ON STABILITY AND GAIN CONVERGENCE IN DISCRETE SIMPLE ADAPTIVE CONTROL

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Abstract: Successful implementations of simple direct adaptive control techniques in various domains of application have been presented over the last two decades in the technical literature. The theoretical background concerning basic conditions needed for stability of the controller and the open questions relating the convergence of the adaptive gains have been recently clarified, yet only for the continuous-time algorithms. Apparently, asymptotic tracking in discrete time systems is possible only with step input commands and the scope of the so called "almost strictly positive real" condition is also not clear. This paper will expand the feasibility of discrete simple adaptive control methodology to include any desired input commands and almost all real-world systems. The proofs of stability are also rigorously revised to solve the ultimate adaptive gain values question that has remained open until now.

Keywords: Control Systems, Discrete Systems, Stability, Passivity, Adaptive Control.

1. INTRODUCTION

Successful implementations of simple direct adaptive control (SAC) techniques in various domains of application have been presented over the last two decades in the technical literature. This methodology has been introduced by Sobel, Kaufman and Mabus (1982) and further developed by Barkana, Kaufman and Balas (1983) and Barkana and Kaufman (1984, 1985). Initially restricted to step input commands and to the so-called "almost strictly positive real (ASPR)" systems (Barkana and Kaufman, 1985), the feasibility of the continuous-time SAC has been extended to any desired input-commands and to any stabilizable real-world plant. Many works (Fradkov, 1976; Owens et al., 1987; Teixeira, 1988; Gu, 1990; Huang et al., 1999) have contributed to define those special systems that not only can be stabilized, but also rendered SPR via constant output feedback. Simply summarized (Barkana, 2004a), any minimum-phase LTI systems $\{A, B, C\}$ is ASPR if the matrical product CB is positive definite *symmetric*. We mention that only the transfer function should be called SPR, while the system should be called Strictly Passive, although it is customary to use either name in LTI systems. The applicability of low-order adaptive controllers to large scale examples has led to successful implementations of SAC in such diverse applications as flexible structures (Bayard et al, 1987; Ih et al, 1987; Lee et al, 1988; Shimada, 1998), flight control (Sanchez, 1986; Morse and Ossman, 1990), power systems (Barkana and Fischl, 1992), robotics (Barkana. and Guez, 1991), motor control (Sun et al, 2000), drug infusion (Palerm et al, 2002) and other.

Discrete-time versions of SAC have also been developed (Barkana 1983 and 1989; Barkana and Kaufman, 1983; Ohtsuka et al, 1997). However, asymptotically perfect following has apparently remained restricted to step inputs, while more general input commands seems to allow only bounded rather than vanishing errors. Moreover, the extent of the ASPR condition in discrete systems has not been clarified, as attempts at directly extending the continuous systems results to discrete systems have failed until recently. Recently, Barkana (2005b) managed to extend the previous results, and thus to establish some useful relations related to the passivity of discrete systems with the realization

$$x_p(t+1) = A_p x_p(t) + B_p u_p(t) \tag{1}$$

$$y_p(t) = C_p x_p(t) + D_p u_p(t) \tag{2}$$

The main result of Barkana (2005b) is the proof of the following lemma:

<u>Lemma 1</u>: Any proper but not strictly proper, minimum-phase, discrete linear system with positive definite and not necessarily symmetric D_p is ASPR, namely, it can be stabilized and rendered SPR via constant feedback.

Furthermore, if the controller $H:\{A_{fb}, B_{fb}, C_{fb}, D_{fb}\}$ stabilizes the system G, then the augmented system $G_a = G + H^{-1}$ is minimum-phase, and if D_{fb} is positive definite, the augmented system is ASPR Barkana (2005b). This way, basic stabilizability properties of systems can be used to implement ASPR configurations, thus extending the feasibility of adaptive and nonlinear control via parallel feedforward to real-world systems.

2. PASSIVITY IN DISCRETE LINEAR SYSTEMS

A system is called ASPR if there exists a positive definite output feedback gain K_e (unknown and not needed for implementation) such that the fictitious closed-loop system is SPR. In other words one could use the K_e in the control signal

$$\begin{aligned} u_p(t) &= -K_e y_p(t) + v_p(t) = -K_e C_p x_p(t) \\ &\quad - K_e D_p u_p(t) + v_p(t) \end{aligned} \tag{3}$$

