NONLINEAR MODEL PREDICTIVE CONTROL WITH MOVING HORIZON STATE AND DISTURBANCE ESTIMATION - WITH APPLICATION TO MSW COMBUSTION

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Abstract: This paper presents a nonlinear model predictive control (NMPC) strategy that can be used to tackle model predictive control problems that involve relatively simple nonlinear dynamic models, as for example obtained with first-principles modeling. The main feature of the proposed NMPC strategy is the usage of a moving horizon estimator (MHE) for the estimation of the states and disturbances (and, if desired, parameters). The closed-loop performance properties of the proposed NMPC strategy are demonstrated by applying it to a model of a municipal solid waste (MSW) combustion plant under a realistic disturbance realization. In addition, a comparison is made with extended Kalman filter (EKF) based NMPC.

Keywords: nonlinear model predictive control, moving horizon estimation, extended Kalman filter, municipal solid waste combustion

1. INTRODUCTION

Due to lack of space, combustion of municipal solid waste (MSW, i.e. household waste) represents a suitable alternative to dumping in many densily populated parts of the world, despite the associated (assumed) negative effects on the environment. MSW is typically combusted at a plant as depicted in figure 1. After having been collected from households and transported to the MSW combustion (MSWC) plant, for example by truck, it is stored in a large bunker from which it is transported by cranes into a large chute. At the bottom of the chute the waste is pushed onto a moving grate by a ram. The waste is combusted while it is traveling on this grate using oxygen from air flows

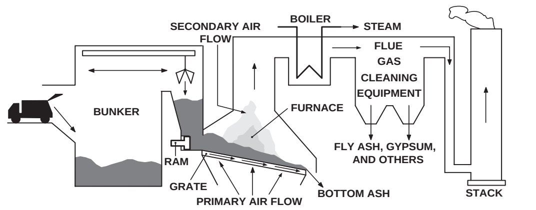

Fig. 1. A typical MSW combustion plant.

that are fed through holes in the grate (primary air flow) and furnace side walls (secondary air flow) to the solid waste layer and gas phase above it. The resulting flue gas enters a boiler delivering heat which is transformed into steam and, subsequently, into energy in the form of heat and/or electricity. Having passed the boiler, the flue gas is cleaned from residues that are not allowed to enter the surroundings. A MSWC plant is controlled by several control systems which can be divided in (i) a combustion control system, controlling the furnace part of the MSWC plant, and (ii) flue gas cleaning equipment (also referred to as air pollution or "post-combustion" control systems). In this paper only the combustion control system of the MSWC plant is considered. (See (Leskens *et al.*, 2005) for a more extended discussion on control of MSWC plants). The control objectives for this system can be stated as to maximize the waste throughput and energy output (steam), both being important from an economic point of view, while exceeding certain bounds that arise from environmental and lifetime considerations as few times as possible. (These objectives are most of the times not directly reflected by the control systems and control behavior encountered in practice, however). In addition, it needs to fulfill these objectives in the presence of large fluctuations in the process variables due to the variation in waste composition. Suppressing these fluctuations is an important objective, in particular to obtain a minimum *back-off* to the bounds mentioned above. One control strategy that seems to be very well suited to fulfill these objectives is model predictive control (MPC), in particular because of the presence of constraints and the multivariable nature of the MSWC plant. This paper presents results from research on the applicability of MPC on MSWC plants. The strategy that is followed in this research is that of applying a relatively simple dynamic model of the MSWC process, obtained from first-principles, directly within an MPC strategy, which, as a result of the nonlinearity of the model, is a *nonlinear* model predictive control (NMPC) strategy. Motivations for this research strategy are that (i) recent results, among which (Leskens *et al.*, 2002), suggest that the dynamics of the MSWC process can be modeled with a relatively simple first-principles model, i.e. with a small number of differential and algebraic equations, and that (ii) an NMPC strategy applied to such a model is able to deliver a solution for the manipulated variables (MVs) well within the chosen sample time with the current generation of computers (at least when the optimization problem to be solved by the NMPC controller (i.e. the time horizon) is not overly large).

