SENSITIVITY ANALYSIS OF LOW COST FUZZY CONTROLLED SERVO SYSTEMS

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Abstract: The paper performs the sensitivity analysis with respect to the parametric variations of the controlled plant in the case of low cost fuzzy control systems dedicated to servo systems, with focus on solving the tracking control problem for a class of wheeled mobile robots with two degrees of freedom used in mining technologies. A new development method for Takagi-Sugeno PI-fuzzy controllers is proposed, based on applying the Extended Symmetrical Optimum method to the basic linear PI controllers in a cascaded control system structure. Original sensitivity models are derived. The approaches are validated by a case study.

Keywords: sensitivity analysis, fuzzy control, second-order systems, PI controllers, servo systems, dynamics.

1. INTRODUCTION

The considered class of controlled plants (abbreviated CPs) is characterized by the transfer functions $H_P(s)$:

$$H_P(s) = k_P / [s(1 + sT_\Sigma)], \qquad (1)$$

where k_P is the plant gain and T_Σ represents the small time constant or the sum of parasitic time constants.

The CPs with the transfer functions (t.f.s) of the second-order systems in (1) can approximate well enough the servo systems used in several applications including the control of mobile robots.

Low cost automation (LCA) involves the use of low cost control equipment and of control solutions that can be developed and implemented relatively easy. LCA solutions employ control structures and algorithms with dynamics that can ensure good control system (CS) performance in many situations. A second-order system (1) can be placed on the lower hierarchical level of complex, large-scale systems. The assurance of good CS performance for these systems by means of LCA solutions represents a necessity.

One way to fulfil the goal of good CS performance by means of LCA solutions for the CP (1) is represented by conventional control under the form of PI controllers (Åström and Hägglund, 1995).

Another way to fulfil the mentioned goal is represented by the use of fuzzy control with dynamics due to the flexible nonlinear input-output static map ensured by the fuzzy controllers (FCs) that can compensate (based on the designers' experience) the model uncertainties, nonlinearities and parametric variations of the CP. Actual applications of fuzzy control in the field of mobile robots are those reported in (Saffiotti, 2001; Hwang and Liu, 2004). But, although the reported simulation and experimental results are acceptable, relatively small research effort has been focussed on the systematic analysis of these fuzzy control systems (FCSs) including their stability or sensitivity analysis due to the nonlinearity of the FCs and of the CPs (Gartner and Astolfi, 2000; Cuesta and Ollero, 2002; Precup and Preitl, 2004).

The development of fuzzy controllers for the linear CPs (1) is considered as a first step in the development of complex control structures including these plants.

The sensitivity analysis of the FCSs with respect to the parametric variations of the CP is useful because the behaviour of these systems is generally reported as "robust" or "insensitive" without offering any systematic analysis tools. The sensitivity analysis performed in this paper is based on the approximate equivalence between the FCSs and the linear control systems. This approach is fully justified because of two reasons: firstly, concerning the controller part of the FCS, where the approximate equivalence between linear and fuzzy controllers is generally accepted in certain conditions (Moon, 1995); secondly, concerning the CP part of the FCS, where the support in using an FC to control a plant with linear or linearized model is to consider this CP model as a simplified model of a relatively complex one that can employ nonlinearities or variable parameters (in the particular case of servo systems).

This paper is organized as follows. The next Section is dedicated to the problem setting in tracking control of a class of wheeled mobile robots with two degrees of freedom. Then, a new development method for the Takagi-Sugeno PI-fuzzy controllers (TS-PI-FCs) used as tracking controllers is presented in Section 3. Section 4 addresses the sensitivity analysis by deriving four sets of original sensitivity models for the FCSs involved. Section 5 contains simulation results for a case study to validate the theoretical approach, and Section 6 draws the conclusions.

2. PROBLEM SETTING IN TRACKING CONTROL OF WHEELED MOBILE ROBOTS

The dynamic model of a class of wheeled mobile robots with two degrees of freedom can be expressed in terms of (Precup and Preitl, 2004):

$$\begin{aligned} \dot{x} &= v\cos\theta \\ \dot{y} &= v\sin\theta \\ \dot{v} &= a_v \\ \dot{\theta} &= \omega \\ T_{\Sigma 1}\dot{a}_v + a_v &= k_{P1}(u_1 + d_1) \\ T_{\Sigma 2}\dot{\omega} + \omega &= k_{P2}(u_2 + d_2) \end{aligned} , \qquad (2)$$

where: (x, y) – coordinates of the centre of the rear axis of the mobile robot; v and a_v – forward velocity and acceleration, respectively; θ – angle between the heading direction and the x-axis; ω – angular velocity; u_1 and u_2 – control signals, proportional to the generalized force variables; d_1 and d_2 – disturbance inputs due to the contact with the robot environment; k_{P1} and k_{P2} – gains; $T_{\Sigma 1}$ and $T_{\Sigma 2}$ – small time constants or time constants equivalent to the cumulative effects of the actuators dynamics, the measuring devices dynamics, the control equipment dynamics and of the parasitic time constants.

