NONLINEAR OBSERVER DESIGN FOR LATERAL VEHICLE DYNAMICS

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Abstract: The vehicle sideslip angle (VSSA) is determined using a nonlinear observer with Adaption of a Quality Function. The observer design is based on an adapted nonlinear double track model. By validation with real measurement data, the model accuracy is proven to be sufficient for observer design. The observer is derived and validated with real measurement data of representative test drives. It is shown that the observer is capable to determine the VSSA with high accuracy up to the stability limit of the vehicle.

Keywords: Automotive control, nonlinear models, state estimation, state observers, vehicle dynamics

1. INTRODUCTION

Describing the deviation between the vehicle's longitudinal axis and its direction of motion, the vehicle sideslip angle (VSSA) is a key variable in vehicle dynamics. In electronic control systems like the Electronic Stability Program (ESP) or the Dynamic Stability Control (DSC) the VSSA is used as a control reference. However, the VSSA cannot directly be measured with standard sensors. Several approaches can be found in literature for the estimation of the VSSA by means of state space observers. In (Stéphant *et al.*, 2004) and (Tseng, 2002) a bicycle model is used as a basis for the observer design. For small lateral acceleration these observers show good results, for larger lateral acceleration, however, the bicycle model is no longer capable to describe the VSSA properly. Consequently the observers do not provide a good estimation any more.

In this paper an adaptive nonlinear double track model is introduced. Parameters crucial for lateral vehicle dynamics such as the cornering stiffnesses are adapted according to the driving situation. Since the Observer with Adaption of a Quality Function (AQF-Observer) is restricted to systems of a specific structure, the vehicle model is restructured accordingly. The restructured vehicle model is validated in the paper and it is proven that the model is capable to describe the vehicle dynamics up to the stability limit with an accuracy sufficient for nonlinear observer design. Then, the Observer with Adaption of a Quality Function is derived. A validation with real measurement data of representative test drives shows that the observer is capable to estimate the VSSA with high accuracy up to the stability limit.

2. VEHICLE MODEL

In order to describe the vehicle dynamics up to the stability limit, a nonlinear double track model is derived. Fig. 1 shows the vehicle model including the most important forces and vehicle parameters.

The Center of Gravity (CoG) as well as the lateral and longitudinal vehicle axis are regarded to be on

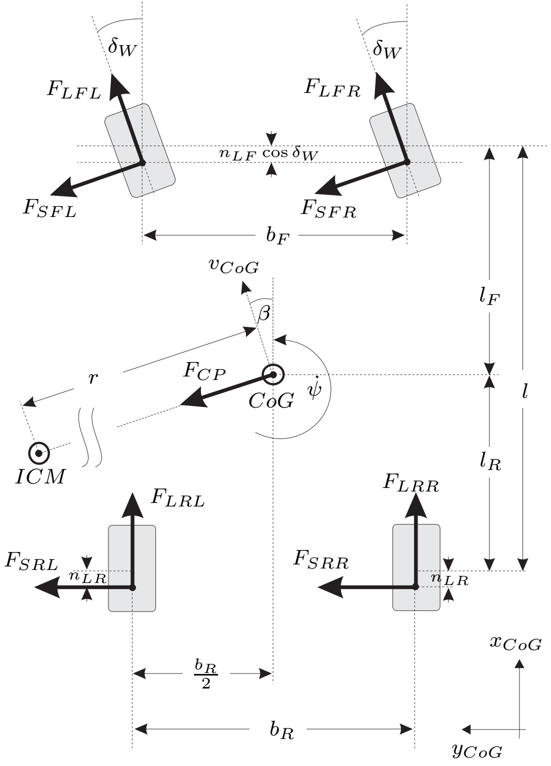

Fig. 1. Nonlinear double track model

the road surface. F_{LFL}, F_{LFR}, F_{LRL} and F_{LRR} are the longitudinal forces at the front left (FL), front right (FR), rear left (RL) and rear right (RR) wheel, F_{Sij} the lateral forces accordingly. F_{CP} is the centripetal force, F_{WX} the wind force and n_{LF}, n_{LR} are the wheel casters (see Fig. 1). The yaw rate $\dot{\psi}$ describes the rotation around the vertical axis. The VSSA β is the deviation between the velocity in the center of gravity v_{CoG} and the vehicle's longitudinal axis.

