SUB-OPTIMAL SENSOR SCHEDULING WITH ERROR BOUNDS

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Abstract: In this paper the problem of sub-optimal sensor scheduling with a guaranteed distance to optimality is considered. Optimal in the sense that the sequence that minimizes the estimation error covariance matrix for a given time horizon is found. The search is done using relaxed dynamic programming. The algorithm is then applied to one simple second order system and one sixth order model of a fixed mounted model lab helicopter.

Keywords: Sensor fusion, Sequences, Estimation, Dynamic Programming, Hybrid Modes

1. INTRODUCTION

Recently a great deal of attention has been given to the subject of wireless sensor networks. As the number of sensors in an area increases, the communication limitations imposed by bandwidth constraints will be more and more evident. Thus not allowing all sensors to communicate their measurements at each sampling interval could be very useful. Also, sensors might be battery powered and thus saving power is an essential factor. To save power a sensor can be put in stand-by mode and then woken by the estimator at certain points in time. There could also be situations where it is impossible to use two sensors at the same time due to the nature of the sensors, ultra sonic sensors is one example.

All these problems urge for algorithms that not only decide how to weight different sensors at different points in time, but also which sensors to use. How to choose which sensor or sensors to use at a specific moment is a nontrivial task studied by many others. In (Meier III *et al.*, 1967) the problem of discrete time sensor scheduling is solved by enumeration of all possible sensor schedules. The combinatorial explosion limits this approach to very short sensor schedules. A local gradient search is also proposed, but this approach doesn't guarantee that the global optimum is found. It is also shown that if the state estimates are to be used for state feedback, the plant control policy can be determined separately from the measurement schedule. The optimal sensor schedule on the other hand depends on the plant control policy. In (Chung *et al.*, 2004) an effort is made to prune the search tree by the use of a sliding window algorithm and a thresholding algorithm.

The sensor scheduling problem has also been approached from a continuous time direction. In (Athans, 1972) it is shown that the sensor scheduling problem can be transformed to a discrete-valued optimal control problem. This problem is then solved using a gradient search algorithm. In (Lee *et al.*, 2001) the discrete-valued optimal control problem of sensor scheduling is transformed into a continuous-valued optimal control problem by the use of a control parameterization enhancing transform (CPET). A method for robust sensor scheduling is developed in (Savkin *et al.*, 2001). Here the problem with growing complexity is tackled in a model predictive way.

Another related problem studied by many others is that of choosing the time distribution of measurements with one sensor given a measurement budget. This problem is studied for discrete time systems in (Shakeri *et al.*, 1995) and for continuous time systems in (Skafidas and Nerode, 1998).

In this paper a method of choosing the sensor switching strategy as well as the Kalman estimator gain for a discrete time system is presented. The objective is to minimize a function of the estimation error covariance matrix at the final time step. The method finds a sub-optimal strategy within a pre-specified distance to optimality. The complexity of the algorithm typically increases rapidly in the first iterations, but then levels out below a constant level. The paper is organized as follows. In Section 2 the class of system to which the algorithm applies is presented and an estimator structure is proposed. In Section 3 the optimization algorithm is presented and the connection to the work by (Lincoln, 2003) and (Lincoln and Rantzer, 2002) is developed. Section 4 presents two examples and finally Section 5 talks about problems and future extensions.

2. PROBLEM FORMULATION

Consider a discrete time system described by

$$\begin{cases} x(n+1) = Ax(n) + Bu(n) + v(n) \\ y_i(n) = C_i x(n) + e_i(n) \end{cases}$$

where $x(n) \in \mathscr{R}^n$ is the state of the process, $v(n) \in \mathscr{R}^n$ a white Gaussian stochastic process with zero mean. The system is observed through M sensor groups $i \in I = \{1 \dots M\}$ with outputs $y_i \in \mathscr{R}^{p_i}$ all disturbed by zero mean white Gaussian noise $e_i(n)$. The process noise and measurement noise has the following correlation matrix.

$$E \begin{bmatrix} v(n) \\ e_i(n) \end{bmatrix} \begin{bmatrix} v(n) \\ e_i(n) \end{bmatrix}^T = R_i$$

At each time instant the system can only be observed through one sensor group. To estimate the state of the system a Kalman filter of the following form will be used.