Define

$$K_{ec} = (I + K_e D_p)^{-1} K_e \tag{4}$$

to get

$$u_p(t) = -K_{ec} C_p x_p(t) + (I + K_e D_p)^{-1} v_p(t) \tag{5}$$

Substituting in (1)-(2) gives

$$x_p(t+1) = A_{pc} x_p(t) + B_{pc} v_p(t) \tag{6}$$

$$y_p(t) = C_{pc} x_p(t) + D_{pc} v_p(t) \tag{7}$$

where

$$B_{pc} = B_p (I + K_e D_p)^{-1} \tag{8}$$

$$D_{pc} = D_p (I + K_e D_p)^{-1} = (D_p^{-1} + K_e)^{-1} \tag{9}$$

$$C_{pc} = \left(I + D_p K_e\right)^{-1} C_p \tag{10}$$

$$A_{pc} = A_p - B_p K_{ec} C_p \tag{11}$$

The closed-loop system is strictly passive and its transfer function is strictly positive real (SPR) if there exist three positive definite symmetric (PDS) matrices of appropriate dimensions, P, Q and Q_0 that satisfy the relations

$$A_{pc}^T P A_{pc} - P = -Q - L^T L \tag{12}$$

$$A_{pc}^T P B_{pc} - C_{pc}^T = L^T W \tag{13}$$

$$D_{pc} + D_{pc}^T = W^T W + B_{pc}^T P B_{pc} + Q_0 \tag{14}$$

Because the original plant is separated from strict positive realness only by a constant output feedback, it is called "almost strictly positive real (ASPR)" (Barkana and H. Kaufman, 1985; Barkana, 1987). Relations (12)-(14) can also be written in a more concise form. Substituting L^T from (13) into (12) and using (14) gives

$$\begin{aligned} &A_{pc}^T P A_{pc} - P + \\ &\left(A_{pc}^T P B_{pc} - C_{pc}^T\right)\left(D_{pc} + D_{pc}^T - B_{pc}^T P B_{pc} - Q_0\right)^{-1} \\ &\bullet \left(B_{pc}^T P A_{pc} - C_{pc}\right) = -Q \end{aligned} \tag{15}$$

3. PRIOR CONDITIONS FOR ASYMPTOTIC TRACKING OF SIMPLE ADAPTIVE CONTROL:

The adaptive control approach assumes that the plant parameters are basically unknown and only some of the plant properties are known. Therefore, adaptive control procedures are devised that are called to construct the control gains on-line. It will be shown that it is sufficient to know that the basic plant (1)-(2) is ASPR, even if one does not know the gain that can make it SPR.

The adaptive control methodology presented here does not just use an output feedback; it instead assumes that the controlled plant is required to follow a desired behaviour represented by an ideal model reference. Because the adaptive system attempts to bring the plant to the ideal situation of perfect following, it is reasonable to check first whether the proposed model following configuration has a perfect following solution, and this is the topic of next section.

The plant (1)-(2) is required to follow the output of the asymptotically stable model

$$x_m(t+1) = A_m x_m(t) + B_m u_m(t) \tag{16}$$

$$y_m(t) = C_m x_m(t) + D_m u_m(t) \tag{17}$$

In the beginning of SAC, the model was considered to be excited by step inputs only. In order to extend its feasibility, we assume that the input command itself can be represented as the output of an unknown command generating system (Barkana, 1983)

$$x_u(t+1) = A_u x_u\left(t\right) \tag{18}$$

$$u_m(t) = C_u x_u(t) \tag{19}$$

When the reference model is fed with an input of form (18)-(19), the solution is the sum of the steady state solution and the transient.

$$x_m(t) = E x_u(t) + A_m^t \delta_0 \tag{20}$$

Substituting (20) into (16) gives:

$$A_m E - E A_u + B_m C_u = 0 \tag{21}$$

$$\delta_0 = x_m(0) - E x_u(0) \tag{22}$$

Notice that the solution (20) always exists for the stable model (16)-(17). Therefore, (21)-(22) are not conditions; they only show the relations between the various values involved. In general, there is an error between the model output and the output of the plant.

$$e_y(t) = y_m(t) - y_p(t) \tag{23}$$

The controller uses the available measurable values to compute the control signal

$$u_p(t) = K_e e_y(t) + K_x x_m(t) + K_u u_m(t) \tag{24}$$

One wants to check if the desired asymptotically perfect tracking is possible.

$$\begin{aligned} y_p(t) &= C_p x_p(t) + D_p u_p(t) \\ &= C_m x_m(t) + D_m u_m(t) = y_m(t) \end{aligned} \tag{25}$$