According to Fig. 1 the force balances in the direction of the longitudinal and lateral axis as well as the torque balance around the vertical axis yield:

$$m_{CoG}\dot{v}_{CoG}\cos\beta = \sum F_L - F_{CP}\sin\beta \qquad (1)$$

$$m_{CoG}\dot{v}_{CoG}\sin\beta = \sum F_S + F_{CP}\cos\beta \qquad (2)$$

$$\begin{aligned} J_Z\ddot{\psi} = &(F_{LFL}+F_{LFR})(l_F - n_{LF}\cos\delta_W)\sin\delta_W \\ &+ (F_{SFL}+F_{SFR})(l_F - n_{LF}\cos\delta_W)\cos\delta_W \\ &- (F_{SRL}+F_{SRR})\cdot(l_R + n_{LR}) \\ &+ (F_{LFR}-F_{LFL})\cdot\cos\delta_W\cdot\frac{b_F}{2} \\ &- (F_{SFR}-F_{SFL})\cdot\sin\delta_W\cdot\frac{b_F}{2} \\ &+ (F_{LRR}-F_{LRL})\cdot\frac{b_R}{2} \end{aligned} \qquad (3)$$

with

$$\begin{aligned} \sum F_L = &(F_{LFL}+F_{LFR})\cos\delta_W + F_{LRL} \\ &- (F_{SFL}+F_{SFR})\sin\delta_W + F_{LRR} + F_{WX} \end{aligned}$$

and

$$\begin{aligned} \sum F_S = &(F_{LFL}+F_{LFR})\sin\delta_W + F_{SRL} \\ &+ (F_{SFL}+F_{SFR})\cos\delta_W + F_{SRR}\,. \end{aligned}$$

According to (Kiencke and Nielsen, 2000), the centripetal force F_{CP} holds

$$F_{CP} = -\frac{m_{CoG}v_{CoG}^2}{r} = -\,m_{CoG}\,v_{CoG}\,(\dot{\beta}+\dot{\psi})\,,$$

and the wind force F_{WX} holds

$$F_{WX} = c_{aer}A_L\frac{\rho}{2}v_{CoG}^2\,,$$

with c_{aer} being the coefficient of the aerodynamic drag, A_L the front vehicle area and ρ the air density. Inserting these two equations into the two force balances (1) and (2), the time derivatives $\dot{v}_{CoG}$ and $\dot{\beta}$ can be isolated:

$$\dot{v}_{CoG} = \frac{1}{m_{CoG}}\left(\cos\beta\cdot\sum F_L + \sin\beta\cdot\sum F_S\right) \qquad (4)$$

$$\begin{aligned} \dot{\beta} = &\frac{1}{m_{CoG}v_{CoG}}\left(\cos\beta\cdot\sum F_S - \sin\beta\cdot\sum F_L\right) \\ &-\dot{\psi}\,. \end{aligned} \qquad (5)$$

The lateral wheel forces F_{Sij} can be expressed by the linear relation between the cornering stiffnesses c_{ij} and the tire side slip angle (TSSA) α_{ij}

$$F_{Sij} = c_{ij}\cdot\alpha_{ij}\,, \qquad (6)$$

with

$$\alpha_{Fj} = \delta_W - \beta - \frac{l_F\cdot\dot{\psi}}{v_{CoG}} \qquad (7)$$

$$\alpha_{Rj} = -\beta + \frac{l_R\cdot\dot{\psi}}{v_{CoG}}\,, \qquad (8)$$

see (Kiencke and Nielsen, 2000). For changing wheel loads and large TSSA the relation is no longer linear, though. To obtain an accurate model even at high lateral acceleration the cornering stiffnesses are therefore adapted. In (Hiemer *et al.*, 2004) the lateral forces are approximated by the function

$$F_{Sij}(t) = \left(1 - \frac{F_{Zij}(t)}{k_1}\right)F_{Zij}(t)\arctan\big(k_2\alpha_{ij}(t)\big)$$

