APPLICATION OF GENEALOGICAL DECISION TREES FOR OPEN-LOOP TRACKING CONTROL

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Abstract: A new approach based on a genealogical decision tree is suggested for solving an open-loop tracking problem. The algorithm associates Gaussian distributions to both the norms of the control actions and the tracking errors. It solves the optimization problem sequentially, using random resampling from a population of solutions. This stochastic search model can be interpreted as a simple genetic particle evolution model with a natural birth and death interpretation. It converges in probability. Two numerical examples, dealing with rapid thermal processing and robotics, illustrate the feasibility and the performance of this control algorithm.

Keywords: Monte Carlo method, optimal control, optimization problems, population-based search.

1. INTRODUCTION

Advances in population-based evolutionary algorithms have introduced new tools for optimimizing and controlling complex systems (Ikonen and Najim, 2002) (Zalzala and Fleming, 1997). This paper considers an algorithm that falls within the area of mathematical population genetics and interacting particle systems.

Genealogical decision trees belong to stochastic search models (Najim *et al.*, 2004). Their flexibility, robustness, and applicability to both linear and non linear systems, and for both continuous and batch processes, make them particularly attractive for applications to optimization and control of complex systems. The first heuristic schemes of these control particle models first appeared in (Del Moral, 1997). Their asymptotic behavior with respect to the time horizon or the particle system size is today well understood (Del Moral, 2004); allowing us to quantify with precision their ability to solve any optimal control problem which can be interpreted in terms of a

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nonlinear filtering problem. Lately, genealogical decision trees have been introduced for solving open-loop tracking control problems (Ikonen *et al.*, 2004).

This paper suggests a control approach based on the extensive use of a process model. This approach is potential, e.g., for processes for which there exist no on-line sensors for the controlled outputs (semiconductors manufacturing, chemistry, biotechnology, etc.), or on-line analysers (concentration measurement, etc.) are very expensive and have high maintenance costs. Soft sensors and inferential control approaches have been dedicated to solve this type of process control problems where the outputs measurements are available only off-line. Soft sensors are based on models relating the desired controlled variable to some easily measured variables. The essence of the inferential control approach is in the prediction of the process outputs over the interval separating two successive measurements. As a consequence, the efficiency of soft sensors and inferential control approaches is limited by the adopted model for the sensor, or the characteristics of the predictor of the outputs.

This paper is organized as follows. The control problem is formulated in the next section; followed by a description of the optimization and control algorithms under consideration, and an analysis of the optimization scheme. Finally, we illustrate the performance of these schemes with two simple numerical examples related to trajectory following. Some concluding remarks end this paper.

2. PROBLEM FORMULATION

Consider any non-linear time-varying dynamic system, described by the following state equations:

$$\begin{cases} \mathbf{X}_n = F_n\left(\mathbf{X}_{n-1}, \mathbf{U}_n\right) \\ \mathbf{Y}_n = h_n\left(\mathbf{X}_n\right) \end{cases}, \tag{1}$$

where $\mathbf{X}_n = [X_{n,1}, X_{n,2}, ..., X_{n,S}]^T \in \mathcal{R}^S$, $\mathbf{U}_n = [U_{n,1}, U_{n,2}, ..., U_{n,P}]^T \in \mathcal{R}^P$, $\mathbf{Y}_n = [Y_{n,1}, Y_{n,2}, ..., Y_{n,Q}]^T \in \mathcal{R}^Q$. Index $n = \overline{1,T}$ represents the sampling instant. $\mathbf{X}_0$ represents the fixed initial state at instant $n = 0$. Let $\mathbf{A}_n$ and $\mathbf{B}_n$ be symmetric and semi-definite positive covariance matrices. The control objective in a finite horizon of length T is given by