The servo system structure as CP (Fig. 1) contains the kinematical subsystem (KS) and the linear dynamic subsystems with the t.f.s $H_{P1}(s)$ and $H_{P2}(s)$:

$$H_{Pi}(s) = k_{Pi}/[s(1+sT_{\Sigma i})],\ i = \overline{1,2}. \qquad (3)$$

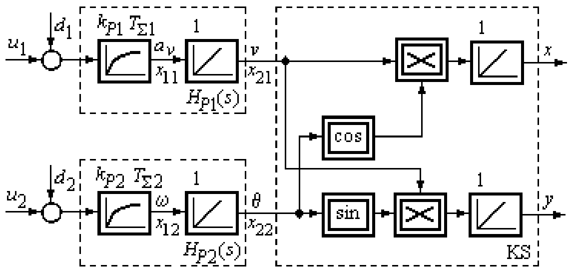

Fig. 1. Servo system structure as controlled plant.

The control aim is to solve the tracking control problem, tracking a reference trajectory (Jiang and Nijmeijer, 1997). With this respect, the CS structure employed for the considered class of mobile robots is presented in Fig. 2, where: C1 and C2 – forward velocity controller and angle controller, respectively; F1 and F2 – feed forward filters. To fulfil this aim the CS structure is a cascaded one, with two inner loops to control the controlled outputs $y_1 = v$ and $y_2 = \theta$, and the outer loops to provide the reference inputs for the inner loops by using the blocks Cx and Cy operating as:

$$\hat{x}_{r,k} = \begin{cases} \Delta x_{r,k}/h & \text{if } |e_{x,k}| \le \varepsilon_x \\ e_{x,k}/h & \text{otherwise} \end{cases}, \quad \hat{y}_{r,k} = \begin{cases} \Delta y_{r,k}/h & \text{if } |e_{y,k}| \le \varepsilon_y \\ e_{y,k}/h & \text{otherwise} \end{cases} \qquad (4)$$

where: h – sampling interval; k – index of the current sampling interval; $\Delta x_{r,k}$ and $\Delta y_{r,k}$ – increments of the reference positions x_r and y_r, respectively; $\varepsilon_x > 0$ and $\varepsilon_y > 0$ – maximum accepted absolute values of the tracking errors e_x and e_y, respectively. The CS designer must specify the values of ε_x and ε_y, as function of the desired CS performance.

To calculate the reference inputs r_1 and r_2 fed to the two inner control loops, there can be used the first two equations in (2), transformed into (5):

$$r_1 = [(\hat{x}_r)^2 + (\hat{y}_r)^2]^{1/2},\ r_2 = \tan^{-1}(\hat{y}_r / \hat{x}_r), \qquad (5)$$

and the nonlinear blocks denoted by (5) in Fig. 2 operate on the basis of (5). The generation of the reference trajectory (x_r, y_r) for the CS structure in Fig. 2 can be performed by several approaches (Tsourveloudis, *et al.*, 2001; Precup, *et al.*, 2003). The presented CS structure guarantees the trajectory tracking although there is usually necessary one more controlled variable to ensure the desired robot position in the case of trajectory tracking (Kanayama, *et al.*, 1990).

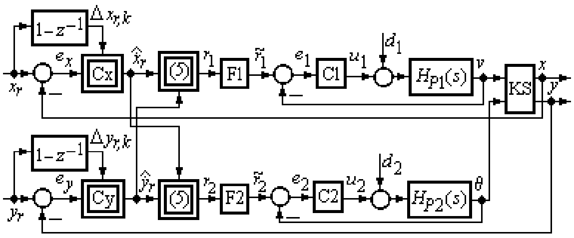

Fig. 2. Control system structure.

3. DEVELOPMENT METHOD FOR TAKAGI-SUGENO PI-FUZZY CONTROLLERS

The two PI-fuzzy controllers playing the role of the controllers C1 and C2 in Fig. 2 are Takagi-Sugeno FCs or type-III fuzzy systems. Both TS-PI-FCs have the structure presented in Fig. 2. This structure is based on adding the dynamics to the fuzzy controllers without dynamics FC_i, $i=\overline{1,2}$, by the numerical differentiation of the control error $e_{i,k}$ as the increment of control error, $\Delta e_{i,k}$, and by the numerical integration of the increment of control signal, $\Delta u_{i,k} = u_{i,k} - u_{i,k-1}$, $i=\overline{1,2}$.