in dependence on the current wheel load $F_{Zij}(t)$ and the TSSA $\alpha_{ij}(t)$. The two parameters k_1 and k_2 are determined by nonlinear least squares techniques (see (Hiemer *et al.*, 2004)). The current cornering stiffnesses then hold:

$$c_{ij}(t) = \frac{F_{Sij}(t)}{\alpha_{ij}(t)}\,. \qquad (9)$$

Under consideration of Eqns. (7) and (8) the two equations for $\dot{v}_{CoG}$ and $\dot{\beta}$ (4) and (5) as well as the torque balance (3) yield three differential equations for the vehicle velocity $\dot{v}_{CoG}$, the VSSA β and the yaw rate $\dot{\psi}$:

$$\begin{aligned}\dot{v}_{CoG} = &\frac{1}{m_{CoG}} \cdot \Big\{(F_{LFL} + F_{LFR})\cos(\delta_W - \beta)\\ &-(c_{FL} + c_{FR})\left(\delta_W - \beta - \frac{l_F\,\dot{\psi}}{v_{CoG}}\right)\sin(\delta_W - \beta)\\ &+\left(F_{LRL} + F_{LRR} - c_{aer}A_L\frac{\rho}{2}\cdot v_{CoG}^2\right)\cdot\cos\beta\\ &+(c_{RL} + c_{RR})\cdot\left(-\beta + \frac{l_R\,\dot{\psi}}{v_{CoG}}\right)\cdot\sin\beta\Big\}\end{aligned} \tag{10}$$

$$\begin{aligned}\dot{\beta} = &\frac{1}{m_{CoG}v_{CoG}} \cdot \Big\{(F_{LFL} + F_{LFR})\sin(\delta_W - \beta)\\ &+(c_{FL} + c_{FR})\cdot\left(\delta_W - \beta - \frac{l_F\,\dot{\psi}}{v_{CoG}}\right)\cdot\cos(\delta_W - \beta)\\ &+(c_{RL} + c_{RR})\cdot\left(-\beta + \frac{l_R\,\dot{\psi}}{v_{CoG}}\right)\cdot\cos\beta\\ &-(F_{LRL} + F_{LRR} - c_{aer}A_L\frac{\rho}{2}\cdot v_{CoG}^2)\cdot\sin\beta\Big\}\\ &-\dot{\psi}\end{aligned} \tag{11}$$

$$\begin{aligned}\ddot{\psi} = &\frac{1}{J_Z}\Big\{(l_F - n_{LF}\cos\delta_W)(F_{LFL} + F_{LFR})\sin\delta_W\\ &+\left(\delta_W - \beta - \frac{l_F\cdot\dot{\psi}}{v_{CoG}}\right)\cos\delta_W\cdot(c_{FL} + c_{FR})\\ &\qquad\cdot(l_F - n_{LF}\cos\delta_W)\\ &+\frac{b_F}{2}\cdot(F_{LFR} - F_{LFL})\cos\delta_W\\ &-\frac{b_F}{2}\cdot(c_{FR} - c_{FL})\cdot\left(\delta_W - \beta - \frac{l_F\cdot\dot{\psi}}{v_{CoG}}\right)\cdot\sin\delta_W\\ &-(l_R + n_{LR})\cdot(c_{RL} + c_{RR})\cdot\left(-\beta + \frac{l_R\,\dot{\psi}}{v_{CoG}}\right)\\ &+\frac{b_R}{2}(F_{LRR} - F_{LRL})\Big\}.\end{aligned} \tag{12}$$

The three differential equations (10) - (12) represent a nonlinear state space model

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}) \tag{13}$$

with three state space variables

$$\mathbf{x} = \left[\, v_{CoG}\ \beta\ \dot{\psi}\,\right]^T$$

and five input variables

$$\mathbf{u} = \left[\, F_{LFL}\ F_{LFR}\ F_{LRL}\ F_{LRR}\ \delta_W\,\right]^T .$$

Two of the three state space variables can be measured, the vehicle velocity and the yaw rate. They represent the output variables

$$\mathbf{y} = \left[\, v_{CoG}\ \dot{\psi}\,\right]^T .$$

Since the Observer with Adaption of a Quality Function is restricted to models with a specific structure, the nonlinear double track model (13) has to be restructured first.