$$J_T\left(\mathbf{U}_1, ..., \mathbf{U}_T\right) = \tag{2}$$
$$\sum_{n=1}^{T} \left\|\mathbf{U}_n\right\|^2_{\mathbf{A}_n} + \sum_{n=1}^{T} \left\|\mathbf{Y}_n - \mathbf{Y}_n^{\mathrm{ref}}\right\|^2_{\mathbf{B}_n}$$

where $\mathbf{Y}_n^{\mathrm{ref}} \in \mathcal{R}^Q$ represents the reference (desired) trajectories, and $\left\|\mathbf{U}\right\|^2_{\mathbf{A}} = \mathbf{U}^T\mathbf{A}^{-1}\mathbf{U}$. Our objective is to find the sequence of control actions that will minimize this control objective for open-loop control.

3. OPTIMIZATION OF THE CONTROL SEQUENCE

Having formulated the control problem, the control algorithm can now be presented. Gaussian distributions are associated to both the norms of the control actions and the tracking errors, i.e.,

$$\left\|\mathbf{U}_n\right\|^2_{\mathbf{A}_n} \longrightarrow \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\beta}{2}\left\|\mathbf{U}_n\right\|^2_{\mathbf{A}_n}\right)$$
$$\left\|\mathbf{Y}_n - \mathbf{Y}_n^{\mathrm{ref}}\right\|^2_{\mathbf{B}_n} \longrightarrow$$
$$\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\beta}{2}\left\|\mathbf{Y}_n - \mathbf{Y}_n^{\mathrm{ref}}\right\|^2_{\mathbf{B}_n}\right).$$

The parameter β is similar to the inverse of the temperature in simulated annealing optimization algorithms. Indeed, for large value of β, the probability distribution will have a form of a hair pin.

Let us consider the following scheme. At instant n, carry out N independent and identically distributed normal vectors $\mathbf{U}_n^i \sim \mathcal{N}\left(\mathbf{0}, \mathbf{A}_n\right)$, $i = \overline{1,N}$:

$$\mathbf{U}_n^1, \mathbf{U}_n^2, ..., \mathbf{U}_n^N.$$

Using the model (F_n, h_n) we evaluate that these control values lead to N outputs:

$$\begin{cases} \mathbf{X}_n^i = F_n\left(\widehat{\mathbf{X}}_{n-1}^i, \mathbf{U}_n^i\right) \\ \mathbf{Y}_n^i = h_n\left(\mathbf{X}_n^i\right) \end{cases}$$

for $i = \overline{1,N}$. At $n = 1$, the initial states $\widehat{\mathbf{X}}_0^i$ are given by $\mathbf{X}_0$.

In order to simplify the notations, let us introduce the following term

$$p_n^i = \frac{\exp\left(-\frac{\beta}{2}\left\|\mathbf{Y}_n^{\mathrm{ref}} - \mathbf{Y}_n^i\right\|^2_{\mathbf{B}_n}\right)}{\sum_{j=1}^{N} \exp\left(-\frac{\beta}{2}\left\|\mathbf{Y}_n^{\mathrm{ref}} - \mathbf{Y}_n^j\right\|^2_{\mathbf{B}_n}\right)}. \tag{3}$$

We have $\sum_{i=1}^{N} p_n^i = 1$, and p_n can be interpreted as a probability measure. Let us then generate N independent and identically distributed random vectors $\widehat{\mathbf{U}}_n^1, \widehat{\mathbf{U}}_n^2, ..., \widehat{\mathbf{U}}_n^N$ according to the distribution

$$p_n\left(\mathbf{u}\right) = \sum_{i=1}^{N} p_n^i \delta_{\mathbf{U}_n^i},$$

where $\delta_{\mathbf{u}}$ is the Dirac measure at the control value $\mathbf{u} \in \mathcal{R}^P$. In other words, for each $j = \overline{1,N}$, each random control $\widehat{\mathbf{U}}_n^j$ takes the value $\mathbf{U}_n^i$ with probability equal to p_n^i. This can be seen as a resampling procedure. The implementation of these control actions leads to

$$\widehat{\mathbf{X}}_n^j = F_n\left(\widehat{\mathbf{X}}_{n-1}^i, \mathbf{U}_n^i\right) \Longrightarrow \widehat{\mathbf{Y}}_n^j = h_n\left(\widehat{\mathbf{X}}_n^j\right)$$

for $j = \overline{1, N}$. The procedure is repeated for all $n = \overline{2, T}$. A mechanization of the procedure is given in the appendix.