Two of the three main aims of this paper are (i) to present an NMPC strategy that is able to fulfill the latter computational requirement and (ii) to show its closed-loop performance properties by applying the NMPC strategy to a simple first-principles model of the MSWC process under a realistic disturbance realization. The proposed NMPC strategy is characterized by the usage of a moving horizon estimator (MHE) for the estimation of the states and disturbances (which, if desired, can be easily extended to include parameters). Although MHE has been given much attention in the literature, see e.g. (Rao and Rawlings, 2002), its application as part of an NMPC strategy still seems to be rare. Typically, an extended Kalman filter (EKF) is used instead of a MHE. See e.g. (Lee and Ricker, 1994). A third main aim of this paper is to compare the closed-loop performance properties of MHE and EKF based NMPC. This is done on the basis of the same MSWC test case referred to above. In addition to these three main aims, also the following issues will be discussed briefly (i) the advantage of NMPC of being able to deal with a much wider range of control problem formulations than most of the other control strategies, (ii) the exploitation of this advantage by applying the proposed MHE based NMPC strategy to an *a*-typical MSWC control problem formulation that reflects more directly its true control objectives, as stated above, and (iii) the back-off properties of the proposed NMPC strategy.

The contents of this paper is as follows. First, in section 2, the proposed NMPC strategy will be discussed in more detail. The focus here is on the basic idea behind MHE. Then, in section 3 the closed-loop performance properties of this NMPC strategy will be discussed and compared to those of an EKF based NMPC strategy on the basis of the MSWC test case referred to above. A comparison of only the MHE and EKF estimation performances is also included here. The final part of this section is devoted to the three issues mentioned just above. Finally, in section 4, conclusions are given.

2. MHE BASED NMPC

2.1 Global set-up

The NMPC strategy proposed here follows the usual decomposition into (i) an estimation problem, where states and, if desired, disturbances and parameters are estimated, and (ii) a problem where the MVs are computed, using the estimated states (parameters, disturbances) as the true initial states. Here, the parameters are assumed to be known exactly and, therefore, are excluded from the discussion. Extension of the estimation part with uncertain time varying parameters is, however, straightforward. Typically, an EKF is used for this estimation part. Here, in contrast, an MHE approach is chosen for that purpose. For the purpose of the presentation of the proposed MHE based NMPC approach models of the form

$$x_{k+1} = f(x_k, u_k, d_k) \tag{1}$$

$$y_k = h(x_k, u_k, d_k) \tag{2}$$

are considered here although it is not restricted to this type of models. Here, x_k represent the states of the model, u_k the inputs/MVs, d_k the disturbances and y_k the measured outputs/controlled variables (CVs). Models of the type (1), (2) can be obtained in different ways, e.g. through system identification techniques or via first-principles modeling. In the latter case a model (1), (2) is typically obtained implicitly via integration of its continuous-time counterpart over e.g. piecewise constant inputs. The latter approach is also used in this paper in the MSWC examples. In terms of the model equations (1), (2), then, the following two steps are performed during every sample interval starting from the present sample instant k and ending before the next sample instant $k+1$:

(1) estimation of the states and disturbances at time $k+1$, $\hat{x}_{k+1}$ resp. $\hat{d}_{k+1}$, using a MHE.
(2) computation of the optimal MVs over a certain future time horizon N_{mpc}, i.e. computing u_{k+1}^{opt}, u_{k+2}^{opt}, ... , $u_{k+N_{mpc}}^{opt}$, using the estimates $\hat{x}_{k+1}$ and $\hat{d}_{k+1}$ and assuming the disturbances constant and equal to this latter value over the future time horizon.

After that, at sample instant $k+1$, the computed values for u_{k+1}^{opt} are implemented on the real plant and the two steps are repeated. The MHE step is discussed in detail in section 2.2. The second step in the NMPC strategy proposed here is not essentially different from the moving horizon formulations typically found in the literature, minimizing at each sample instant a criterion that reflects the control objectives for the plant under a set of constraints on the MVs, states and CVs. Apart from some remarks in section 3.1 on how the underlying dynamic optimization problem was solved in the MSWC examples, this second step is not discussed here.