2.1 Restructuring of the nonlinear double track model

For the observer design the underlying process model has to hold the specific structure

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{A}(\mathbf{y}, \mathbf{u})\,\mathbf{x} + \mathbf{b}(\mathbf{y}, \mathbf{u}),\\ \mathbf{y} &= \mathbf{C}\,\mathbf{x}\end{aligned} \tag{14}$$

with $\mathbf{u}$ and $\mathbf{y}$ being the measured inputs and outputs. The variables $\mathbf{x}$ represent the unknown state space variables to be determined.

In order to restructure the nonlinear double track model (13) accordingly, the differential equation (10) to (12) for the three state space variables are linearized with respect to the unknown VSSA β. Equation (12) for the yaw rate is already linear in β. The effect of the linearization of the other to equations for v_{CoG} and $\dot{\psi}$ was analyzed by means of simulations for several representative test drives. For the VSSA the linearized and the original nonlinear function are almost identical. For the velocity, however, there are significant deviations. Consequently, the velocity is no longer regarded as a state space variable but as an input variable. Then, the corresponding differential equation is no longer required and the system order reduces from $n = 3$ to $n = 2$.

The restructured nonlinear double track model reads

$$\begin{aligned}\dot{\mathbf{x}} &= \underbrace{\begin{bmatrix} a_{11}(\dot{\psi}, \mathbf{u}^*) & a_{12}(\mathbf{u}^*) \\ a_{21}(\mathbf{u}^*) & a_{22}(\mathbf{u}^*)\end{bmatrix}}_{\mathbf{A}(y, \mathbf{u}^*)}\cdot\mathbf{x} + \underbrace{\begin{bmatrix} b_1(\mathbf{u}^*) \\ b_2(\mathbf{u}^*)\end{bmatrix}}_{\mathbf{b}(\mathbf{u}^*)},\\ y &= \dot{\psi} = \left[\,0\ \ 1\,\right]\cdot\mathbf{x} = \mathbf{C}\cdot\mathbf{x}\end{aligned} \tag{15}$$

with two state space variables

$$\mathbf{x} = \left[\,\beta\ \dot{\psi}\,\right]^T$$

and six input variables

$$\mathbf{u}^* = \left[\, F_{LFL}\ F_{LFR}\ F_{LRL}\ F_{LRR}\ \delta_W\ v_{CoG}\,\right]^T .$$

The elements of $\mathbf{A}$ and $\mathbf{b}$ are

$$a_{11}(\dot{\psi}, \mathbf{u}^*) = \frac{1}{m_{CoG} v_{CoG}} \Big\{ (c_{FL} + c_{FR})[-\cos\delta_W + \sin\delta_W (\delta_W - \frac{l_F\,\dot{\psi}}{v_{CoG}})] - (c_{RL} + c_{RR}) - (F_{LRL} + F_{LRR} - c_{aer} A_L \frac{\rho}{2} v_{CoG}^2) \Big\} - (F_{LFL} + F_{LFR}) \cos\delta_W \tag{16}$$

$$a_{12}(\mathbf{u}^*) = \frac{1}{m_{CoG} v_{CoG}^2} \Big\{ l_R (c_{RL} + c_{RR}) - l_F \cos\delta_W (c_{FL} + c_{FR}) \Big\} - 1 \tag{17}$$

$$a_{21}(\mathbf{u}^*) = \frac{1}{J_Z} \Big\{ -\frac{b_F}{2} \sin\delta_W (c_{FL} - c_{FR}) - (c_{FL} + c_{FR})(l_F - n_{LF}\cos\delta_W)\cos\delta_W + (c_{RR} + c_{RL})(l_R + n_{LR}) \Big\} \tag{18}$$

$$a_{22}(\mathbf{u}^*) = \frac{1}{J_Z v_{CoG}} \Big\{ -\frac{l_F\, b_F}{2} \sin\delta_W (c_{FL} - c_{FR}) - l_F (c_{FL} + c_{FR})(l_F - n_{LF}\cos\delta_W)\cos\delta_W - l_R (c_{RR} + c_{RL})(l_R + n_{LR}) \Big\} \tag{19}$$