3.1 *A genealogical decision tree model*

The stochastic search model suggested in the previous section can be interpreted as a simple genetic particle evolution model. These evolutionary algorithms have a natural birth and death interpretation.

More precisely, each controlled state $\widehat{\mathbf{X}}_n^i$ results from the selection of a random control action, say $\widehat{\mathbf{U}}_n^i = \mathbf{U}_n^j$ for some j. That is, we have that

$$\widehat{\mathbf{X}}_n^i = F_n\left(\widehat{\mathbf{X}}_{n-1}^j, \mathbf{U}_n^j\right).$$

In this case, we can interpret the state $\widehat{\mathbf{X}}_{n-1}^j$ as the parent of the individual $\widehat{\mathbf{X}}_n^i$ at level $n-1$, and we denote $\widehat{\mathbf{X}}_{n-1,n}^i = \widehat{\mathbf{X}}_{n-1}^j$.

In the same way, the parent individual $\widehat{\mathbf{X}}_{n-1}^j$ results from the selection of a random control action, say $\widehat{\mathbf{U}}_{n-1}^j = \mathbf{U}_{n-1}^k$, for some k, that is we have that

$$\widehat{\mathbf{X}}_{n-1}^j = F_{n-1}\left(\widehat{\mathbf{X}}_{n-2}^k, \mathbf{U}_{n-1}^k\right).$$

Arguing as above, we can interpret the state $\widehat{\mathbf{X}}_{n-2}^k$ as the parent of the individual $\widehat{\mathbf{X}}_{n-1}^j$, and therefore, as the ancestor of the individual $\widehat{\mathbf{X}}_n^i$ at level $n-2$, $\widehat{\mathbf{X}}_{n-2,n}^i = \widehat{\mathbf{X}}_{n-2}^k$.

Running back in time, we can trace the complete ancestral line of the current individual

$$\begin{aligned}
\widehat{\mathbf{X}}_{0,n}^i \longleftarrow & \ldots \\
& \longleftarrow \widehat{\mathbf{X}}_{n-2,n}^i = \widehat{\mathbf{X}}_{n-2}^k \\
& \longleftarrow \widehat{\mathbf{X}}_{n-1,n}^i = \widehat{\mathbf{X}}_{n-1}^j \\
& \longleftarrow \widehat{\mathbf{X}}_{n,n}^i = \widehat{\mathbf{X}}_n^i.
\end{aligned}$$

We define, in the same way, the ancestral decision line of the corresponding control actions

$$\begin{aligned}
\widehat{\mathbf{U}}_{1,n}^i \longleftarrow & \ldots \\
& \longleftarrow \widehat{\mathbf{U}}_{n-2,n}^i = \widehat{\mathbf{U}}_{n-2}^k \\
& \longleftarrow \widehat{\mathbf{U}}_{n-1,n}^i = \widehat{\mathbf{U}}_{n-1}^j \\
& \longleftarrow \widehat{\mathbf{U}}_{n,n}^i = \widehat{\mathbf{U}}_n^i.
\end{aligned}$$