2.2 Moving horizon estimation

For ease of explanation, the discussion of the basic idea behind MHE given here follows a least squares (LS) or system identification approach. (Typically, one starts this discussion with a probabilistic interpretation as e.g. in (Rao and Rawlings, 2002)). Assume that an optimal estimate for the state vector at time $k+1$, i.e. x_{k+1}, is desired. From the state equation(s) (1) it follows that this optimal estimate, denoted as $\hat{x}_{k+1}$, can be obtained as

$$\hat{x}_{k+1} = f(\hat{x}_k, u_k, \hat{d}_k)$$

if the optimal estimates for $\hat{x}_k$ and $\hat{d}_k$ are given (u_k is, obviously, known). Following the same line of thought, an optimal estimate for $\hat{x}_k$ can be obtained as

$$\hat{x}_k = f(\hat{x}_{k-1}, u_{k-1}, \hat{d}_{k-1})$$

if, indeed, the optimal estimates for $\hat{x}_{k-1}$ and $\hat{d}_{k-1}$ are given. Repeating this line of thought N_{mhe} times, it follows that an optimal estimate $\hat{x}_{k+1}$ can be obtained, from integrating the state equations, when $\hat{x}_{k-N_{mhe}}$ and $\{\hat{d}_{k-N_{mhe}}, ..., \hat{d}_k\}$ are known. How to obtain these latter estimates ? The answer is by fitting them to the data, i.e. by minimizing, over these estimates, a criterion that is a function of the differences between the measured outputs y_{k-i} and the predicted outputs

$$\hat{y}_{k-i} = h(\hat{x}_{k-i}, u_{k-i}, \hat{d}_{k-i})$$

at the time instants defined by $i = 0, ..., N_{mhe}$ (using the state equations to obtain the intermediate state estimates $\hat{x}_{k-i}$, $i = 0, ..., (N_{mhe}-1)$). Choosing an LS criterion, the following dynamic optimization problem to be solved is obtained for the MHE:

$$\min_{\hat{x}_{k-N_{mhe}}, \{\hat{d}_{k-N_{mhe}}, ..., \hat{d}_k\}} \sum_{i=0}^{N_{mhe}} \|y_{k-i} - \hat{y}_{k-i})\|^2$$

subject to the model equations (1) and (2). Having obtained the estimates $\hat{x}_{k-N_{mhe}}$ and $\{\hat{d}_{k-N_{mhe}}, ..., \hat{d}_k\}$, an estimate for $\hat{x}_{k+1}$ is then obtained via integrating the model equations.
For obvious computational reasons the time horizon N_{mhe} cannot be chosen arbitrarely large. Instead, a moving horizon formulation is chosen where the dynamic optimization problem is solved repeatedly every new sample instant for a fixed chosen time horizon N_{mhe}.
The MHE criterion formulations found in the literature are, generally, extensions of the LS formulation given above: they generally contain, motivated by probabilistic arguments (see e.g. (Rao and Rawlings, 2002)), an additional term that punishes the difference between $\hat{x}_{k-N_{mhe}}$ and a previously obtained estimate $\overline{x}_{k-N_{mhe}}$ for this initial state vector:

$$\|\hat{x}_{k-N_{mhe}} - \overline{x}_{k-N_{mhe}}\|_Q^2 + \sum_{i=0}^{N_{mhe}} \|y_{k-i} - \hat{y}_{k-i})\|_R^2 \tag{3}$$

In addition, one could also add a term punishing the rate of change of the disturbance estimates $\{\hat{d}_{k-N_{mhe}}, ..., \hat{d}_k\}$. In the MSWC examples to be discussed, a criterion of the form (3) is used together with a term punishing the rate of change of the one disturbance assumed to be acting on the MSWC plant.
A very important characteristic of the MHE approach, in particular with respect to the EKF,

is that *constraints* can be included, i.e. on disturbance, state and output estimates. It is this ability of including constraints together with the ability to include (all) nonlinear dynamics what distinguishes MHE from other estimation strategies (Rao and Rawlings, 2002). In the latter reference, examples show the significant improvement in estimation performance that can be obtained by including constraints into the estimation problem.

The MHE discussed so far does not deliver an estimate for the disturbances at time $k+1$, i.e. $\hat{d}_{k+1}$; only disturbance estimates up to k are delivered. One obvious and, assumably, best way to obtain an estimate for $\hat{d}_{k+1}$ is to incorporate a (state space) model of the disturbances in the MHE. Another simple way, one that can be employed if no such disturbance model is available and one that is employed here in the MSWC example(s) to be discussed, is to simply set $\hat{d}_{k+1} = \hat{d}_k$.

