closed-loop system is

$$x_p(t+1) = \left(A_{pc} - B_{pc}K(x_p,t)C_{pc}\right)x_p(t) \quad (57)$$

Select the Lyapunov function

$$V(t) = x_p^T(t)Px_p(t) \quad (58)$$

The Lyapunov difference function is

$$\Delta V(t) = x_p^T(t+1)Px_p(t+1) - x_p^T(t)Px_p(t) \quad (59)$$

$$\begin{aligned}\Delta V(t) &= x_p^T(t)\left(A_{pc} - B_{pc}K(x_p,t)C_{pc}\right)^T P \\ &\bullet\left(A_{pc} - B_{pc}K(x_p,t)C_{pc}\right)x_p(t) - x_p^T(t)Px_p(t)\end{aligned} \quad (60)$$

From (60) one gets (Appendix B)

$$\begin{aligned}\Delta V(t) &= -x_p^T(t)Qx_p(t) \\ &-x_p^T(t)C_{pc}^T\left[K(x_p,t) + K^T(x_p,t)\right]C_{pc}x_p(t) \\ &-x_p^T(t)\left[L - WK(x_p,t)C_{pc}\right]^T \\ &\bullet\left[L - WK(x_p,t)C_{pc}\right]x_p(t) \\ &+x_p^T(t)C_{pc}^TK^T(x_p,t)\left[D_{pc} + D_{pc}^T - Q_0\right] \\ &\bullet K(x_p,t)C_{pc}x_p(t)\end{aligned} \quad (61)$$

Because of the last (positive semidefinite) term in (61), the Lyapunov difference may not be negative definite if $K(x_p,t)$ becomes very large, and therefore high-gain stability of the closed-loop system cannot be guaranteed.

Now, try the same nonlinear gain with the output signal $y_p(t)$

$$\begin{aligned}u_p(t) &= -K(x_p,t)y_p(t) = -K(x_p,t)C_{pc}x_p(t) \\ &- K(x_p,t)D_{pc}u_p(t)\end{aligned} \quad (62)$$

Define

$$K_D(x_p,t) = -\left(I + K(x_p,t)D_{pc}\right)^{-1}K(x_p,t) \quad (63)$$

to get from (62)

$$u_p(t) = -K_D(x_p,t)C_{pc}x_p(t) \quad (64)$$

The closed-loop system is now

$$x_p(t+1) = \left(A_{pc} - B_{pc}K_D(x_p,t)C_{pc}\right)x_p(t) \quad (65)$$

Select again the Lyapunov function (58). The Lyapunov difference function (59) gives (Appendix C)

$$\begin{aligned}\Delta V(t) &= -x_p^T(t)Qx_p(t) - x_p^T(t)\left(L^T - WK_D(x_p,t)C_{pc}\right)^T \\ &\bullet\left(L^T - WK_D(x_p,t)C_{pc}\right)x_p(t) - x_p^T(t)C_{pc}^TK_{DD}C_{pc}x_p(t)\end{aligned} \quad (66)$$

where

$$\begin{aligned}K_{DD} &= K_D(x_p,t) + K_D^T(x_p,t) \\ &-K_D^T(x_p,t)\left(D_{pc}^T + D_{pc} - Q_0\right)K_D(x_p,t)\end{aligned} \quad (67)$$

Equation (66) looks similar to (61), and may apparently imply that the last term could still become positive and adversely affect the stability of the system. We will show that this is not the case and that now the last term in (66) is negative semidefinite. To this end, substitute $K_D(x_p,t)$ in (67) and get by using the inverse matrix lemma and after some algebra

$$\begin{aligned}K_{DD} &= \left[I + D_pK(x_p,t)\right]^{-T}K_D(x_p,t) \\ &+ K_D^T(x_p,t)\left[I + D_pK(x_p,t)\right]^{-1} \\ &+ K_D^T(x_p,t)Q_0K_D(x_p,t) > 0\end{aligned} \quad (68)$$

Using (68), one finally gets

$$\Delta V(t) < -x_p^T(t)Qx_p(t) < 0 \quad \forall x_p > 0 \quad (69)$$

10. CONCLUSIONS

This paper extends the strict passivity results of LTI systems to discrete systems. In particular, it is shown that any proper but not strictly proper, minimum-phase, system with D >0 becomes strictly passive via positive output feedback. This result has been further extended, showing that if the controller H stabilized the system G, then the augmented system $G_a = G + H^{-1}$ is minimum-phase. This way, basic stabilizability properties of systems can be used to implement ASP configurations, thus extending the feasibility of adaptive and nonlinear control to real-world systems that are not necessarily minimum-phase.

APPENDIX A. THE LYAPUNOV DIFFERENCE FUNCTION FOR THE STRICTLY PROPER SYSTEM

From (59) one gets

$$\begin{aligned}\Delta V(t) &= x_p^T(t)\left(A_{pc}^TPA_{pc} - P\right)x_p(t) \\ &- x_p^T(t)A_{pc}^TPB_{pc}K(x_p,t)C_{pc}x_{pc}(t) \\ &- x_p^T(t)C_{pc}^TK^T(x_p,t)B_{pc}^TPA_{pc}x_p(t) \\ &+x_p^T(t)C_{pc}^TK^T(x_p,t)B_{pc}^TPB_{pc}K(x_p,t) \\ &\bullet C_{pc}x_p(t)\end{aligned} \quad (A.1)$$

and after some algebra

$$\begin{aligned}\Delta V(t) &= -x_p^T(t)Qx_p(t) \\ &-x_p^T(t)C_{pc}^T\left[K(x_p,t) + K^T(x_p,t)\right]C_{pc}x_p(t) \\ &-x_p^T(t)\left[L - WK(x_p,t)C_{pc}\right]^T \\ &\bullet\left[L - WK(x_p,t)C_{pc}\right]x_p(t) \\ &+x_p^T(t)C_{pc}^TK^T(x_p,t)\left[W^TW + B_{pc}^TPB_{pc}\right] \\ &\bullet K(x_p,t)C_{pc}x_p(t)\end{aligned} \quad (A.2)$$

From (A.2) one finally gets (61).

APPENDIX B. THE LYAPUNOV DIFFERENCE FUNCTION IN STRICTLY PROPER SYSTEMS

Using (65) with (59) gives

$$\Delta V(t) = x_p^T(t)\left(A_{pc} - B_{pc}K_D(x_p,t)C_{pc}\right)^T P \bullet\left(A_{pc} - B_{pc}K_D(x_p,t)C_{pc}\right)x_p(t) - x_p^T(t)Px_p(t) \quad (B.1)$$

Substituting $K_D(x_p,t)$ in (B.1) and using the SPR relations (20)-(22) leads after some algebra to

$$\begin{aligned}\Delta V(t) &= -x_p^T(t)Qx_p(t) \\ &- x_p^T(t)C_{pc}^T\left(K_D(x_p,t) + K_D^T(x_p,t)\right)\bullet C_{pc}x_p(t) \\ &- x_p^T(t)\left(L^T - WK_D(x_p,t)C_{pc}\right)^T \bullet\left(L^T - WK_D(x_p,t)C_{pc}\right)x_p(t) \\ &+ x_p^T(t)C_{pc}^T K_D^T(x_p,t)\bullet\left(D_{pc}^T + D_{pc} - Q_0\right) \bullet K_D(x_p,t)C_{pc}x_p(t)\end{aligned} \quad (B.2)$$

From (B.2) one finally gets (66).

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