$$\begin{aligned} \hat{x}(n+1) &= A\hat{x}(n) + Bu(n) + K(n)\tilde{y}_{k(n)} \\ \tilde{y}_{k(n)} &= y_{k(n)} - C_{k(n)}\hat{x}(n) \end{aligned}$$

Here $k \in \kappa(n)$ denotes a specific sequence in the set $\kappa(n)$ of all possible sequences and $K \in \Lambda(n)$ a specific gain sequence in the set $\Lambda(n)$ of all possible gain sequences.. For a fixed sequence k and gain sequence K the estimation error covariance is given by equation (1) see (Åström and Wittenmark, 1997).

$$\begin{aligned} P(n+1,k,K) = \\ (A - K(n)C_{k(n)})P(n,k)(A - K(n)C_{k(n)})^T \\ + \begin{bmatrix} I \\ -K^T(n) \end{bmatrix}^T R_{k(n)} \begin{bmatrix} I \\ -K^T(n) \end{bmatrix} \end{aligned} \quad (1)$$

The initial estimation error covariance $P(0) = P_0$ which should reflect the knowledge of the initial state. The aim is to find a switching sequence k and a gain sequence K such that the following function is minimized.

$$J(N) = \min_{\substack{k(0)\dots k(N-1) \\ K(0)\dots K(N-1)}} \operatorname{tr}(P(N)W) \quad (2)$$

The problem can be solved by iterating equation (1) and expanding the search tree for all possible sequences. The size of the search tree $\|\kappa(n)\|$ will however grow as M^n which makes this procedure impossible in practice.

3. FINDING AN α-OPTIMAL SEQUENCE

To address the problem of increasing complexity a way of pruning the search tree has to be developed. In (Lincoln, 2003) and (Lincoln and Rantzer, 2002) a way of pruning the search three for the problem of choosing a switching *control law* is developed. As expected that problem turns out to be the dual of the estimation problem addressed here and thus the algorithm could be used with only small modifications.

Let $\Pi(n)$ denote the set of all potentially α-optimal estimation error covariances at time step n. α-optimal means that the estimation error covariance matrix associated with the found sequence fulfills

$$\underline{\alpha} \min_{\underline{\pi}^* \in \underline{\Pi}^*} \underline{\pi}^* \le \min_{\pi \in \Pi(n)} \pi \le \overline{\alpha} \min_{\overline{\pi}^* \in \overline{\Pi}^*} \overline{\pi}^*$$

where $\pi^* \in \Pi^*(n)$ denotes an element in the optimal set. The initial set $\Pi(0)$ is equal to the initial estimation error covariance P_0. Further let $\kappa_{opt}(n-1)$ denote the set of corresponding sequences and $\Lambda_{opt}(n-1)$ the set of corresponding gain sequences.

To continue the iteration first define

$$\begin{aligned} \overline{P}_i(n+1) = (A - K(n)C_i)\pi(n)(A - K(n)C_i)^T \\ + \begin{bmatrix} I \\ -K^T(n) \end{bmatrix}^T \overline{\alpha} R_i \begin{bmatrix} I \\ -K^T(n) \end{bmatrix} \end{aligned}$$

and

$$\begin{aligned} \underline{P}_i(n+1) = (A - K(n)C_i)\pi(n)(A - K(n)C_i)^T \\ + \begin{bmatrix} I \\ -K^T(n) \end{bmatrix}^T \underline{\alpha} R_i \begin{bmatrix} I \\ -K^T(n) \end{bmatrix} \end{aligned}$$

where $\pi(n) \in \Pi(n)$. Then an upper $\overline{\Pi}$ and lower bound $\underline{\Pi}$ for $\Pi(n+1)$ is calculated as

$$\begin{aligned} \overline{\Pi} &= \left\{ \min_{K(n)} \overline{P}_i(n+1) \,|\, \pi(n) \in \Pi(n), i \in I \right\} \\ \underline{\Pi} &= \left\{ \min_{K(n)} \underline{P}_i(n+1) \,|\, \pi(n) \in \Pi(n), i \in I \right\}. \end{aligned} \quad (3)$$

That is for each $\pi \in \Pi$ and $i \in I$ the estimation error covariance matrix is computed by minimizing over $K(n)$. Next the set of possible sequences

$$\kappa_{cand} = \left\{ \begin{bmatrix} k & i \end{bmatrix} \mid k \in \kappa_{opt}(n-1), i \in I \right\}$$

is computed. The set of corresponding gain matrix sequences is then computed using Procedure 1.