The fuzzification in the fuzzy controllers without dynamics FC_i, $i=\overline{1,2}$, can be solved in the initial phase by using the input membership functions illustrated in Fig. 4. Other shapes of membership functions can contribute to CS performance enhancement. Both blocks FC_i, $i=\overline{1,2}$, use the max and min operators in the inference engine and employ the weighted average method for defuzzification (Babuška and Verbruggen, 1996).

The development of the two TS-PI-FCs starts with the development of the two basic linear PI controllers. For the class of servo systems with the t.f.s (1) or (3) the use of PI controllers with the t.f.:

$$H_{Ci}(s) = k_{ci}(1+sT_{ci})/s = k_{Ci}[1+1/(sT_{ci})], \qquad (6)$$

with the gains k_{ci} (or k_{Ci}) and the integral time constants T_{ci}, $i=\overline{1,2}$, tuned by the Extended Symmetrical Optimum (ESO) method (Preitl and Precup, 1999), can ensure good CS performance. The ESO method guarantees for the two inner control loops (in Fig. 2) maximum phase margins for constant CP parameters and minimum phase margins for variable CP gains, k_{Pi}. The tuning equations specific to the ESO method are:

$$k_{ci} = 1/(\beta_i^{3/2} T_{\Sigma i}^2 k_{Pi}),\ T_{ci} = \beta_i T_{\Sigma i},\ k_{Ci} = k_{ci} T_{ci}, \qquad (7)$$

where β_i, $i=\overline{1,2}$, represent design parameters, only one for each controller.

By the choice of the design parameters β_i in the recommended domain $1 < \beta_i < 20$, the CS performance indices {σ_{1i} – overshoot, $\hat{t}_{si} = t_{si}/T_{\Sigma i}$ – settling time, $\hat{t}_{ri} = t_{ri}/T_{\Sigma i}$ – rise time, φ_{mi} – phase margin} can be accordingly modified. A compromise to these indices can be reached by using the diagrams in Fig. 5. These performance indices can be corrected by adding the feed forward filters F1 and F2 (Fig. 2).

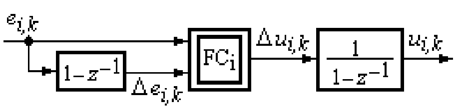

Fig. 3. TS-PI-FC structure, $i=\overline{1,2}$.

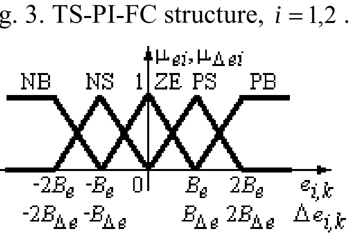

Fig. 4. Input membership functions of FC_i, $i=\overline{1,2}$.

For the development of the TS-PI-FCs, the continuous-time PI controllers (6) are discretized with the result in the incremental versions of the quasi-continuous digital PI controllers:

$$\begin{aligned}\Delta u_{i,k} &= K_{Pi}\,\Delta e_{i,k} + K_{Ii}\,e_{i,k} = \\ &= K_{Pi}(\Delta e_{i,k} + \delta_i \cdot e_{i,k}),\ i=\overline{1,2},\end{aligned} \qquad (8)$$

where $\{K_{Pi}, K_{Ii}, \delta_i\}$ are functions of $\{k_{ci}, T_{ci}\}$:

$$\begin{aligned}&K_{Pi} = k_{ci}T_{ci}[1-(h/2T_{ci})],\ K_{Ii} = k_{ci}\,h, \\ &\delta_i = K_{Ii}/K_{Pi} = 2h/(2T_{ci}-h),\ i=\overline{1,2}.\end{aligned} \qquad (9)$$

The rule bases of the blocks FC_i can be expressed in terms of the decision table presented in Table 1.

The strictly positive parameters of the TS-PI-FCs are $\{B_{ei}, B_{\Delta ei}, B_{\Delta ui}, m_i, n_i, p_i\}$, tuned by the development method to be presented as follows. The parameters B_{ei}, $B_{\Delta ei}$ and $B_{\Delta ui}$ are connected to the shapes of the input membership functions (Fig. 4), and the parameters m_i, n_i and p_i (Table 1), $m_i < n_i < p_i$, have been added to the standard version of TS-PI-FCs to improve the CS performance by modifying the input-output static maps of the blocks FC_i.