At the final horizon time $n = T$, we obtain the approximating optimal open-loop control actions as our solution to minimize the control objective:

$$\begin{aligned}
\widehat{\mathbf{U}}_{1,T}^I \longleftarrow & \ldots \\
& \longleftarrow \widehat{\mathbf{U}}_{T-2,T}^I \\
& \longleftarrow \widehat{\mathbf{U}}_{T-1,T}^I \\
& \longleftarrow \widehat{\mathbf{U}}_{T,T}^I = \widehat{\mathbf{U}}_T^I,
\end{aligned}$$

where the index label I is chosen so that

$$I = \arg \inf_{i=\overline{1,N}} J_n\left(\widehat{\mathbf{U}}_{1,n}^i, \widehat{\mathbf{U}}_{2,n}^i, \ldots, \widehat{\mathbf{U}}_{n,n}^i\right).$$

4. ASYMPTOTIC ANALYSIS

There exist many results on the asymptotic analysis of the genetic evolutionary models presented above. In advanced signal processing, for instance, these interacting particle algorithms and their genealogical tree models provide a powerful stochastic and adaptive grid approximation for solving nonlinear filtering and smoothing problems.

The main idea is to translate the cost function as the likelihood of a conditional probability. This gives a duality between optimal regulation and optimal filtering. This idea can be seen as an extension of the fact that the optimal Kalman–Bucy filter gives the right answer in both situations, as soon as the models are linear and quadratic.

The signal disturbance filtering problem is given by the equations

$$\left\{\begin{array}{l}
\mathbf{X}_n = F_n\left(\mathbf{X}_{n-1}, \mathbf{W}_n\right) \\
\mathbf{Y}_n = h_n\left(\mathbf{X}_n\right) + \mathbf{V}_n
\end{array}\right.,$$

where $\mathbf{W}_n$ and $\mathbf{V}_n$ are independent centered Gaussian random vectors, with respective covariance matrices $\mathbf{A}_n/\beta$ and $\mathbf{B}_n/\beta$. In our optimal control/tracking context –and in some sense– we can prove that, for any time horizon n, we have for any bounded and measurable function φ_n on $\mathcal{R}^{P \times n}$:

$$\begin{aligned}
& \frac{1}{N} \sum_{i=1}^{N} \varphi_n\left(\widehat{\mathbf{U}}_{1,n}^i, \widehat{\mathbf{U}}_{2,n}^i, \ldots, \widehat{\mathbf{U}}_{n,n}^i\right) \\
& \underset{N \to \infty}{\longrightarrow} E\left\{\varphi_n\left(\mathbf{W}_1, \mathbf{W}_2, \ldots, \mathbf{W}_n\right) \mid\right. \\
& \qquad \left. \mathbf{Y}_1 = \mathbf{Y}_1^{\mathrm{ref}}, \ldots, \mathbf{Y}_n = \mathbf{Y}_n^{\mathrm{ref}}\right\}.
\end{aligned} \tag{4}$$

To find the control actions which minimize the considered control objective, it is equivalent to look for the most likely actions $\mathbf{W}$.

To understand the duality between this filtering model and the control problem discussed in this work, we observe that:

$$
\begin{aligned}
&\Pr\left\{(\mathbf{W}_1, \mathbf{W}_2, ..., \mathbf{W}_n) \in d(\mathbf{w}_1, ..., \mathbf{w}_n) \middle| \right. \\
&\qquad\qquad \left. \mathbf{Y}_1 = \mathbf{Y}_1^{\text{ref}}, ..., \mathbf{Y}_n = \mathbf{Y}_n^{\text{ref}} \right\} \\
&= \frac{1}{Z_n} \exp\left[-\frac{\beta}{2} \left(\sum_{k=1}^{n} \|\mathbf{w}_k\|_{\mathbf{A}_k}^2 + \right.\right. \qquad (5) \\
&\qquad \left.\left. \sum_{k=1}^{n} \left\| \mathbf{Y}_k^{\text{ref}} - h_k(\mathbf{X}_k) \right\|_{\mathbf{B}_k}^2 \right) \right] d\mathbf{w}_1 ... d\mathbf{w}_n \\
&= \frac{1}{Z_n} \exp\left(-\frac{\beta}{2} \left(J_n(\mathbf{w}_1, ..., \mathbf{w}_n) \right) \right) d\mathbf{w}_1 ... d\mathbf{w}_n,
\end{aligned}
$$

