FINITE HORIZON ROBUST MODEL PREDICTIVE CONTROL USING LINEAR MATRIX INEQUALITIES

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Abstract: In this paper, we develop a finite horizon model predictive control algorithm which is robust to model uncertainties. A moving average system matrix is constructed to capture model uncertainties and facilitate future output predictions. The paper is focused on step tracking control. Using linear matrix inequality techniques, the design is converted into a semi-definite optimization problem. Closed-loop stability is treated by adding extra terminal cost constraints. The simulation results demonstrate that the approach can be useful for practical applications.

Keywords: Model predictive control, linear matrix inequalities, robust stability, terminal cost constraints.

INTRODUCTION

Since the first version of model predictive control (MPC), known as dynamic matrix control (DMC), was published in 1978, various MPC algorithms have been developed. However, the performance and stability of MPC controllers are greatly dependent on the accuracy of process models. To overcome such a limitation, Kothare and co-workers established a comprehensive infinite horizon robust model predictive control (IH-RMPC) algorithm (Kothare *et al.*, 1996), which is superior in closed-loop stability, in 1996, and since then, IH-RMPC has drawn considerable attention (Rodrigues and Odloak, 2003; Wan and Kothare, 2003; Hu and Linnemann, 2002). Despite its popularity, the fixed infinite horizon reduces the tuning freedom of MPC schemes, and originates a potential problem — feasibility.

In this paper, we will focus on the finite horizon robust MPC (FH-RMPC) problem, namely, setting both the prediction horizon and control horizon as finite integers, still remains open. This is due to two obstacles: (1) It is difficult to perform state predictions if there exist uncertain terms in the state space matrices, specially in the matrix A of the linear discrete state space model; (2) robust stability is another barrier in FH-RMPC study (Mayne *et al.*, 2000). In this paper, a moving average system matrix, capturing model uncertainties, is constructed, and extra terminal cost constraints in the form of LMIs for closed-loop stability are imposed in the FH-RMPC formulation. Based on a property of linear matrix inequalities of uncertain matrix terms (Ghaout and Lebret, 1997; Löfberg, 2001), the FH-RMPC formulation is finally recast into a semi-definite optimization problem.

1. NOMINAL MPC USING LMIS

Consider a nominal model,

$$x(k+1) = Ax(k) + Bu(k),$$
$$y(k) = Cx(k) + Du(k), \qquad (1)$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the input vector and $y \in \mathbb{R}^q$ is the output vector. A, B, C and D are constant system matrices with compatible dimensions. To obtain step tracking control, the objective function is defined as

$$J = \sum_{i=1}^{N_p-1} \|r - y(k+i|k)\|_Q^2 + \sum_{i=0}^{N_u-1} \|u(k+i|k)\|_R^2 + \|r - y(k+N_p|k)\|_{Q_{N_p}}^2, \qquad (2)$$

where r is the given reference signal, $u(\cdot|k)$, $y(\cdot|k)$ are the predicted input and output signals over the control horizon and prediction horizon starting at instant k, and Q, R and Q_{N_p} are output, input and terminal positive semi-definite weightings. Both N_p and N_u are integers, which represent the prediction horizon and control horizon, respectively. The norms in J are defined as

$$\|r - y(k+i|k)\|_Q^2 = (r - y(k+i|k))^T Q (r - y(k+i|k)),$$

similarly for the other ones. Based on the model in (1), the predicted states, $x(k+i|k)$ can be calculated by:

$$A^i x(k) + A^{i-1} Bu(k|k) + \cdots + Bu(k+i-1|k), \qquad (3)$$

where $u(k+i|k) \equiv u(k+N_u-1|k)$ if $N_u < i \le N_p$. Rewrite the objective function in (2) into the augmented matrix form described in (Maciejowski, 2002)

$$J = \|(\mathcal{T} - \mathcal{Y}(k))\|_{\mathcal{Q}}^2 + \|\mathcal{U}(k)\|_{\mathcal{R}}^2, \qquad (4)$$