The development method for the two TS-PI-FCs consists of the development steps (1) and (2):

(1) The continuous controller design:

- express the mathematical model of the servo system as CP in its simplified form (1) and compute the CP parameters, k_P and T_Σ; in the case of robot control, the CP is characterized by $H_{P1}(s)$ and $H_{P2}(s)$ in (3), with the CP parameters k_{P1}, k_{P2}, $T_{\Sigma 1}$ and $T_{\Sigma 2}$;

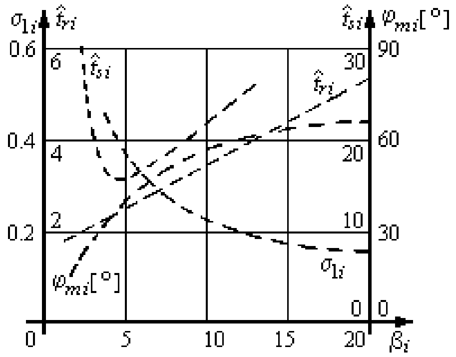

Fig. 5. CS performance indices versus β_i, $i=\overline{1,2}$.

Table 1 Decision table of FC_i

$\Delta e_{i,k}$	$e_{i,k}$				
	NB	NS	ZE	PS	PB
PB	$\Delta u_{i,k}$	$m_i\Delta u_{i,k}$	$n_i\Delta u_{i,k}$	$p_i\Delta u_{i,k}$	$p_i\Delta u_{i,k}$
PS	$m_i\Delta u_{i,k}$	$\Delta u_{i,k}$	$m_i\Delta u_{i,k}$	$n_i\Delta u_{i,k}$	$p_i\Delta u_{i,k}$
ZE	$n_i\Delta u_{i,k}$	$m_i\Delta u_{i,k}$	$\Delta u_{i,k}$	$m_i\Delta u_{i,k}$	$n_i\Delta u_{i,k}$
NS	$p_i\Delta u_{i,k}$	$n_i\Delta u_{i,k}$	$m_i\Delta u_{i,k}$	$\Delta u_{i,k}$	$m_i\Delta u_{i,k}$
NB	$p_i\Delta u_{i,k}$	$p_i\Delta u_{i,k}$	$n_i\Delta u_{i,k}$	$m_i\Delta u_{i,k}$	$\Delta u_{i,k}$

- choose the values of the design parameters β_1 and β_2 as function of the desired CS performance indices by using the diagrams in Fig. 5 and – in the case of robot control – the parameters ε_x and ε_y;
- use (7) to tune the parameters of the basic continuous-time PI controllers $\{k_{c1}, T_{c1}\}$ (for C1) and $\{k_{c2}, T_{c2}\}$ (for C2);
- add to the CS structure the accordingly chosen feed forward filters F1 and F2 (Preitl and Precup, 1999).

(2) The fuzzification of the basic linear PI controllers:
- set a sufficiently small sampling period, h, accepted by quasi-continuous digital control and take into account the presence of zero-order hold blocks, discretize the two continuous-time PI controllers and compute the parameters of the two quasi-continuous digital PI controllers, $\{K_{Pi}, K_{Ii}, \delta_i\}$, $i=\overline{1,2}$, by (9);
- set the values of the parameters m_i, n_i, p_i and B_{ei} in accordance with the experience of the CS designer, and apply the modal equivalence principle:

$$B_{\Delta\, ei} = \delta_i B_{ei}\,, \quad i = \overline{1,2}\,. \tag{10}$$

4. SENSITIVITY ANALYSIS

The sensitivity analysis offers useful information to the parameter setting of m_i, n_i, p_i and B_{ei} to obtain the desired FCS performance. But, the FCS performance can be improved also by the adequate choice of the inference method and of the defuzzification method.

The variation of CP parameters (k_{Pi} and $T_{\Sigma i}$, $i=\overline{1,2}$, for the considered servo systems) due to the change of the steady-state operating points or to other conditions leads to additional motion of the CSs. This motion is usually undesirable under uncontrollable parametric variations. Therefore, for the development of the FCs to alleviate the effects of parametric disturbances it is useful to perform the sensitivity analysis with respect to the parametric variations of the CP.

The sensitivity models (SMs) enable the sensitivity analysis of the FCSs accepted, as mentioned in Section 1, to be approximately equivalent with the linear CSs (with the continuous-time PI controllers). In this context, it is necessary to obtain the SMs of the two linear CSs, the inner control loops in Fig. 2 with the PI controllers (6) used as the controllers C1 and C2. The SMs of the FCSs will be identical to the SMs of the linear CSs. The only difference between these SMs is in the generation of the nominal trajectory of the CSs involved, by using either the TS-PI-FCs or the linear PI controllers.