3. APPLICATION TO A MSWC MODEL

3.1 Implementation issues

The NMPC strategy presented in section 2 was applied to a relatively simple first-principles model of the MSWC process derived from (Van Kessel, 2003) and the so called calorific value sensor (CVS) equations given in (Van Kessel *et al.*, 2004). This model contained 4 inputs (MVs), 4 differential states, 1 algebraic state and 2 outputs (CVs). The inputs were (i) a measure for the waste inlet flow U_{wif}, (ii) a measure for the speed of the grate U_{sog}, (iii) the primary air flow Φ_{prim} and (iv) the secondary air flow Φ_{sec} (see also figure 1). The states of the model were (i) a state related to the dynamics of the grate, (ii) the waste mass on the grate, (iii) the temperature of this waste mass T_s and (iv) the produced steam flow Φ_{st}. The outputs were (i) (again) the steam flow and (ii) the concentration of O_2 in the flue gas. Finally, the algebraic state was the gas phase temperature. The model contained one disturbance z which is a measure for the waste composition. The model consisted largely of mass and energy balances and contained severe nonlinearities, among which radiation terms containing state variables to the power four. The model was (only) validated with respect to its capability of reproducing a set of measured steady-state values corresponding to an operating point of a real-life MSWC plant and was thought to be at least a reasonable representation of the MSWC dynamics. The (largest) dominant time constant of the model was approximately half an hour while the fastest time constant of the model was smaller than 1 minute. The latter was also the sample time for the NMPC controllers employed in the MSWC examples to be discussed.

Most closed-loop simulations to be discussed were performed with a realization for z that was obtained from the real-life MSWC plant mentioned just above using the also already mentioned CVS equations. This realization was assumed to be a realistic representation of the true waste composition variation. No other disturbances were acting on the plant during the simulations. Also, in the design of the NMPC controllers employed in the MSWC examples z was the only disturbance assumed to be acting on the plant.

The two MHE based NMPC controllers employed in the MSWC examples used the same MHE whose purpose was to estimate the past trajectory for z and an initial state vector as explained in section 2.2. (Note that, in a real-life setting, z can equally well be estimated, though not predicted, with the CVS equations. One reason for not considering this situation here is that not all MSWC plants have all measurements available for applying the CVS equations and, in addition, some of these measurements are not always reliable.)

The EKF used in the EKF based NMPC controller employed in the MSWC examples was equipped with a random walk model for z, assuming no (more specific) disturbance model to be available for this disturbance. It was optimally tuned using simulations. The part of this NMPC controller that computed the MVs was the same as used in the MHE based NMPC controller to which it was compared.

All programming was done with FORTRAN on a 2.4 GHz Pentium 4 computer. Both the dynamic optimization problem underlying the MHE and the one underlying the computation of the MVs were solved according to the approach of (Tousain and Bosgra, 2000): the nonlinear programming problem underlying both these dynamic optimization problems was solved with a sequential quadratic programming (SQP) method which was derived from (Boggs and Tolle, 1995) and which used an l_1-merit function and BFGS Hessian updates. Both MHE based NMPC controllers employed in the MSWC examples used time horizons $N_{mhe} = N_{mpc} = 5$. Also worth mentioning is the usage of the, so called, sensitivity equations to obtain gradient information for the SQP method. These equations remove the sometimes difficult choice of perturbation step size in finite-difference gradient computation methods. Sensitivity equations were effectively obtained from the model equations using Maple™ (V).

Tuning of the two NMPC controllers proved to be not that difficult. The ability to play around with the accuracy levels inherent to the SQPs was found to be very useful as this allowed for making a good compromise between desired accuracy and computational speed. The latter also applied to the ability to set a maximum number of iterations for each of the SQPs and (sub)iterations therein.

It is worth mentioning that during initialization of the NMPC controllers the computational speed could be much lower than after the initialization phase where each of the SQPs could be "warm-started" properly with the values for the solution, Lagrange multipliers and Hessian computed at the previous step. Finally, the computations made by the NMPC controllers employed in the MSWC examples ended all well within the sample time of 1 minute.