where $d(\mathbf{w}_1,...,\mathbf{w}_n)$ stands for an infinitesimal neighborhood of the perturbation sequence $(\mathbf{w}_1, ..., \mathbf{w}_n) \in \mathcal{R}^{P\times n}$, Z_n is a normalizing constant, and $d\mathbf{w}_k$ represents the Lebesgue measure on $\mathcal{R}^P$. Loosely speaking, the above expression indicates that the conditional probability mass is concentrated around the optimal control sequence. The approximation result (4) can be used to estimate the desired control sequence.

One can prove easily the following convergence in probability

$$
\begin{aligned}
&\lim_{N\to\infty} \underline{\inf}_{i=1,q(N)} J_n\left(\widehat{\mathbf{U}}_{1,n}^i, \widehat{\mathbf{U}}_{2,n}^i, ..., \widehat{\mathbf{U}}_{n,n}^i \right) \\
&= \inf_{\mathbf{U}_1,...,\mathbf{U}_n} J_n(\mathbf{U}_1, ..., \mathbf{U}_n)
\end{aligned}
$$

for any increasing sequence of block sizes $q(N) = o\left(\sqrt{N}\right)$, where $\inf J_n$ stands for the essential infinimum of the control objective function J_n with respect to the Lebesgue measure on $R^{P\times n}$.

5. NUMERICAL EXAMPLES

Let us here illustrate the operation of the control algorithm with two simulated examples. The first example concerns Rapid Thermal Processing (RTP); the second case examines a single link manipulator. In both cases, given a desired trajectory and the initial state at zero, the task was to find (in open loop) the optimal control sequence that would minimize the control objective (2). Incremental control was considered in both cases (i.e., the sequence of changes in the applied control action was optimized), and the process model was assumed to be known. Some guidelines for the selection of algorithm parameters $\mathbf{A}_n$, $\mathbf{B}_n$, N and β are discussed in (Ikonen *et al.*, 2004).

5.1 *Example 1 – Rapid thermal processing*

Reduction of product and process variation has become a critical component of competitiveness in today's manufacturing industries. RTP systems play an important role in semiconductors manufacturing such as annealing, oxidation, chemical vapor deposition, etc. Many phenomenological models have been developed and used for the design, optimization and control of RTP systems (Edgar *et al.*, 2000) (Bordeneuve *et al.*, 1991) (Norman, 1992).

Chemical Vapour Deposition (CVD) is a technique for creating thin films on silicon wafers. Gaseous reactants are deposited onto a substrate, and we are interested in the depth of the deposited material onto the substrate. There exists no online sensors for this depth measurement. Instead, very precise phenomenological models (Fayolle *et al.*, 1996)(Kleijn, 1996) have been developed for CVD reactors. The simulation of these models is very time consuming (several hours on big computers), consequently, they can not be used as soft sensors. Neverthless, based on these models, the presented control algorithm can be easily used for the control of the depth of the deposited material onto the substrate.

In order to achieve uniform processing and a high level of reproducibility of phenomena, the wafer temperature has to track a pre-specified temperature trajectory. Many control approaches ranging from adaptive predictive to iterative learning control have been experienced (Bordeneuve *et al.*, 1991) (Schaper *et al.*, 1999) (Choi and Do, 2001). These approaches are based on linear or on feedforward neural models.