To derive the SMs it is necessary to specify firstly the state mathematical models (MMs) of the CP and of the linear PI controllers. By taking the state variables x_{1i} and x_{2i} according to Fig. 1, if the linear PI controllers are tuned by (7) with respect to the nominal values of controlled plant parameters (k_{Pi0} and $T_{\Sigma i0}$, $i=\overline{1,2}$), the state MMs of the CP and of the PI controllers will result in terms of (11) and (12), respectively:

$$\begin{aligned}
&\dot{x}_{1i}(t) = x_{2i}(t),\\
&\dot{x}_{2i}(t) = -(1/T_{\Sigma i})x_{2i}(t) + (k_{Pi}/T_{\Sigma i})u_i(t) +\\
&\qquad + (k_{Pi}/T_{\Sigma i})d_i(t),\\
&y_i(t) = x_{1i}(t),\ i=\overline{1,2}\,,
\end{aligned} \tag{11}$$

$$\begin{aligned}
&\dot{x}_{3i} = [(1/\beta_i T_{\Sigma i0})]e_i,\\
&u_i = [1/(\beta_i^{1/2} k_{Pi0} T_{\Sigma i0})](x_{3i}+e_i),\ i=\overline{1,2}\,.
\end{aligned} \tag{12}$$

The state MMs of the closed-loop systems can be obtained by connecting the MMs in (11) and (12):

$$\begin{aligned}
&\dot{x}_{1i}(t) = x_{2i}(t),\\
&\dot{x}_{2i}(t) = -[k_{Pi}/(\beta_i^{1/2} k_{Pi0} T_{\Sigma i0} T_{\Sigma i})]x_{1i}(t) -\\
&- (1/T_{\Sigma i})x_{2i}(t) + [k_{Pi}/(\beta_i^{1/2} k_{Pi0} T_{\Sigma i0} T_{\Sigma i})]x_{3i}(t) +\\
&+ [k_{Pi}/(\beta_i^{1/2} k_{Pi0} T_{\Sigma i0} T_{\Sigma i})]r_i(t) + (k_{Pi}/T_{\Sigma i})d_i(t),\\
&\dot{x}_{3i}(t) = -[1/(\beta_i T_{\Sigma i0})]x_{1i}(t) + [1/(\beta_i T_{\Sigma i0})]r_i(t),\\
&y_i(t) = x_{1i}(t),\ i=\overline{1,2}\,.
\end{aligned} \tag{13}$$

For the MMs (13) there can be derived the sensitivity functions $\{\lambda_{1i}, \lambda_{2i}, \lambda_{3i}\}$ and the output sensitivity functions, σ_i (Rosenwasser and Yusupov, 2000):

$$\begin{aligned}
&\lambda_{ji}(t) = [\partial x_{ji}(t)/\partial\alpha]_{\alpha 0},\\
&\sigma_i(t) = [\partial y_i(t)/\partial\alpha]_{\alpha 0},\ j=\overline{1,3}\,,\ i=\overline{1,2}\,,
\end{aligned} \tag{14}$$

where the subscript "0" stands for the nominal values of the CP parameters, and $\alpha \in \{k_{Pi}, T_{\Sigma i}\}$.

Accepting the dynamic regimes characterized by the step modifications of the reference inputs r_i for $d_i(t)$=0, or the step modifications of the disturbance inputs d_i for $r_i(t)$=0, there will result four sets of SMs by computing the partial derivatives with respect to k_{Pi} and $T_{\Sigma i}$ in (13).

These SMs are presented in Table 2 as follows: SMs 1 – with respect to the variation of k_{Pi}, the step modification of d_i, and $r_i(t)$=0; SMs 2 – with respect to the variation of $T_{\Sigma i}$, the step modification of d_i, and $r_i(t)$=0; SMs 3 – with respect to the variations of k_{Pi}, the step modifications of r_i, and $d_i(t)$=0; SMs 4 – with respect to the variation of $T_{\Sigma i}$, the step modification of r_i, and $d_i(t)$=0.

The following notations were used in Table 2: $\{x_{1i0}, x_{2i0}, x_{3i0}\}$ – nominal values of the state variables, r_{i0} – nominal values of the reference inputs, and d_{i0} – nominal values of the disturbance inputs, $i=\overline{1,2}$.

5. CASE STUDY. DIGITAL SIMULATION RESULTS

To validate the proposed development method for TS-PI-FCs and the SMs dedicated to fuzzy controlled servo systems it is considered a case study characterized by a servo system dedicated to mobile robot control with the CP (3) having the parameters $k_{P1} = k_{P2} = 1$ and $T_{\Sigma 1} = T_{\Sigma 2} = 1$ s. It is accepted that the simplified dynamic models (2) characterize well the considered class of wheeled mobile robots with two degrees of freedom. The obstacles are placed in (3, 3), (9, 3), (6, 7), with the potentials 0.3.