3.2 Application to MSWC model for a typical combustion control problem formulation

The combustion control problem formulation typically used at MSWC plants is that of minimizing the deviations from the measured Φ_{st} and O_2 from their setpoints. A continuously and severely changing waste composition makes this a challenging problem. Generally, a multivariable controller is used consisting of several proportional and, sometimes, integrating actions. Application of the MHE and EKF based NMPC controllers to tackle this conventional MSWC control problem led to the results depicted in figures 2 and 3 and those given in table 1. Figure 2 focuses on the closed-loop responses whereas figure 3 focuses on the estimation results obtained with the MHE and EKF. The realization for the ("real") disturbance z used to obtain these simulation results is depicted in the upper part of figure 3. From these results it can be concluded that (i) both the proposed MHE based NMPC strategy and the EKF based NMPC strategy exhibit good disturbance rejection properties, (ii) the EKF based NMPC strategy exhibits worse disturbance rejection properties than the MHE based NMPC strategy and (iii) the MHE gives better estimation results than the EKF, which is thought to explain the second conclusion. The closed-loop performance and estimation differences are thought to be the result of only the linearization steps performed in the EKF as constraints did not come into play during the simulations: if they do, the EKF is expected to perform much more worse than the MHE.

Note the rather large difference in the computed

Table 1. MHE vs. EKF based NMPC.

	PERF. MEASURE (in standard dev.) (sp = setpoint)	*MHE*	*EKF*
closed-loop	std(Φ_{st}^{sp} - $\Phi_{st,k}$)	0.19	0.31
performance	std(O_2^{sp} - $O_{2,k}$)	0.58	0.63
state	std($\hat{z}_{k+1}$-z_{k+1})	0.043	0.046
and	std($\hat{x}^1_{k+1}$-x^1_{k+1})	0	$1.2 * 10^{-19}$
disturbance	std($\hat{x}^2_{k+1}$-x^2_{k+1})	0.12	0.24
estimation	std($\hat{x}^3_{k+1}$-x^3_{k+1})	3.3	19.7
performance	std($\hat{x}^4_{k+1}$-x^4_{k+1})	0.05	0.18

values for the third MV, Φ_{prim}: see lower par of figure 2. Also note that the MHE estimates

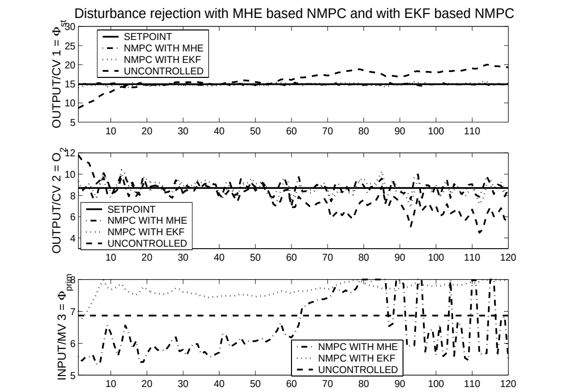

Fig. 2. NMPC applied to MSWC test case: disturbance rejection properties (x-axes: minutes).

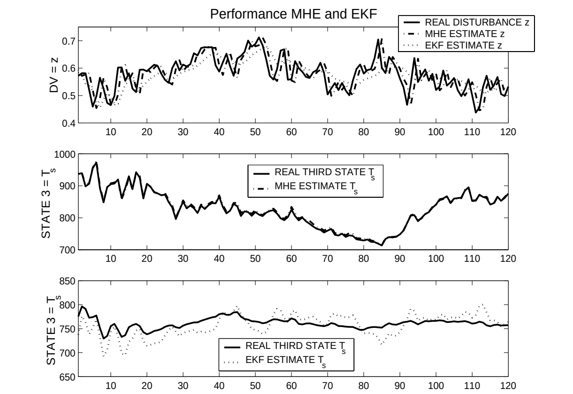

Fig. 3. NMPC applied to MSWC test case: estimation results.

for z represent an almost perfect one-step-ahead-shifted version of their real counterparts, see upper part of figure 3, implying that the MHE is very well capable of providing good estimates $\hat{d}_k$ (note that the MHE sets $\hat{d}_{k+1} = \hat{d}_k$).