Ideally, at the ideal steady-state perfect tracking the plant moves such the tracking error is zero, and the control signal is now the ideal control $u_p^*(t)$ that allows perfect tracking

$$u_p^*(t) = K_x x_m(t) + K_u u_m(t) \tag{26}$$

The plant must move along such ideal trajectories that allow perfect tracking. It can be shown that these ideal trajectories also get the form

$$x_p^*(t) = X_x x_m(t) + X_u u_m(t) \tag{27}$$

Substituting (26) and (27) in (25) gives

$$\begin{aligned} &C_p X_x x_m(t) + C_p X_u u_m(t) + D_p K_x x_m(t) \\ &+ D_p K_u u_m(t) = C_m x_m(t) + D_m u_m(t) \end{aligned} \tag{28}$$

As one may want to require satisfaction of the perfect tracking conditions at any moment, one gets the first set of conditions:

$$C_p X_x + D_p K_x = C_m \tag{29}$$

$$C_p X_u + D_p K_u = D_m \tag{30}$$

On the other hand, the ideal trajectories must satisfy the plant differential equations:

$$x_p^*(t+1) = A_p x_p^*(t) + B_p u_p^*(t) \tag{31}$$

Also, one gets from the trajectory equation (27):

$$x_p^*(t+1) = X_x x_m(t+1) + X_u u_m(t+1) \tag{32}$$

Equating (31) and (32), and using the model equations (16)-(17) and the input command equations (18)-(19) finally gives

$$\begin{aligned} &x_p^*(t+1) = A_p X_x x_m(t) + A_p X_u C_u x_u(t) \\ &+ B_p K_x x_m(t) + B_p K_u C_u x_u(t) = X_x A_m x_m(t) \\ &+ X_x B_m C_u x_u(t) + X_u C_u A_u x_u(t) \end{aligned} \tag{33}$$

Identifying corresponding coefficients gives

$$A_p X_x + B_p K_x = X_x A_m \tag{34}$$

$$A_p X_u C_u + B_p K_u C_u = X_x B_m C_u + X_u C_u A_u \tag{35}$$

The four conditions for perfect following are thus (29)-(30) and (34)-(35). One gets from (29) and (30)

$$K_x = D_p^{-1} C_m - D_p^{-1} C_p X_x \tag{36}$$

$$K_u = D_p^{-1} D_m - D_p^{-1} C_p X_u \tag{37}$$

Therefore, if there exist solutions for X_x and X_u, there also exist solutions for the control gains K_x and K_u. Substitute in (34) and (35) to get

$$\begin{aligned} &\left(A_p - B_p D_p^{-1} C_p\right) X_x - X_x \left(A_m + B_m C_u\right) \\ &= -B_p D_p^{-1} C_m \end{aligned} \tag{38}$$

$$\begin{aligned} &\left(A_p - B_p D_p^{-1} C_p\right) X_u C_u - X_u C_u A_u \\ &= X_x B_m C_u - B_p D_p^{-1} D_m \end{aligned} \tag{39}$$

Equation (39) has a unique solution for the matrix

$$X = X_u C_u \tag{40}$$

if the matrices $A_z = A_p - B_p D_p^{-1} C_p$ and A_u share no eigenvalue, yet even if these conditions are satisfied, one needs explicit solutions for X_u. As (40) has $n_p * n_u$ equations with $n_p * m$ variables, a solution exists in general only if

$$n_u \le m \tag{41}$$

Condition (41) seems to imply that this model following configuration cannot deal with rich input commands. For this reason, in the first presentations of SAC (Sobel et al, 1982) only step input commands were treated. We will show below that this limitation is only apparent. If the adaptive control can be shown to maintain stability of the system and to ultimately bring the plant along those trajectories that satisfy perfect tracking, one is entitled to assume that the steady-state values of the adaptive gains must belong to the solutions of (29)-(30) and (34)-(35). However, experiments had shown that this is not the case and the adaptation may end with totally different gain values than those predicted. This result forces one to reconsider the asymptotic tracking conditions. As one only expects the adaptive controller to achieve perfect tracking after the adaptation process elapses, one is led to think of those conditions that the control gains must only ultimately satisfy. To this end, substitute the solution (20) in (28) and (33) to get

$$\begin{aligned} &C_p X_x E x_u(t) + C_p X_x A_m^t \delta(t) + C_p X_u C_u x_u(t) \\ &+ D_p K_x E x_v(t) + D_p K_x A_m^t \delta(t) \\ &+ D_p K_u C_u x_u(t) = C_m E x_u(t) \\ &+ C_m A_m^t \delta(t) + D_m C_u x_u(t) \end{aligned} \tag{42}$$

$$\begin{aligned} &x_p^*(t+1) = A_p X_x E x_u(t) + A_p X_x A_m^t \delta_0 \\ &+ A_p X_u C_u x_u(t) + B_p K_x E x_u(t) + B_p K_x A_m^t \delta_0 \\ &+ B_p K_u C_u x_u(t) = X_x A_m E x_u(t) + X_x A_m A_m^t \delta_0 \\ &+ X_x B_m C_u x_u(t) + X_u C_u A_u x_u(t) \end{aligned} \tag{43}$$