Table 2 Derived sensitivity models

SMs 1: $\dot{\lambda}_{1i}(t) = \lambda_{2i}(t)$, $\dot{\lambda}_{2i}(t) = -[1/(\beta_i^{1/2}T_{\Sigma i0}^2)]\lambda_{1i}(t) - (1/T_{\Sigma i0})\lambda_{2i}(t) +$ $+ [1/(\beta_i^{1/2}T_{\Sigma i0}^2)]\lambda_{3i}(t) - [1/(\beta_i^{1/2}k_{Pi0}T_{\Sigma i0}^2)]x_{1i0}(t) +$ $+ [1/(\beta_i^{1/2}k_{Pi0}T_{\Sigma i0}^2)]x_{3i0}(t) + (1/T_{\Sigma i0})d_{i0}(t)$, $\dot{\lambda}_{3i}(t) = -[1/(\beta_i T_{\Sigma i0})]\lambda_{1i}(t)$, $\sigma_i(t) = \lambda_{1i}(t),\ i = \overline{1,2}$.	SMs 2: $\dot{\lambda}_{1i}(t) = \lambda_{2i}(t)$, $\dot{\lambda}_{2i}(t) = -[1/(\beta_i^{1/2}T_{\Sigma i0}^2)]\lambda_{1i}(t) - (1/T_{\Sigma i0})\lambda_{2i}(t) +$ $+ [1/(\beta_i^{1/2}T_{\Sigma i0}^2)]\lambda_{3i}(t) + [1/(\beta_i^{1/2}T_{\Sigma i0}^3)]x_{1i0}(t) +$ $+ (1/T_{\Sigma i0}^2)x_{2i0}(t) - [1/(\beta_i^{1/2}T_{\Sigma i0}^3)]x_{3i0}(t) -$ $- (k_{Pi0}/T_{\Sigma i0}^2)d_{i0}(t)$, $\dot{\lambda}_{3i}(t) = -[1/(\beta_i T_{\Sigma i0})]\lambda_{1i}(t)$, $\sigma_i(t) = \lambda_{1i}(t),\ i = \overline{1,2}$.
SMs 3: $\dot{\lambda}_{1i}(t) = \lambda_{2i}(t)$, $\dot{\lambda}_{2i}(t) = -[1/(\beta_i^{1/2}T_{\Sigma i0}^2)]\lambda_{1i}(t) - (1/T_{\Sigma i0})\lambda_{2i}(t) +$ $+ [1/(\beta_i^{1/2}T_{\Sigma i0}^2)]\lambda_{3i}(t) - [1/(\beta_i^{1/2}k_{Pi0}T_{\Sigma i0}^2)]x_{1i0}(t) +$ $+ [1/(\beta_i^{1/2}k_{Pi0}T_{\Sigma i0}^2)]x_{3i0}(t) + [1/(\beta_i^{1/2}k_{Pi0}T_{\Sigma i0}^2)]r_{i0}(t)$, $\dot{\lambda}_{3i}(t) = -[1/(\beta_i T_{\Sigma i0})]\lambda_{1i}(t)$, $\sigma_i(t) = \lambda_{1i}(t),\ i = \overline{1,2}$.	SMs 4: $\dot{\lambda}_{1i}(t) = \lambda_{2i}(t)$, $\dot{\lambda}_{2i}(t) = -[1/(\beta_i^{1/2}T_{\Sigma i0}^2)]\lambda_{1i}(t) - (1/T_{\Sigma i0})\lambda_{2i}(t) +$ $+ [1/(\beta_i^{1/2}T_{\Sigma i0}^2)]\lambda_{3i}(t) + [1/(\beta_i^{1/2}T_{\Sigma i0}^3)]x_{i10}(t) +$ $+ (1/T_{\Sigma i0}^2)x_{2i0}(t) - [1/(\beta_i^{1/2}T_{\Sigma i0}^3)]x_{3i0}(t) -$ $- [1/(\beta_i^{1/2}T_{\Sigma i0}^3)]r_{i0}(t)$, $\dot{\lambda}_{3i}(t) = -[1/(\beta_i T_{\Sigma i0})]\lambda_{1i}(t)$, $\sigma_i(t) = \lambda_{1i}(t),\ i = \overline{1,2}$.

The initial position of the robot is in the point (10, 4), and the goal, representing the desired / final position, is placed in the point (6, 11) with the potential –1. For these obstacles and robot positions the application of the artificial potential method yields the reference trajectory (x_r, y_r) presented in Fig. 6 (with dotted line).

Applying the steps of the development method (Section 3) involves the choice of the design parameters $\beta_1 = \beta_2 = 4$, and the parameters of the linear PI controllers will be $k_{c1} = k_{c2} = 0.125$ and $T_{c1} = T_{c2} = 4$ s. For h=0.1 s, the parameters of the quasi-continuous digital PI controllers will obtain the values $K_{P1} = K_{P2} = 0.4938$, $K_{I1} = K_{I2} = 0.0125$, $\delta_1 = \delta_2 = 0.0253$. Setting $B_{e1} = 0.3$, $B_{e2} = 0.45$, $m_1 = m_2 = 1$, $n_1 = n_2 = 2$ and $p_1 = p_2 = 2.4$, the values of the other parameters of the TS-PI-FCs will be $B_{\Delta e1} = 0.0076$, $B_{\Delta u1} = 0.0037$, $B_{\Delta e2} = 0.0114$ and $B_{\Delta u2} = 0.0056$.