where the augmented vectors are defined as follows

$$\mathcal{U}(k) = \left[\, u^T(k|k) \ \cdots \ u^T(k+N_u-1|k) \,\right]^T,$$
$$\mathcal{Y}(k) = \left[\, y^T(k+1|k) \ \cdots \ y^T(k+N_p|k) \,\right]^T,$$
$$\mathcal{T} = \left[\, r^T \ \cdots \ r^T \,\right]^T; \qquad (5)$$

the augmented weightings are

$$\mathcal{Q} = \mathrm{diag}\left(Q, \cdots, Q, Q_{N_p}\right), \ \mathcal{R} = \mathrm{diag}(R, \cdots, R). \qquad (6)$$

Inserting the predicted states in (3) into (1), setting $i = 1$ to $i = N_p$ iteratively, and then utilizing the augmented vectors and weightings in (5) and (6), the predicted output sequence $\mathcal{Y}(k)$ can be also expressed in the term of the state measurement $x(k)$, i.e.,

$$\mathcal{Y}(k) = \mathcal{C}(\mathcal{A}x(k) + \mathcal{B}\mathcal{U}(k)) + \mathcal{D}\mathcal{U}(k), \qquad (7)$$

where

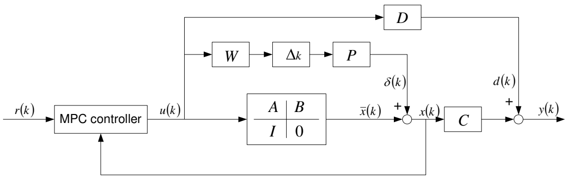

Fig. 1. Framework of the FH-RMPC system

$$\mathcal{A} = \left[A^T \ \cdots \ \left(A^{N_u}\right)^T \ \cdots \ \left(A^{N_p}\right)^T \right]^T,$$

$$\mathcal{B} = \begin{bmatrix} B & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ A^{N_u-1}B & A^{N_u-2}B & \cdots & B \\ \vdots & \vdots & \vdots & \vdots \\ A^{N_p-1}B & A^{N_p-2}B & \cdots & A^{N_p-N_u}B + \\ & & & \cdots + B \end{bmatrix},$$

$$\mathcal{C} = \begin{bmatrix} C & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & C \end{bmatrix}, \text{ and } \mathcal{D} = \begin{bmatrix} 0 & D & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & D \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & D \end{bmatrix}. \qquad (8)$$

Substituting (7) into (4), taking advantage of an auxiliary positive scalar t, and applying Schur complements, the nominal MPC controller can be solved by minimizing a linear objective function, i.e.,

$$J_o = \min_{t,\, \mathcal{U}(k)} t,$$

subject to

$$t > 0,$$

$$\begin{bmatrix} t & (\mathcal{T} - (\mathcal{C}\mathcal{A}x(k) + (\mathcal{C}\mathcal{B} + \mathcal{D})\mathcal{U}(k)))^T & \mathcal{U}^T(k) \\ * & \mathcal{Q}^{-1} & 0 \\ * & * & \mathcal{R}^{-1} \end{bmatrix} \ge 0, \qquad (9)$$

where the symbol "$*$" indicates symmetric terms in the matrix, and $x(k)$ is the state measurement at instant k.

2. FH-RMPC

How to configure a system framework to represent the influence of model uncertainties on controller design as well as facilitate state/output predictions, is the key point in the FH-RMPC synthesis.

2.1 Framework for model uncertainties

Fig. 1 shows the framework adopted by this paper. It is composed of the nominal model paralleling the model uncertainty block, and in the form of input to state, and then to output. In Fig. 1,

Δ_k stands for the model uncertainties over the prediction horizon starting at instant k, i.e.,

$$\Delta_k = \begin{bmatrix} \Delta_k(k,k) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ \Delta_k(k+N_p,k) & \cdots & \Delta_k(k+N_p,k+N_p) \end{bmatrix}, \tag{10}$$

with $\|\Delta_k\| = \bar{\sigma}(\Delta_k) \le 1$, i.e., the maximal singular value of Δ_k is less than 1. W and P are weighting matrices. To simplify the FH-RMPC formulation, we assume that C is known precisely, $x(k)$, the state measurement at instant k, is fully measurable, and the predicted state values are independent of the previous model uncertainties due to the monotonicity of the prediction horizon. Taking advantage of such an assumption, the controller design may be significantly simplified.