There is no assumption on the RTP model in the open loop tracking control approach presented in this paper, given that the model can be expressed in the form given by equation (1). Indeed, any type of model can be easily used for control purposes. A simple nonlinear continuous time model (Gorinevsky, 2002) for such thermal processing has two states: furnace temperature T_{F} and part temperature T_{P}:

$$
\begin{aligned}
\dot{T}_{\text{F}} &= b_u u - c_1\left(T_{\text{F}}^4 - T_{\text{P}}^4\right) - c_2\left(T_{\text{F}} - T_{\text{A}}\right) \\
\dot{T}_{\text{P}} &= c_3\left(T_{\text{F}}^4 - T_{\text{P}}^4\right),
\end{aligned}
$$

where u is the heating intensity (control input), T_{P} is the part temperature (the system output to be controlled), and T_{A} is the ambient temperature. The parameter values were taken from (Gorinevsky, 2002): $b_u = 1000$, $c_1 = 1.1 \cdot 10^{-10}$, $c_2 = 0.8$ and $c_3 = 1.5 \cdot 10^{-9}$, as well as the desired control goal (a trajectory consisting of constant and ramp phases). T_{A} was taken to be 20 °C.

Figure 1 illustrates the solution found by the genealogical decision tree optimization procedure ($N = 5000$, $A_n = 0.03^2$, $B_n = 20^2$, $\beta = 1$). The part temperature follows closely the desired trajectory. The furnace temperature is within acceptable limits, and the control manipulations appear realizable. Overall, the results resemble those

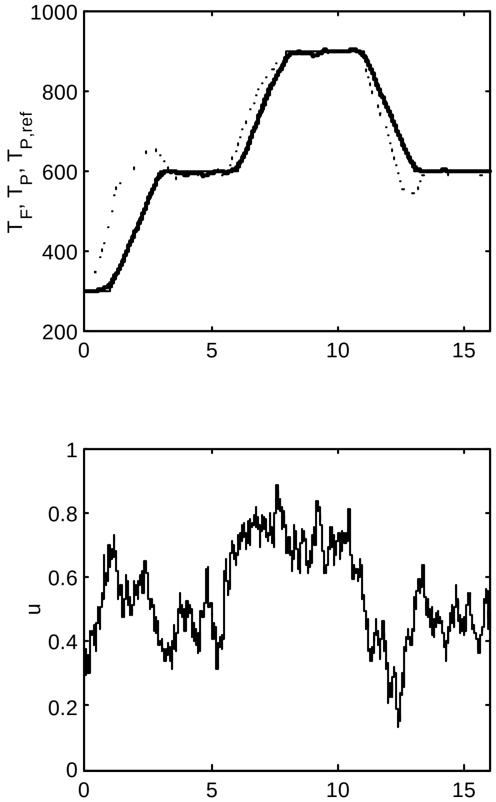

Fig. 1. Rapid thermal processing. The part temperature ($T_{\rm P}$ [°C], solid line) follows very closely the desired trajectory (thin solid line), consisting of ramps followed by constant output phases. The two lines are almost indistinguishable. The upper plot shows also the furnace temperature ($T_{\rm F}$, dashed line); the lower plot shows the sequence of control actions (heating intensity u).

obtained in (Gorinevsky, 2002) using an iterative learning scheme.

5.2 *Example 2 – Single link manipulator*

Among other areas, the genealogical decision tree control algorithm can find wide applications in robotics. Note that based on the Lagrangian and Newton–Euler formalisms, models of robots can easily be derived.

The following model for a single link manipulator was considered (Becerra, 2004)

$$\ddot{\theta} = -\frac{g}{l}\sin\theta - \frac{v}{ml^2}\dot{\theta} + \frac{1}{ml^2}u,$$

where θ is the angular position, m is the mass of the end of rod element, l is the length of the rod, v is the friction coefficient at the pivot point, and u is the applied torque at the pivot point. Let $m = 2$ kg, $l = 1$ m, $v = 6$ kgm^2/s. Denoting $x_1 := \theta$ and $x_2 := \dot{\theta}$ the model can be written in the state space form, with states x_1 and x_2, and output $y := x_1$.