The actual trajectory of the mobile robot is shown in Fig. 6 (with continuous line) in the conditions of the disturbance inputs $d_1 = 0.1\sigma(t)$, $d_2 = 0.01\sigma(t)$ (σ – the unit step signal), acceptable to model the contact of the robot with its environment.

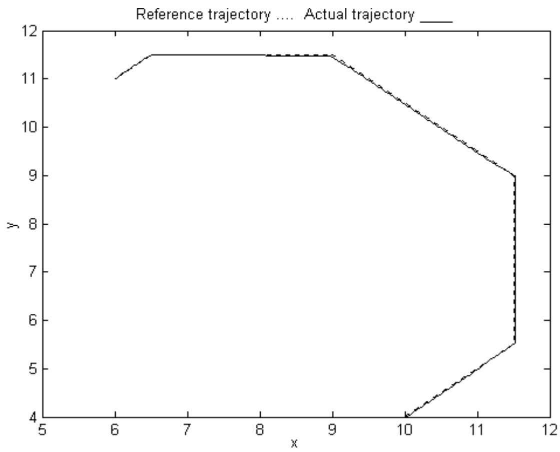

Fig. 6. Mobile robot reference and actual trajectory.

By setting the initial conditions $\lambda_{11}(0) = \lambda_{12}(0) = 0.2$, $\lambda_{21}(0) = \lambda_{22}(0) = 0.1$, $\lambda_{31}(0) = \lambda_{32}(0) = -0.2$, the behaviours of the SMs 1 and 4 are illustrated in Fig. 7 and Fig. 8, respectively.

The behaviours of the SMs 2 and 3 are similar to these behaviours.

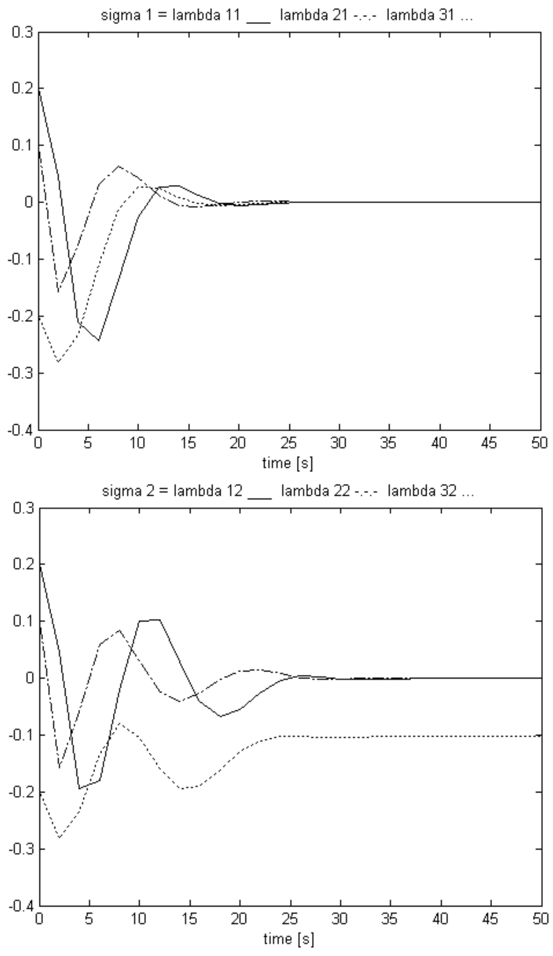

Fig. 7. Sensitivity functions versus time for SMs 1.