2.2 FH-RMPC formulation

Based on the uncertainty block defined in (10), we do the state predictions. The key point is exploiting the monotonicity of the prediction horizon, say, at every prediction horizon starting at instant k, predictions are independent of the previous horizon uncertainty block Δ_{k-1}.

The nominal model is given by

$$\bar{x}(k+i+1|k) = A\bar{x}(k+i|k) + Bu(k+i|k), \tag{11}$$

where $\bar{x}(\cdot)$ denotes the nominal value, corresponding to the real state $x(\cdot)$, and the uncertain term $\delta(k)$ caused by model uncertainties can be computed from

$$\delta(k+i|k) = \sum_{j=k}^{k+i} \hat{\Delta}(k+i,j)\, u(j|k), \tag{12}$$

where the uncertainty matrix $\hat{\Delta}$ is defined, for convenience, as

$$\hat{\Delta} = P\Delta_k W.$$

From (11) and (12), we can derive the real state space model for state/output predictions,

$$\begin{aligned} x(k+1+i|k) &= Ax(k+i|k) + Bu(k+i|k) \\ &\quad + \sum_{j=k}^{k+1+i} \hat{\Delta}(k+1+i,j)\, u(j|k) \\ &\quad - A\sum_{j=k}^{k+i} \hat{\Delta}(k+i,j)\, u(j|k), \quad (13) \\ y(k+i|k) &= Cx(k+i|k) + Du(k+i|k). \quad (14) \end{aligned}$$

Without loss of generality, the uncertainty block Δ_k is presumed strictly causal, i.e., it is lower triangular matrix with the identical zero diagonal. Consequently, $\hat{\Delta}(k,k) = 0$ because weightings P and W are block diagonal matrices. From (13), the predicted state $x(k+i|k)$ can be calculated by:

$$\begin{aligned} &A^i x(k) + A^{i-1}Bu(k|k) + \cdots + Bu(k+i-1|k) \\ &+ \sum_{j=k}^{k+i} \hat{\Delta}(k+i,j)\, u(j|k), \end{aligned} \tag{15}$$

where $u(k+i|k) \equiv u(k+N_u-1|k)$ if $N_u \le i \le N_p$. In the same fashion of state predictions in nominal MPC schemes, the above predicted states and outputs can be represented in the form of augmented matrices,

$$\begin{aligned} \mathcal{X}(k) &= \mathcal{A}x(k) + \mathcal{B}\mathcal{U}(k) + M_l P\Delta_k W M_r \mathcal{U}(k), \\ \mathcal{Y}(k) &= \mathcal{C}\mathcal{X}(k) + \mathcal{D}\mathcal{U}(k), \end{aligned} \tag{16}$$

where constant matrices M_l and M_r are defined as the left- and right-multipliers of the uncertainty block $\hat{\Delta}$,

$$M_l = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & I & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & I \end{bmatrix}, \text{ and } M_r = \begin{bmatrix} I & 0 & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & I & 0 \\ 0 & \cdots & 0 & I \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & I \end{bmatrix}, \tag{17}$$

and $\mathcal{X}(k)$ is for the augmented, predicted state vector,

$$\mathcal{X}(k) = \left[\, x^T(k+1|k) \;\cdots\; x^T(k+N_p|k) \,\right]^T.$$

Therefore, the approach to the nominal MPC scheme for step tracking control, could be extended into FH-RMPC cases:

The finite horizon robust MPC system can be represented by its corresponding nominal model in parallel with a weighed unity-norm uncertainty block. Based on such a framework, robust step tracking control, say, step tracking in the presence of model uncertainties, can be achieved by solving a robust semi-definite optimization problem (if solutions exist) whose constraints contain uncertain matrix terms:

$$J_o = \min_{t,\, \mathcal{U}(k)} t, \tag{18}$$

subject to

$$t > 0,$$

$$\begin{aligned} &\max_{\Delta_k} J \le t \;\; (with \;\; \|\Delta_k\| = \bar{\sigma}(\Delta_k) \le 1), \\ &J = \|T - \mathcal{Y}(k)\|_{\mathcal{Q}} + \|\mathcal{U}(k)\|_{\mathcal{R}}, \\ &\mathcal{X}(k) = \mathcal{A}x(k) + \mathcal{B}\mathcal{U}(k) + M_l P\Delta_k W M_r \mathcal{U}(k), \\ &\mathcal{Y}(k) = \mathcal{C}\mathcal{X}(k) + \mathcal{D}\mathcal{U}(k). \end{aligned} \tag{19}$$

2.3 FH-RMPC using LMIs

Due to the presence of model uncertainties, Eq. (19) comprises the uncertain terms of Δ_k. Therefore, we cannot apply Schur complements directly, and use existing software packages to solve it numerically. In order to overcome such a barrier, we first introduce the following Lemma:

Lemma 1. (Ghaout and Lebret, 1997; Löfberg, 2001) Let $T_1 = T_1^T$, and T_2, T_3, T_4 be real matrices of appropriate sizes. Then $\det(I - T_4\Delta) \neq 0$ and

$$T_1 + T_2\Delta\left(I - T_4\Delta\right)^{-1} T_3 + T_3^T\left(I - T_4\Delta\right)^{-T}\Delta^T T_2^T \geq 0 \tag{20}$$

for every Δ, $\|\Delta\| = \bar{\sigma}(\Delta) \leq 1$, if and only if $\|T_4\| < 1$ and there exists a scalar $\tau \geq 0$ such that

$$\begin{bmatrix} T_1 - \tau T_2 T_2^T & T_3^T - \tau T_2 T_4^T \\ T_3 - \tau T_4 T_2^T & \tau\left(I - T_4 T_4^T\right) \end{bmatrix} \geq 0.$$

Theorem 2. The robust step tracking performance for the MPC system in Fig. 1 is achievable if the following semi-definite optimization problem is solvable:

$$J_o = \min_{t,\, \mathcal{U}(k),\, \tau} t,$$

subject to

$$t > 0,\ \tau \geq 0,$$

and

$$\begin{bmatrix} t & (T - \mathcal{CA}x(k) - (\mathcal{CB} + \mathcal{D})\mathcal{U}(k))^T & \mathcal{U}^T(k) & (WM_r\mathcal{U}(k))^T \\ * & \mathcal{Q}^{-1} - \tau\mathcal{C}M_lP(\mathcal{C}M_lP)^T & 0 & 0 \\ * & * & \mathcal{R}^{-1} & 0 \\ * & * & * & \tau I \end{bmatrix} \geq 0, \tag{21}$$

where I is an identity matrix. The augmented reference input T, predicted input sequence $\mathcal{U}(k)$ and weighting matrices $\mathcal{Q}$, $\mathcal{R}$, are defined in (5) and (6). The constant augmented matrices $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, and left-, right- matrices M_l, M_r of the uncertainty block Δ_k in Fig. 1 are introduced in (8) and (17).

PROOF. Applying Schur complements and rewriting constraints in (19), we have

$$\begin{bmatrix} t & (T - \mathcal{CA}x(k) - (\mathcal{CB} + \mathcal{D})\mathcal{U}(k))^T - (\mathcal{C}M_lP\Delta_kWM_r\mathcal{U}(k))^T & \mathcal{U}^T(k) \\ * & \mathcal{Q}^{-1} & 0 \\ * & * & \mathcal{R}^{-1} \end{bmatrix} \geq 0. \tag{22}$$