$$b_1(\mathbf{u}^*) = \frac{1}{m_{CoG} v_{CoG}} \Big\{ \delta_W \cos\delta_W (c_{FL} + c_{FR}) + \sin\delta_W (F_{LFL} + F_{LFR}) \Big\} \tag{20}$$

$$b_2(\mathbf{u}^*) = \frac{1}{J_Z} \Big\{ \frac{b_F}{2} \cos\delta_W (F_{LFR} - F_{LFL}) + \delta_W \cos\delta_W (c_{FL} + c_{FR})(l_F - n_{LF}\cos\delta_W) + (F_{LFR} + F_{LFL}) \sin\delta_W (l_F - n_{LF}\cos\delta_W) + (c_{FL} - c_{FR})\delta_W \frac{b_F}{2} \sin\delta_W + (F_{LRR} - F_{LRL}) \frac{b_R}{2} \Big\}. \tag{21}$$

Before the observer can be designed, the observability of the model has to investigated. Criteria for the observability of nonlinear systems can e.g. be found in (Birk, 1992) or (Zeitz, 1987). For the restructured nonlinear double track model the proof of global observability was carried out.

Since the quality of the observer significantly depends on the accuracy of the underlying model, the restructured nonlinear double track models is validated with real measurement data to ensure that the model describes the vehicle dynamics with sufficient accuracy.

If the measured velocity is taken as an input variable, the modeled state space variables significantly deviate from the measured values. Therefore, instead of using the measured signals, the velocity is simulated using the original nonlinear differential equation (DE) according to Eqn. (10). Fig. 2 shows the resulting structure for the observers to be designed on basis of the restructured model.

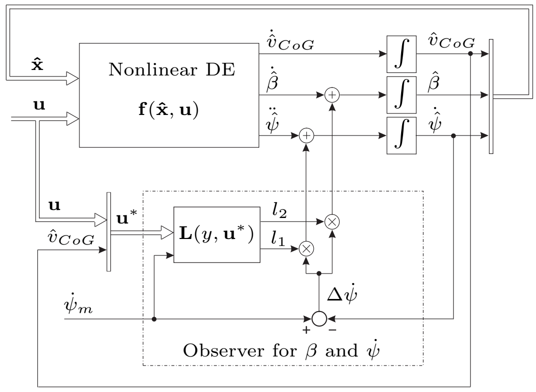

Fig. 2. Observer structure on basis of the restructured nonlinear double track model

The observer gain $\mathbf{L}(y, \mathbf{u}^*)$ is calculated on basis of the restructured double track model (15). As the process model, though, the original nonlinear double track model (13) is used.

2.2 Model Validation

The restructured nonlinear double track model (15) was simulated with real measurement data of a variety of test drives. The results will be shown for one of these representative test drives. Starting with a straight forward drive, the steering wheel angle δ_W is slowly increased up to 32° and is then reduced again. This results in an instationary circle. Fig. 3 compares the measured values for the velocity, the VSSA and the yaw rate with the values obtained simulating the nonlinear double track model. For comparison, the simulation results obtained from a linear bicycle model are also shown.

While the VSSA obtained from the bicycle model significantly deviates from the measured values, the nonlinear double track model is capable to describe the VSSA with high accuracy. The test drive presented describes a driving situation right at the stability limit of the vehicle. The measured VSSA increases up to almost 15°. For this test drive a linear model is no longer sufficient as the vehicle dynamics are highly nonlinear. The nonlinear double track model, however, is capable to describe the vehicle dynamics up to the stability limit with an accuracy that is sufficient for observer design.