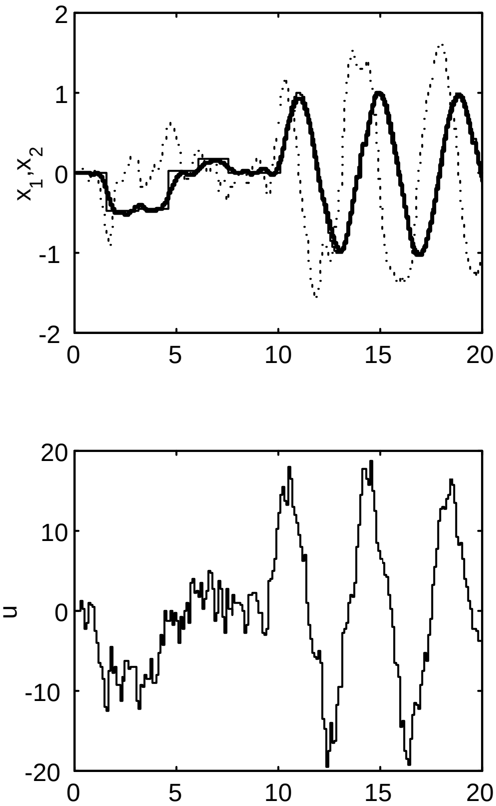

Fig. 2. Single link manipulator control. The angle of the robot arm (x_1 [rad], solid line) follows closely the desired trajectory (thin solid line), consisting of steps of random amplitude (from 0 to 10s) followed by a sinusoid wave (from 10 to 20s). The upper plot shows also the state x_2 (dashed line); the lower plot shows the sequence of control actions (applied torque).

Figure 2 illustrates the solution found by the genealogical decision tree optimization procedure ($N = 500$, $A_n = 2^2$, $B_n = 0.1^2$, $\beta = 1$). The angle of the robot arm follows closely the desired trajectory and the control actions remain within acceptable limits.

6. CONCLUSIONS

In this paper, a novel algorithm for open-loop tracking control of complex systems was presented, and illustrated by two numerical examples. This control algorithm is based on a powerful approach: the genealogical decision tree. The idea behind the control strategy consists of associating Gaussian distributions to both the norms of the control actions and the tracking errors. The resulting stochastic search technique can be interpreted as a simple genetic particle evolution model with a natural birth and death interpretation. It converges in probability.

The considered open-loop control strategy is well suited for a multitude of batch processes for which on-line sensors are not available, are too expensive, or can not be adapted for the measurement of different controlled variables. Indeed, models of these processes are often readily available, whereas the complex dynamics commonly exhibited by the models make the application of standard optimization routines unsatisfactory. Two simulation examples illustrated the approach in semiconductor manufactoring (rapid thermal processing) and robotics (single link manipulator control).

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APPENDIX

Let us assume that the plant model is given by

$$\mathbf{x}_{n+1} = f(\mathbf{x}_n, \mathbf{u}_n), \ \mathbf{y}_n = g(\mathbf{x}_n).$$

The following pseudo-code implements the algorithm:

for $n = 1 : T$

  for $i = 1 : N$

    if $n == 1$, Initialize $\mathbf{x}^i = \mathbf{x}_0$, $\mathbf{y}^i = g(\mathbf{x}_0)$ and $J_T^i = 0$; end

    Generate random $\mathbf{u}^i \sim N(\mathbf{0}, \mathbf{A}_n)$.

    Store action to a list $\mathbf{v}_n^i = \mathbf{u}^i$.

    Evaluate $J_Y = \left\|\mathbf{y}_n^{\text{ref}} - \mathbf{y}^i\right\|_{\mathbf{B}_n}^2$, $J_U = \left\|\mathbf{u}^i\right\|_{\mathbf{A}_n}^2$ and $J_T^i = J_T^i + J_Y + J_U$.