> Procedure 1
> For each $\pi \in \Pi(n)$
> (1) Pick the corresponding sequence $K \in \Lambda_{opt}(n-1)$
> (2) For each $i \in I$
> - Calculate the minimizing
> $$K(n) = \arg\min_{K(n)} \underline{P}_i(n+1)$$
> - Add the concatenated sequence $\begin{bmatrix} K & K(n) \end{bmatrix}$ to Λ_{cand}.

Now the objective is to find a set $\Pi(n+1)$ such that

$$\min_{\underline{\pi} \in \underline{\Pi}} \underline{\pi} \leq \min_{\pi \in \Pi(n+1)} \pi \leq \min_{\overline{\pi} \in \overline{\Pi}} \overline{\pi} \tag{4}$$

Together with the set $\Pi(n+1)$, a set of corresponding sequences $\kappa_{opt}(n)$ and a set of matrix gain sequences $\Lambda_{opt}(n)$ are also needed. This problem is solved by Procedure 2 which is a more detailed version of Procedure 3.2 in (Lincoln, 2003).

> Procedure 2
> (1) Sort $\overline{\Pi}$ so that
> $$\mathrm{tr}\overline{\pi}_i \leq \mathrm{tr}\overline{\pi}_j \;\; \forall i < j$$
> Λ_{cand} and κ_{cand} are ordered in the same way.
> (2) Let $\Pi(n+1) = \kappa_{opt}(n) = \Lambda_{opt}(n) = \oslash$
> (3) Pick the first $\overline{\pi} \in \overline{\Pi}$ and remove it from $\overline{\Pi}$.
> (4) If there exists x s.t.
> $$x^T \overline{\pi} x < x^T \pi x \;\; \forall \pi \in \Pi(n+1)$$
> then
> - Pick the first $\underline{\pi} \in \underline{\Pi}$, $k \in \kappa_{cand}$ and $K \in \Lambda_{cand}$.
> - Add this $\underline{\pi}$ to $\Pi(n+1)$ and remove $\underline{\pi}$ from $\underline{\Pi}$.
> - Add this k to κ_{opt} and remove k from κ_{cand}.
> - Add this K to Λ_{opt} and remove K from Λ_{cand}.
> - Go to step 3
>
> (5) Remove the first $\underline{\pi}$ from $\underline{\Pi}$.
> Remove the first k from κ_{cand}.
> Remove the first K from Λ_{cand}.
> If $\overline{\Pi} \neq \oslash$ go to step 3.

The new set of possibly α-optimal estimation error covariance matrices can now be used to compute upper and lower bounds for the next iteration. The iteration procedure can be ended when the old set fulfills (4) that is

$$\min_{\underline{\pi} \in \underline{\Pi}} \underline{\pi} \leq \min_{\pi \in \Pi(n)} \pi \leq \min_{\overline{\pi} \in \overline{\Pi}} \overline{\pi}$$

or when n=N.

The slack parameters $\overline{\alpha}$ and $\underline{\alpha}$ are used to control the tradeoff between complexity and accuracy. To find the α-optimal sequence it is enough to find the element in $\Pi(N)$ that minimizes (2) and pick the sequence associated with that estimation error covariance matrix.

4. EXAMPLES

In this section two examples will be given to illustrate the optimization algorithm presented in section 3. First a very simple second order system will be used to show the basic principle. Then a sixth order fixed mounted helicopter from the lab at Lund Institute of Technology will be used to illustrate that the procedure is applicable to problems of higher order.

4.1 A Second Order System

Consider the following discrete second order system with two sensors

$$\begin{cases} x(n+1) = \begin{bmatrix} 0.9 & 0.009 \\ 0.009 & 0.9 \end{bmatrix} x(n) + v(n) \\ y_1(n) = \begin{bmatrix} 1 & 0 \end{bmatrix} x + e_1(n) \\ y_2(n) = \begin{bmatrix} 0 & 1 \end{bmatrix} x + e_2(n) \end{cases}$$

with noise covariance matrices

$$R_1 = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 1 \end{bmatrix} \;\; R_2 = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & r \end{bmatrix} \;\; P_0 = I.$$

The system consists of two weakly interconnected states, each observed through a separate sensor. The goal is to minimize (2) with $W = I$ and $N = 50$.