Separating the certain and uncertain terms of (22) and defining

$$T_1 = \begin{bmatrix} t & (T - \mathcal{CA}x(k) - (\mathcal{CB} + \mathcal{D})\mathcal{U}(k))^T & \mathcal{U}^T(k) \\ * & \mathcal{Q}^{-1} & 0 \\ * & * & \mathcal{R}^{-1} \end{bmatrix},$$

$$T_2 = \begin{bmatrix} 0 \\ -\mathcal{C}M_lP \\ 0 \end{bmatrix},\ T_3 = \begin{bmatrix} WM_r\mathcal{U}(k) & 0 & 0 \end{bmatrix},$$

and $T_4 = 0$. (23)

Now that (22) is equivalently rewritten into the form of (20), *Theorem 2* is then proved. □

3. TERMINAL COST CONSTRAINTS

In 1988, Keerthi and Gilbert first proposed a method which employed the objective function of MPC system as a Lyapunov function to solve the nominal stability problem (Keerthi and Gilbert, 1988). In this paper, we will employ a similar idea and develop terminal cost constraints to guarantee robust stability of FH-RMPC systems.

3.1 LMIs for terminal cost constraints

In order to analyze closed-loop stability, we presume that the reference input r is equal to zero. Then the objective function J in (19) can be rewritten as:

$$J = \sum_{j=1}^{N_p-1} \|x(k+j|k)\|_{Q'}^2 + \|u(k+j|k)\|_R^2 + \|\hat{x}(k+N_p|k)\|_{Q'_{N_p}}^2, \tag{24}$$

where

$$Q' = C'QC \text{ and } Q'_{N_p} = C'Q_{N_p}C.$$

Without loss of generality, here we set $N_p = N_u$, otherwise we can enforce the terminal input $u(k+N_u-1|k) = 0$ to resume the following derivation.

Consider a quadratic function

$$V(x) = x^T\Phi x,\ \Phi > 0, \tag{25}$$

satisfying

$$V(x(k+i+1|k)) - V(x(k+i|k)) < -\left(\|x(k+i|k)\|_{Q'}^2 + \|u(k+i|k)\|_R^2\right). \tag{26}$$

Summing (26) from $i = 0$ to $i = N_p$, we get

$$V(x(k+N_p|k)) - V(x(k)) < -J - \|x(k)\|_{Q'}^2 + \|x(k+N_p|k)\|_{Q'_{N_p}}^2.$$

In the sequel, we will employ $V(x(k))$ as a Lyapunov function, satisfying

$$V(x(k)) > \|x(k)\|_{Q'}^2 - \|x(k+N_p|k)\|_{Q'_{N_p}}^2 + V(x(k+N_p|k)) + t, \tag{27}$$

where t is the upper bound of the cost J defined in (19). Then if $\tilde{V}(k) : \mathbb{R}^n \to \mathbb{R}$, the difference of Lyapunov functions of $x(k+1)$ and $x(k)$, is decreasing, i.e.,

$$\begin{aligned}\tilde{V}(k) :&= V(x(k+1)) - V(x(k)) \\ &< \|x(k+1)\|_{\Phi}^2 - t - \|x(k)\|_{Q'}^2 + \\ &\quad \|x(k+N_p|k)\|_{Q'_{N_p}}^2 - \|x(k+N_p|k)\|_{\Phi}^2 \\ &< 0,\end{aligned} \tag{28}$$

closed-loop asymptotic stability of the resulting FH-RMPC system can be guaranteed. From (15), it can be seen that

$$x(k+1) = \left(A + BF_k + \hat{\Delta}(k+1,k)F_k\right)x(k), \tag{29}$$

where F_k is defined as a feedback gain satisfying $u(k|k) = F_k x(k)$. Then, introduce two constant matrices E_1 and E_2 such that

$$\hat{\Delta}(k+1,k) = E_1\hat{\Delta}E_2 = E_1 M_l P \Delta_k W M_r E_2, \tag{30}$$

where

$$E_1 = \begin{bmatrix}0 & I & 0 & \cdots & 0\end{bmatrix}, \text{ and } E_2 = \begin{bmatrix}I & 0 & \cdots & 0\end{bmatrix}^T.$$