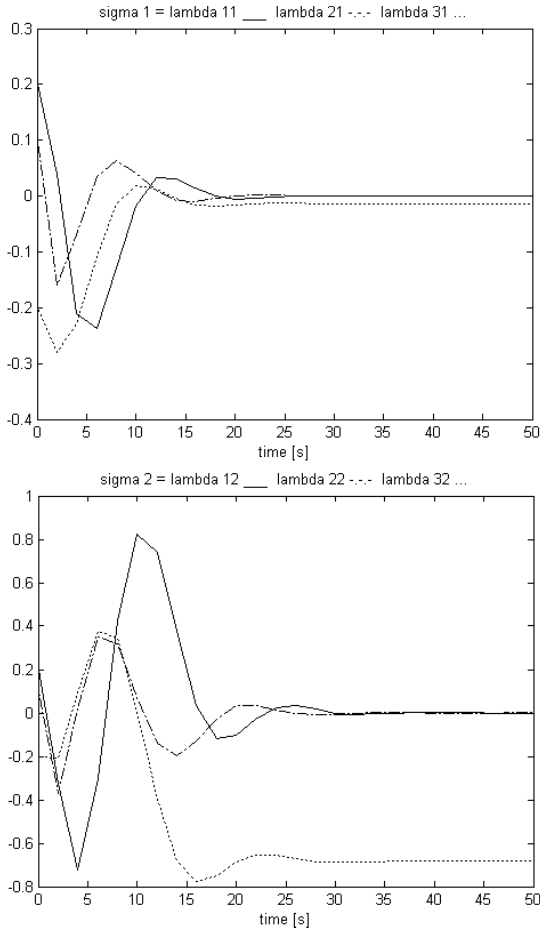

Fig. 8. Sensitivity functions versus time for SMs 4.

6. CONCLUSIONS

The paper derives new sensitivity models that enable the sensitivity analysis of fuzzy control systems dedicated to a class of second-order servo systems. In addition, the paper offers an attractive development method for low cost Takagi-Sugeno PI-fuzzy controllers, expressed in terms of two development steps**.** The SMs derived in the paper represent support to solve control problems concerning wheeled mobile robots with two degrees of freedom.

The simplicity of the TS-PI-FCs structure together with the flexibility and transparency of the development method determines the proposed FCs to be seen as LCA solutions that can be extended with additional features to more complex applications.

The case study presented in the paper deals with FCSs with TS-PI-FCs that can be implemented as tracking controllers in cascade control system structures, and it validates the SMs and the development method. As it has been shown in Fig. 6, the developed low cost FCs provide very good control system performance.

Although the paper is dedicated to tricycle type mobile robots, the future work will be focussed on differential-drive mobile robots and on real-time experiments in the field of mining technologies.

REFERENCES

Åström, K.J. and T. Hägglund (1995). *PID Controllers Theory: Design and Tuning*. Inst. Society of America, Research Triangle Park.

Babuška, R. and H.B. Verbruggen (1996). An overview on fuzzy modeling for control. *Control Engineering Practice*, **4**, 1593-1606.

Cuesta, F. and A. Ollero (2002). Stability analysis of fuzzy reactive navigation. *Preprints of 15th IFAC World Congress b'02*, Barcelona, paper 1799, 6 pp.

Gartner, H. and A. Astolfi (2000). Fuzzy control of a mobile robot with guaranteed stability. In: *Stability Issues in Fuzzy Control* (J. Aracil and F. Gordillo, Eds.). Pp. 323-346. Springer-Verlag, Berlin.

Hwang, C.-L. and H.-Y. Liu (2004). Trajectory tracking of robot using a fuzzy decentralized sliding-mode tracking control. *Proceedings of IEEE International Conference on Fuzzy Systems FUZZ-IEEE 2004*, Budapest, Hungary, paper 1087, 6 pp.

Jiang, Z.-P. and H. Nijmeijer (1997). Tracking control of mobile robots: a case study in backstepping. *Automatica*, **33**, 1393-1399.

Kanayama, Y., Y. Kimura, F. Miyazaki and T. Noguchi (1990). A stable tracking control method for an autonomous mobile robot. *Proceedings of 1990 IEEE International Conference on Robotics and Automation*, Cincinnati, OH, 384-389.

Moon, B.S. (1995). Equivalence between fuzzy logic controllers and PI controllers for single input systems. *Fuzzy Sets and Systems*, **69**, 105-113.

Precup, R.-E. and S. Preitl (2004). Sensitivity analysis of a class of fuzzy controlled mobile robots. *Preprints of 2nd IFAC Workshop on Advanced Fuzzy/ Neural Control*, Oulu, Finland, paper 7025, 6 pp.

Precup, R.-E., S. Preitl, C. Szabo, Z. Gyurko and P. Szemes (2003). Sliding mode navigation control in Intelligent Space. *Proceedings of 2003 IEEE International Symposium on Intelligent Signal Processing WISP 2003*, Budapest, Hungary, 225-230.

Preitl, S. and R.-E. Precup (1999). An extension of tuning relations after Symmetrical Optimum method for PI and PID controllers. *Automatica*, **35**, 1731-1736.

Rosenwasser, E. and R. Yusupov (2000). *Sensitivity of Automatic Control Systems*. CRC Press, Boca Raton, FL.

Saffiotti, A. (2001). Fuzzy logic in autonomous navigation. In: *Fuzzy Logic Techniques for Autonomous Vehicle Navigation* (D. Driankov and A. Saffiotti, Eds.). Pp. 3-24. Physica-Verlag, Heidelberg.

Tsourveloudis, N.C., K.P. Valvanis and T. Hebert (2001). Autonomous vehicle navigation utilizing electro-static potential fields and fuzzy logic. *IEEE Trans. Robotics and Automation*, **17**, 2001, 490-497.