The α-optimal sequence was computed for three different values of the parameter $r = \{10, 50, 110\}$. These values correspond to sequence k_1, k_2 and k_3 in Figure 1. The slack parameters where chosen as $\overline{\alpha} = 1.01$ and $\underline{\alpha} = \overline{\alpha}^{-1}$, which gave a set $\Pi(50)$ of size 5.

As the variance of the measurement noise at sensor 2 increases the number of samples taken from sensor 2 also increases up to a certain point. For this particular example sensor 2 is not used at all for a value of $r > 104$. The singular values of the observability Gramian

$$\sigma(W_o) = \begin{bmatrix} 5.5534 \\ 0.0138 \end{bmatrix}$$

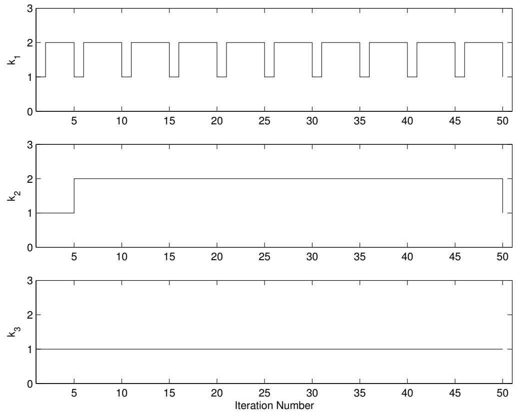

Fig. 1. Sequences for measurement noise variance $r = \{10, 50, 110\}$ at sensor two.

give an indication that x_2 has a low degree of observability from sensor 1. Thus the observer needs to use sensor 2 despite the large amount of measurement noise associated with it. When the noise on sensor two exceeds a certain level, the observer can get more information from sensor one and thus sensor two is not used. This is counterintuitive, but is detected by the algorithm.

4.2 The Model Helicopter

In this section a sensor switching strategy for a fixed mounted model helicopter will be developed. The model helicopter is situated in the Automatic Control Lab at Lund Institute of Technology. For a detailed description of the helicopter see (Gäfvert, 2001). The helicopter can be modeled by a sixth order system with the following state vector.

State	Description
w_1	Angular Velocity of Propeller 1
w_2	Angular Velocity of Propeller 2
ϕ	Yaw angle
$\dot{\phi}$	Yaw rate
θ	Pitch angle
$\dot{\theta}$	Pitch rate

A discrete time model with sample time $h = 0.05$ linearized around the forced equilibrium point $x^0 = \begin{bmatrix} w_1^0 & w_2^0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$ is given by

$$x(n+1) = Ax + v(n).$$

The system is observed through one yaw position sensor, one pitch angle sensor and one angular velocity sensor for propeller 1. The three sensors denoted y_1, y_2 and y_3 are disturbed by white Gaussian noise.

$$\begin{bmatrix} y_1(n) \\ y_2(n) \\ y_3(n) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} x(n) + e(n)$$