Inserting (29) and (30) into (28), we get

$$\left\|\left(A + BF_k + \hat{\Delta}(k+1,k)F_k\right)x(k)\right\|_{\Phi}^2 - \|x(k)\|_{Q'}^2 - t + \|x(k+N_p|k)\|_{\left(Q'_{N_p}-\Phi\right)}^2 < 0. \tag{31}$$

So if the following two inequalities

$$-\left\|\left(A + BF_k + \hat{\Delta}(k+1,k)F_k\right)x(k)\right\|_{\Phi} + \|x(k)\|_{Q'}^2 + t > 0, \tag{32}$$

$$\Phi - Q'_{N_p} \geq 0, \tag{33}$$

hold simultaneously, we can guarantee the condition in (31). Applying Schur complements and the property of robust LMIs (*Lemma 1*), (32) is recast into:

$$\begin{bmatrix} x^T(k)Q'x(k) + t & * & * \\ (A+BF_k)x(k) & X - \lambda_1 E_1 M_l P (E_1 M_l P)^T & * \\ W M_r E_2 F_k x(k) & 0 & \lambda_1 I \end{bmatrix} > 0, \tag{34}$$

where $X = \Phi^{-1}$ and λ_1 is a positive scalar. Then left- and right-multiplying X to both sides of inequality (33) and defining a small positive scale κ which is selected to satisfy the invertibility of $\left(Q'_{N_p} + \kappa I\right)$, we have

$$X - X\left(Q'_{N_p} + \kappa I\right)X \geq 0 \tag{35}$$

It is obvious that if $\kappa \to 0$, (35) is equivalent to (33). Apply Schur complements to Eq. (35) and derive

$$\begin{bmatrix} X & X \\ X & \left(Q'_{N_p} + \kappa I\right)^{-1}\end{bmatrix} \geq 0. \tag{36}$$

Combined with (36), (34) forms a sufficient condition to (28), which is designed for asymptotical stability of the closed-loop FH-RMPC system.

Moreover, in order to use $V(x(k))$ as a Lyapunov function candidate, in the sequel, we will manage to derive another LMI to guarantee (27). To this end, taking advantage of the condition in (33), we can derive a sufficient condition to (27),

$$\|x(k)\|_{Q_{N_p}}^2 - t - \|x(k)\|_{Q'}^2 - \|x(k+N_p|k)\|_{\Phi}^2 > 0. \tag{37}$$

From (15), $x(k+N_p|k)$ is expressed as:

$$x(k+N_p|k) = A^{N_p}x(k) + E_3\mathcal{B}\mathcal{U}(k) + E_3 M_l P \Delta_k W M_r \mathcal{U}(k), \tag{38}$$

where $E_3 = \begin{bmatrix}0 \cdots & 0 & 0 & I\end{bmatrix}$. Therefore, applying Schur complements and using the property of robust LMIs, (37) is equivalent to

$$\begin{bmatrix} x^T(k)Q'_{N_p}x(k) - x^T(k)Q'x(k) - t & * & * \\ A^{N_p}x(k) + E_3\mathcal{B}\mathcal{U}(k) & X - \lambda_2 E_3 M_l P (E_3 M_l P)^T & * \\ W M_r \mathcal{U}(k) & 0 & \lambda_2 I \end{bmatrix} > 0, \tag{39}$$

where λ_2 is a positive scalar.

Theorem 3. To achieve step tracking performance for the FH-RMPC system defined in Fig. 1, the state feedback matrix F_k in the control law $u(k|k) = F_k x(k)$, $k > 0$, can be obtained by minimizing the upper bound of the objective function J in (19), i.e., solving the following optimization problem:

$$J_o = \min_{\mathcal{U}(k),\ F_k} t,$$

subject to (21), (34), (36) and (39), where X, λ_1 and λ_2 are variables of LMIs for terminal cost constraints. The closed-loop system is guaranteed asymptotically stable if the optimal input sequence $\mathcal{U}^o(k)$ does exist.