To highlight only the setpoint tracking properties of the NMPC strategies, simulations have been performed with constant z and setpoint changes for both Φ_{st} and O_2. The results are depicted in figure 4. From this figure it can be concluded that (i) the MHE based NMPC strategy exhibits good setpoint tracking properties and (ii) these properties are considerably better than those corresponding to the EKF based NMPC strategy.

3.3 Application to MSWC model for an a-typical combustion control problem formulation

One advantage of NMPC is that one can apply a much wider range of control problem formulations than with other control strategies. In particular, one can apply other control problem formulations than the quadratic one typically encountered in the control literature and which is also used in the previous section in the MSWC example. This gives the opportunity to employ a MSWC control problem formulation that reflects more directly its true control objectives. These are, as stated

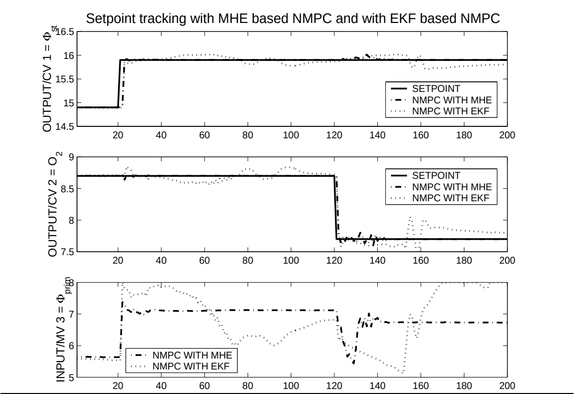

Fig. 4. NMPC applied to MSWC test case: setpoint tracking properties.

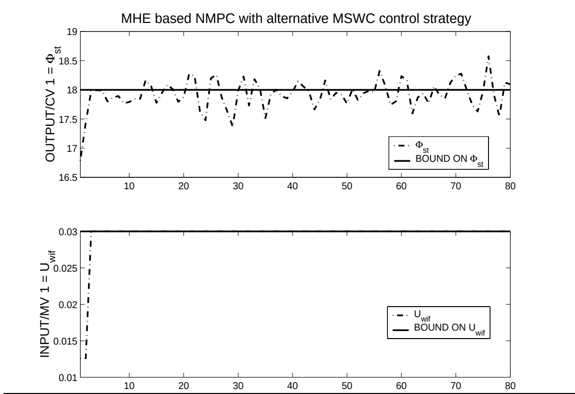

Fig. 5. NMPC applied to MSWC test case: alternative control problem formulation.

in section 1, maximization of both U_{wif} and Φ_{st} while having a back-off to the constraints imposed out of lifetime and environmental considerations that is as small as possible but still large enough to avoid too frequent crossing of these constraints. Changing the control problem formulation of the MHE based NMPC strategy of section 3.2 to one that is maximization based (the rest as before) led, with z again as in figure 3, to the closed-loop simulation results depicted in figure 5. Both U_{wif} and Φ_{st} can now be seen to jump from their initial values right on resp. close to their imposed upper bounds (the one on Φ_{st} imposed out of lifetime considerations), thereby demonstrating the maximizing effect of the new NMPC strategy. Φ_{st}, however, can be seen not to have good back-off properties, frequently crossing the imposed bound. This is thought to be due to the impossibility of the NMPC scheme to predict the exact values of the disturbances at time $k+1$. Remedies to improve the back-off properties are (i) the introduction of an accurate disturbance model in the estimation part of the NMPC controller and (ii) lowering the upper limit on Φ_{st}. A third and perhaps better remedy is to use one of the recently proposed (closed-loop, stochastic) MPC strategies that claim to deal more systematically with disturbances as e.g. proposed in (Van Hessem and Bosgra, 2004).

4. CONCLUSIONS

This paper has presented an NMPC strategy that is characterized by the usage of an MHE for the estimation of the states and disturbances (and, if desired, parameters). This strategy has been shown to be able to tackle NMPC problems employing relatively simple nonlinear dynamic models, as for example obtained with first-principles modeling. It also has been shown, through an MSW combustion example, that the MHE based NMPC strategy has good disturbance rejection and setpoint tracking properties which, in addition, are better than those obtained with an EKF based NMPC strategy.

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