The corresponding covariance matrices are

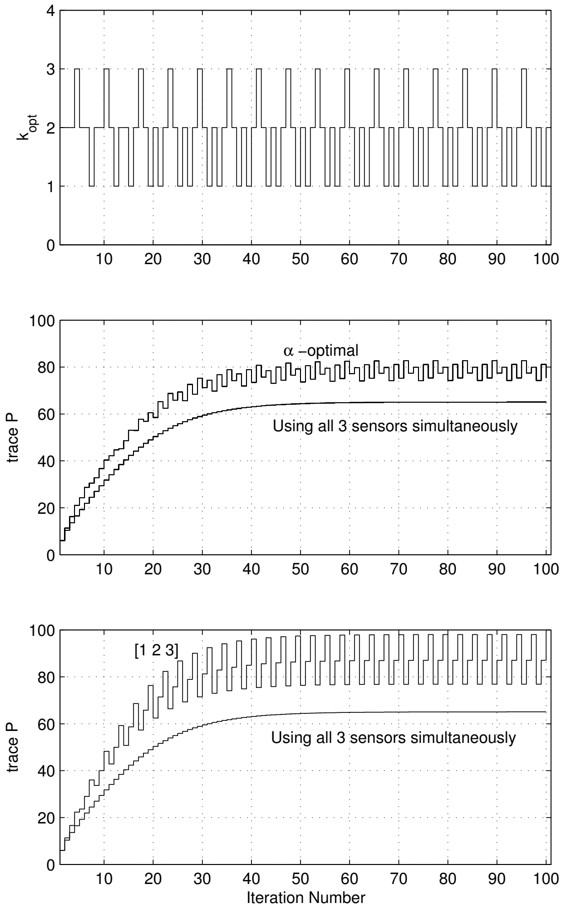

Fig. 2. α-optimal sequence k_{opt} together with traces of $P(n)$ for k_{opt} and for the sequence $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$.

$$R_1 = R_2 = R_3 = P_0 = I.$$

The objective parameters where chosen as $N = 100$ and $W = I$. Because of the higher complexity of this problem the the slack parameters where chosen as $\overline{\alpha} = 1.5$ and $\underline{\alpha} = \overline{\alpha}^{-1}$. This resulted in a set $\Pi(100)$ of size 4.

The α-optimal sequence was computed and is given in part one of Figure 2. The sequence is periodic with the period $\begin{bmatrix} 1 & 2 & 1 & 2 & 3 & 2 \end{bmatrix}$ except for the first 14 samples. The reason for this is the influence of the initial estimation error covariance P_0. The trace of the error covariance matrix $P(n)$ was also calculated for the α-optimal sequence and for a periodic sequence consisting of the triple $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$. As a comparison the trace of $P(n)$ was also computed for a Kalman filter which uses all three sensors at each sampling instant. These traces are found in part two and three of Figure 2.

To illustrate the pruning of sequences the size of the sequence candidate set κ is plotted in Figure 3. Here one can see the rapid increase in size the first 10 or so samples, but then instead of growing exponentially the size levels out. The size of $\kappa(n)$ never exceeds 7. If the slack parameters $\overline{\alpha}$ and $\underline{\alpha}$ are changed the shape

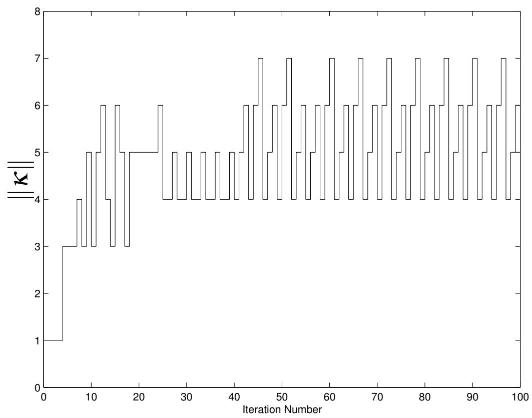

Fig. 3. Complexity in terms of the size of the sequence candidate set $\kappa(n)$.

of the complexity graph will remain the same, but the steady state level will be greater.

5. CONCLUSIONS AND FUTURE WORK

In this paper, the problem of minimizing the estimation error covariance matrix at the final step for a time discrete linear system, when only one sensor group may be used at each sampling interval is considered. The estimation problem was solved by slightly modifying the the procedure given in (Lincoln, 2003) for minimizing the quadratic cost with respect to a *control sequence*. The procedure uses relaxed dynamic programming where the parameters $\overline{\alpha}$ and $\underline{\alpha}$ are used in the tradeoff between complexity and distance to optimality, see (Lincoln and Rantzer, 2002) for details. The procedure was applied to two different examples, one simple second order example and one more complex sixth order example.

One drawback with the procedure presented here is that it minimizes the error covariance matrix at the final step only. Future work would include to extend the procedure to more general cost functions such as

$$J(N) = \min_{\substack{k(0)\dots k(N-1)\\ K(0)\dots K(N-1)}} \sum_{0}^{N} \operatorname{tr}(P(n)W(n)).$$

This is possibly a more difficult task, because the choice of sensor and Kalman gain not only effects the covariance in the next sample, but the value of the cost function for all future samples.

Work is also going on to combine switching observers with switching controllers to achieve sub-optimal performance of control systems with limited communication abilities.

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