4. SIMULATION EXAMPLE

Consider a classical angular positioning system,

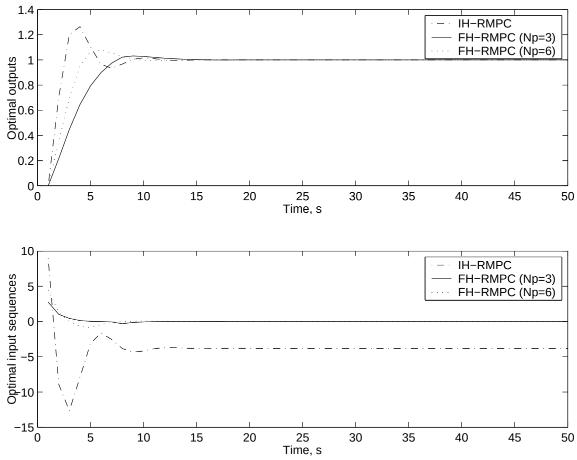

Fig. 2. IH-RMPC controller and FH-RMPC controllers.

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0.1 \\ 0 & 1-0.1\alpha \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 0.787 \end{bmatrix} u(k),$$
$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(k), \qquad (40)$$

where $\alpha \in [0.1,\ 10]$ is the uncertain term. Based on approaches discussed in (Kothare *et al.*, 1996), an IH-RMPC controller for the model uncertainties in the form of the structured feedback loop and two FH-RMPC controllers, are first designed. Comparing control results, it can be seen that FH-RMPC controllers have better tracking performance and smaller overshoot of the optimal input sequences (Fig. 2). Here the tuning parameters are selected as: $r = 1$, $Q = I$, $Q_{N_p} = I$, $R = 0.00002I$, $P = I$, $N_u = 3$ and $W = 0.1$. The simulation length equals to 50. For the simulations presented in Fig. 2, $\alpha = 0.7$.

Fig. 3 demonstrates the influence of the imposed terminal cost constraints on the system performance with the different terminal weighting Q_{N_p}. Here we set $\alpha = 0.9$, $N_p = 3$ and $N_u = 3$. In the figures, solid lines are derived from *Theorem 2*, and dash-dotted lines from *Theorem 3*.

5. CONCLUSION

In this paper, we have discussed finite horizon robust model predictive control (FH-RMPC) problems. Two topics were covered: how to achieve robust step tracking control based on an FH-RMPC scheme, and the closed-loop stability analysis of the resulting FH-RMPC system. Taking advantage of the property of robust LMIs, whose constraints have uncertainty terms, the conventional *min-max* problem was converted into a standard semi-definite optimization problem. Comparing with the infinite horizon model predictive control, FH-RMPC has more tuning freedom, better control performance, and faster online implementation.

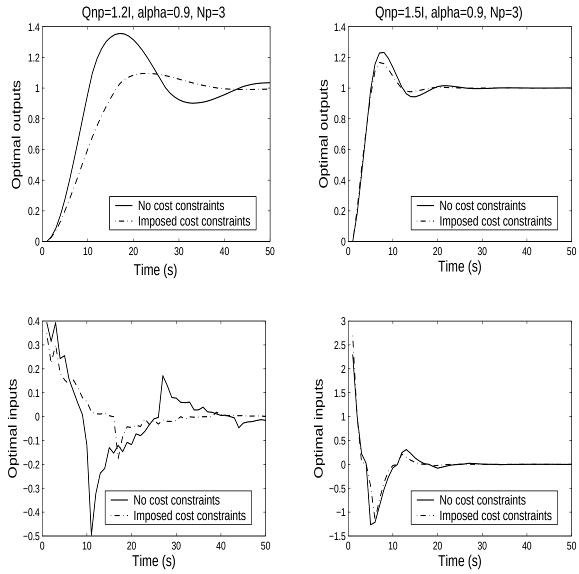

Fig. 3. Influence of terminal cost constraints with terminal weightings.

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