    Set weight $p_{\text{un}}^i = \exp\left(\frac{-\beta}{2} J_Y\right)$.

  end

  For all $i = 1 : N$: Compute resampling probabilities: $p^i = p_{\text{un}}^i / \sum_{i=1}^N p_{\text{un}}^i$.

  Resample: For all $i = 1 : N$: Select $\widehat{I}_i = k$ such that $\Pr(k = j) = p^j$.

  For all $i \in \widehat{I}$: Compute model for next n: $\mathbf{x}^i = f(\mathbf{x}^i, \mathbf{u}^i)$; $\mathbf{y}^i = g(\mathbf{x}^i)$.

  Death and birth. For all $j = 1 : N$: Replace $\mathbf{x}^j$, $\mathbf{y}^j$, $\mathbf{v}_n^j$ and J_T^j by $\mathbf{x}^{\widehat{I}_j}$, $\mathbf{y}^{\widehat{I}_j}$, $\mathbf{v}_n^{\widehat{I}_j}$ and $J_T^{\widehat{I}_j}$.

end

Find $i^* = \arg\min_i J_T^i$. The solution for the optimal control sequence is $\mathbf{v}_T^{i^*}$.

7. REFERENCES

Becerra, V (2004). Advanced nonlinear control (lecture notes). Technical report. http:// www.personal.rdg.ac.uk/ shs99vmb/ notes/anc/ lecture6.pdf.

Bordeneuve, J, K Najim and C Ganibal (1991). Long-range predictive control of a rapid thermal processor. *International Journal of Systems Science* **22**(12), 2377–2391.

Choi, J Y and H M Do (2001). A learning approach of wafer temperature control in rapid thermal processing system. *IEEE Transactions on Semiconductor Manufacturing* **14**(1), 1–10.

Del Moral, P (1997). Idempotency analysis and its applications. In: *Translation of Idempotency Analysis and its Applications in Optimal Control (in Russian)* (V Kolokoltsov and V Maslov, Eds.). Chap. Appendix, pp. 243–302. Kluwer.

Del Moral, P (2004). *Feynman-Kac Formulae Genealogical and Interaction Particle Systems with Applications*. Springer-Verlag. Berlin.

Edgar, T, S Butler, W Campbell, C Pfeiffer, C Bode, S Hwang, K Balakrishnan and J Hahn (2000). Automatic control in microelectronics manufacturing: Practices, challenges, and possibilities. *Automatica* **36**(11), 1567–1603.

Fayolle, F, J-P Couderc and P Duverneuil (1996). Analysis and modeling of SIPOS deposition in a hot wall tubular reactor. part i: Review of previous work and experimental study. *Chem. Vap. Deposition* **2**(6), 255–263.

Gorinevsky, D (2002). Loop-shaping for iterative control of batch processes. *IEEE Control Systems Magazine* **22**(6), 55–65.

Ikonen, E and K Najim (2002). *Advanced Process Identification and Control*. Marcel Dekker Inc. New York.

Ikonen, E, P Del Moral and K Najim (2004). A genealogical decision tree solution to optimal control problems. In: *IFAC AFNC'04*. Oulu, Finland.

Kleijn, C R (1996). Numerical modelling of chemical vapor deposition as a tool for process optimisation and reactor design. *Ultra Clean Technology* **8**(1), 5–9.

Najim, K, E Ikonen and D Aït-Kadi (2004). *Stochastic Processes: Estimation, Optimization and Analysis*. Kogan Page Ltd. London.

Norman, S A (1992). Optimization of transient temperature uniformity in RTP systems. *IEEE Transactions on Electron Devices* **39**(1), 205–207.

Schaper, C D, T Kailath and Y J Lee (1999). Decentralized control of wafer temperature for multizone rapid thermal processing systems. *IEEE Transactions on Semiconductor Manufacturing* **12**(2), 193–199.

Zalzala, A M S and P J Fleming (1997). *Genetic Algorithms in Engineering*. IEE Control Engineer Series. London.