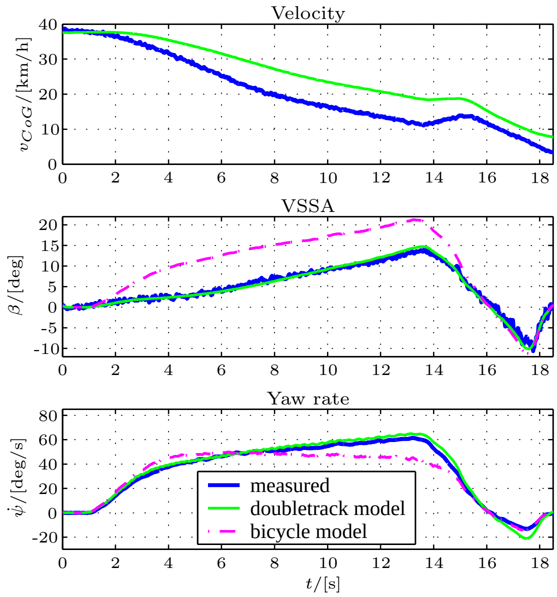

Fig. 3. Simulation of the restructured nonlinear double track model for an instationary circle

3. OBSERVER DESIGN

The basic idea of the Observer with Adaption of a Quality Function (AQF-Observer) is the adaption of the nonlinear estimation error dynamics to the one of a linear reference system. The observer design is only briefly described here, a detailed explanation can be found in (Sieber, 1991) or (Föllinger, 1993).

For the state space model

$$\dot{\mathbf{x}} = \mathbf{A}(y,\mathbf{u}^*)\,\mathbf{x} + \mathbf{b}(\mathbf{y},\mathbf{u}^*), \quad y = \mathbf{C}\,\mathbf{x} \tag{22}$$

an AQF-Observer

$$\begin{aligned}\dot{\hat{\mathbf{x}}} &= \mathbf{A}(y,\mathbf{u}^*)\,\hat{\mathbf{x}} + \mathbf{b}(y,\mathbf{u}^*) + \mathbf{L}(y,\mathbf{u}^*)\cdot(y-\hat{y})\,,\\ \hat{y} &= \mathbf{C}\,\hat{\mathbf{x}}\end{aligned} \tag{23}$$

is introduced. The differential equation for the estimation error $\tilde{\mathbf{x}}(t) = \mathbf{x}(t) - \hat{\mathbf{x}}(t)$ then becomes

$$\dot{\tilde{\mathbf{x}}} = [\mathbf{A}(y,\mathbf{u}^*) - \mathbf{L}(y,\mathbf{u}^*)\mathbf{C}]\,\tilde{\mathbf{x}}\,. \tag{24}$$

For the determination of an appropriate observer gain $\mathbf{L}$, the nonlinear estimation error (24) is adapted to a linear reference model. This reference model is derived by linearizing the nonlinear state space model (22) around an arbitrary equilibrium point $(\mathbf{x}_R, \mathbf{u}_R^*)$. The resulting linear reference model reads:

$$\dot{\mathbf{x}}_{lin} = \mathbf{A}_0\,\mathbf{x}_{lin} + \mathbf{B}_0\mathbf{u}^*_{lin}, \quad y_{lin} = \mathbf{C}\,\mathbf{x}_{lin} \tag{25}$$

with $\mathbf{A}_0$, $\mathbf{B_0}$ and $\mathbf{C}$ being constant matrices. For this linear model a linear observer is set up to

$$\begin{aligned}\dot{\hat{\mathbf{x}}}_{lin} &= \mathbf{A}_0\,\hat{\mathbf{x}}_{lin} + \mathbf{B}_0\,\mathbf{u}^*_{lin} + \mathbf{L}_{lin}\cdot(y_{lin} - \hat{y}_{lin})\,,\\ \hat{y}_{lin} &= \mathbf{C}\,\hat{\mathbf{x}}_{lin}\,.\end{aligned}$$

The differential equation for the linear estimation error $\tilde{\mathbf{x}}_{lin} = \mathbf{x}_{lin} - \hat{\mathbf{x}}_{lin}$ is then given by

$$\dot{\tilde{\mathbf{x}}}_{lin} = \big(\mathbf{A}_0 - \mathbf{L}_{lin}\mathbf{C}\big)\cdot\tilde{\mathbf{x}}_{lin}\,. \tag{26}$$

The poles of the dynamic matrix $\mathbf{A}_0 - \mathbf{L}_{lin}\mathbf{C}$ are placed in the open left half plane. Then, the estimation error $\tilde{\mathbf{x}}_{lin}$ vanishes in time.

For the adaption of the dynamics of the nonlinear estimation error (24) to the one of the linear reference system, the Lyapunov stability criterion is employed:

The state vector $\mathbf{x}(t)$ of the dynamic nonlinear system $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u}^*)$ converges against the equilibrium point $\mathbf{x}_R = \mathbf{0}$ from any initial point $\mathbf{x}(0)$, if a function $V(\mathbf{x})$ can be found with

$$\begin{aligned}&(1)\quad V(\mathbf{x}) > 0 \quad \forall\ \mathbf{x} \neq 0\,,\\ &(2)\quad V(\mathbf{x}) = 0 \quad \text{for } \mathbf{x} = 0\,,\\ &(3)\quad \dot V(\mathbf{x}) \le 0 \quad \forall\ \mathbf{x}\,.\end{aligned} \tag{27}$$

If these conditions are fulfilled, $V(\mathbf{x})$ is called Lyapunov function and the equilibrium point $\mathbf{x}_R = 0$ is called globally stable. In (Föllinger, 1993) a special Lyapunov function is proposed for the linear estimation error:

$$V_{lin} = \tilde{\mathbf{x}}_{lin}^T\mathbf{P}\,\tilde{\mathbf{x}}_{lin} \tag{28}$$

with

$$\mathbf{P} = \sum_{i=1}^{n} \tilde{P}_{ii}\,\bar{\mathbf{w}}_i\,\mathbf{w}_i^T\,. \tag{29}$$

Therein $\mathbf{w}_i$ are the left eigenvalues of the dynamic matrix $\mathbf{A}_0 - \mathbf{L}_{lin}\mathbf{C}$, $\bar{\mathbf{w}}_i$ is the complex conjugate of $\mathbf{w}_i$. The coefficients $\tilde{P}_{ii}$ are arbitrary positive weighting factors. According to (Föllinger, 1993) this Lyapunov function fulfills the conditions $V_{lin}(\tilde{\mathbf{x}}_{lin}) > 0\ \forall\ \tilde{\mathbf{x}}_{lin} \neq 0$ and $V_{lin}(\tilde{\mathbf{x}}_{lin}) = 0$ for $\tilde{\mathbf{x}}_{lin} = 0$. The time derivative of V_{lin} becomes

$$\dot V_{lin} = -\tilde{\mathbf{x}}_{lin}^T\mathbf{R}_{lin}\tilde{\mathbf{x}}_{lin} \tag{30}$$

with

$$\mathbf{R}_{lin} = \big[\mathbf{C}^T\mathbf{L}_{lin}^T - \mathbf{A}_0^T\big]\mathbf{P} + \mathbf{P}\big[\mathbf{L}_{lin}\mathbf{C} - \mathbf{A}_0\big]. \tag{31}$$

If the eigenvalues of $(\mathbf{A}_0 - \mathbf{L}_{lin}\mathbf{C})$ are placed in the open left half plane, $\dot V_{lin}(\tilde{\mathbf{x}}_{lin}) \le 0\ \forall\ \tilde{\mathbf{x}}_{lin}$ is also fulfilled and $\hat{\mathbf{x}}_{lin}$ converges against $\mathbf{x}_{lin}$.

The Lyapunov function (28) is set up for the nonlinear estimation error $\tilde{\mathbf{x}}$, too:

$$V = \tilde{\mathbf{x}}^T\mathbf{P}\,\tilde{\mathbf{x}} \quad \Rightarrow \quad \dot V = -\tilde{\mathbf{x}}^T\mathbf{R}\,\tilde{\mathbf{x}} \tag{32}$$

with

$$\begin{aligned}\mathbf{R} = &\big[\mathbf{C}^T\mathbf{L}(y,\mathbf{u}^*)^T - \mathbf{A}(y,\mathbf{u}^*)^T\big]\cdot\mathbf{P}\\ &+ \mathbf{P}\cdot\big[\mathbf{L}(y,\mathbf{u}^*)\mathbf{C} - \mathbf{A}(y,\mathbf{u}^*)\big]\,.\end{aligned} \tag{33}$$

The time derivative $\dot V$ can be regarded as a measure how fast the estimation error decreases. Since the design of the linear observer makes the linear estimation error decrease fast, $\dot{V}_{lin}$ also decrease fast. Consequently, the dynamics of the nonlinear estimation error is adapted to the one of the linear estimation error by adapting $\dot{V}$ to $\dot{V}_{lin}$. By an appropriate choice of the observer gain $\mathbf{L}(y, \mathbf{u}^*)$, the norm

$$N = \left\| \mathbf{R}_{lin} - \mathbf{R} \right\| \tag{34}$$

has to be minimized. Using equations (31) and (33), this norm can be calculated depending on the observer gain $\mathbf{L}$. The minimization requires an extension of the commonly used matrix operations. Details can for instance be found in (Sieber, 1991).

For the restructured nonlinear double track model (15) the AQF-Observer is set up to

$$\begin{aligned} \dot{\hat{\mathbf{x}}} &= \mathbf{A}(y, \mathbf{u}^*)\,\hat{\mathbf{x}} + \mathbf{b}(\mathbf{u}^*) + \mathbf{L}(y, \mathbf{u}^*) \cdot (y - \hat{y}) \,, \\ \hat{y} &= \mathbf{C}\,\hat{\mathbf{x}} = \dot{\hat{\psi}} \,. \end{aligned} \tag{35}$$

One equilibrium point was determined to

$$\mathbf{x}_R = \left[0.24^\circ,\ 3.54\frac{\circ}{s} \right]^T, \ \mathbf{u}_R^* = \left[0,\ 0,\ 0,\ 0,\ 10\frac{m}{s},\ 1^\circ \right]^T.$$

By linearizing the nonlinear model around this equilibrium point, the linear reference model is derived and a linear observer is calculated. Its eigenvalues are placed at $\lambda_1 = -20$ and $\lambda_2 = -120$. The free coefficients of the Lyapunov function (28) are chosen $P_{11} = 2$ and $P_{22} = 1$. Finally, the observer gain of the nonlinear observer can be calculated:

$$\begin{aligned} \mathbf{L}(\dot{\psi}, \mathbf{u}^*) = & \begin{bmatrix} 1,41 & 0,33 & 1 & 0 \\ -0,10 & 1,03 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} a_{11}(\dot{\psi}, \mathbf{u}^*) \\ a_{21}(\mathbf{u}^*) \\ a_{12}(\mathbf{u}^*) \\ a_{22}(\mathbf{u}^*) \end{bmatrix} \\ & + \begin{bmatrix} 109,9 \\ 117,4 \end{bmatrix} . \end{aligned} \tag{36}$$

For the elements a_{ij} Eqns. (16) to (19) hold.

3.1 Observer Validation

The AQF-Observer was validated with the test drives already used for the evaluation of the nonlinear double tack model. Fig. 4 compares the measured and estimated VSSA for the instationary circle. The AQF-observer follows the measured reference signal very well. The VSSA can be estimated with high accuracy up to the stability limit.

4. CONCLUSION

A nonlinear observer with adaption of a quality function was derived for the determination of the vehicle sideslip angle (VSSA). The observer design is based on a nonlinear adaptive double track model. The model was restructured to meet the specific structure that is required for the observer design. Based on this model the observer was derived. Model and observer were validated with real measurement data of representative test drives. It was shown, that the observer is capable to determine the VSSA very accurately up to the stability limit of the vehicle.

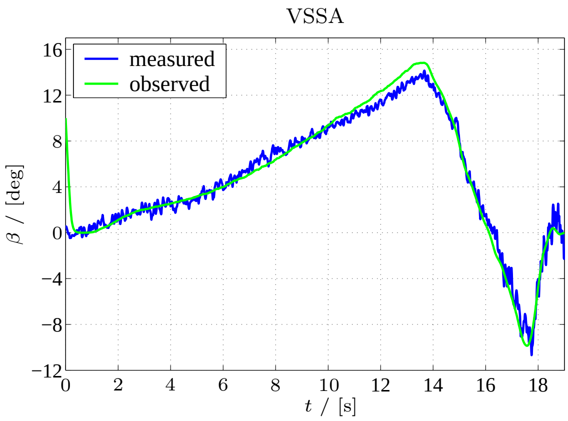

Fig. 4. Validation of the AQF-Observer for the instationary circle

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