that results in

$$\begin{aligned} &\left(C_p X_x E + C_p X_u C_u + D_p K_x E + D_p K_u C_u\right) x_u(t) \\ &+ \left(C_p X_x + D_p K_x\right) A_m^t \delta_0 = \left(C_m E + D_m C_u\right) x_u(t) \\ &+ C_m A_m^t \delta_0 \end{aligned} \tag{44}$$

$$\left(A_p X_x E + A_p X_u C_u + B_p K_x E + B_p K_u C_u\right) x_u(t) + \left(A_p X_x + B_p K_x\right) A_m^t \delta_0 = \left(X_x A_m E + X_x B_m C_u + X_u C_u A_u\right) x_u(t) + X_x A_m A_m^t \delta_0 \quad (45)$$

As we are now interested in the steady-state solutions of (44)-(45), we get a new set of conditions

$$C_p X_x E + C_p X_u C_u + D_p K_x E + D_p K_u C_u = C_m E + D_m C_u \quad (46)$$

$$A_p X_x E + A_p X_u C_u + B_p K_x E + B_p K_u C_u = X_x A_m E + X_x B_m C_u + X_u C_u A_u \quad (47)$$

Denote

$$X_{xu} = X_x E + X_u C_u \quad (48)$$

$$K_{xu} = K_x E + K_u C_u \quad (49)$$

$$C_D = C_m E + D_m C_u \quad (50)$$

and use (21) to finally get

$$A_p X_{xu} + B_p K_{xu} = X_{xu} A_u \quad (51)$$

$$C_p X_{xu} + D_p K_{xu} = C_D \quad (52)$$

From (52) one gets

$$K_{xu} = D_p^{-1} C_D - D_p^{-1} C_p X_{xu} \quad (53)$$

Substitute K_{xu} in (51) to get

$$\left(A_p - B_p D_p^{-1} C_p\right) X_{xu} - X_{xu} A_u = B_p D_p^{-1} C_D \quad (54)$$

Equation (54) has a unique solution for X_{xu} if the matrices $A_z = A_p - B_p D_p^{-1} C_p$ and A_u share no eigenvalue. As A_z is the zero dynamics of the plant, perfect tracking is possible if no natural mode of the input commands can be blocked by a zero of the controlled plant. If (54) has a solution for X_{xu}, then (53) gives the solution for K_{xu}. The equations for the gains K_x and K_u, X_x and X_u are:

$$X_x E + X_u C_u = X_{xu} \quad (55)$$

$$K_x E + K_u C_u = K_{xu} \quad (56)$$

The first equation has now $n_p * n_u$ equations with $n_p * \left(n_m + m\right)$ variables, and the second equation has $m * n_u$ with $m * \left(n_m + m\right)$ variables. Thus, a main condition for existence of solutions is

$$n_m + m \geq n_u \quad (57)$$

Condition (57) implies that if the model reference is of the order of any expected input command, the control configuration can accommodate those commands. The equations of motions and the perfect following equations hold asymptotically. Following (46) one gets the tracking error

$$e_y(t) = y_m(t) - y_p(t) = y_p^*(t) - y_p(t) - \left(C_p X_x + D_p K_x - C_m\right) A_m^t \delta_0 \quad (58)$$

and after some algebra

$$e_y(t) = C_{pc} e_x(t) - D_{pc}\left(K(t) - K\right) r(t) - \left(I + D_p K_e\right)^{-1}\left(C_p X_x + D_p K_x - C_m\right) A_m^t \delta_0 \quad (59)$$

The trajectory equation is

$$x_p^*(t+1) - A_p x_p^*(t) - B_p u_p^*(t) = \left(X_x A_m E + X_x B_m C_u + X_u C_u A_u\right) x_u(t) + X_x A_m A_m^t \delta_0 - A_2 x_u(t) - \left(A_p X_x + B_p K_x - X_x A_m\right) A_m^t \delta_0 \quad (60)$$

where from (47)

$$A_2 = A_p X_x E + A_p X_u C_u + B_p K_x E + B_p K_u C_u - X_x A_m E - X_x B_m C_u - X_u C_u A_u = 0 \quad (61)$$

Therefore

$$x_p^*(t+1) = A_p x_p^*(t) + B_p u_p^*(t) - \left(A_p X_x + B_p K_x - X_x A_m\right) A_m^t \delta_0 \quad (62)$$

4. IMPLEMENTATION OF SIMPLE ADAPTIVE CONTROL:

The adaptive controller assumes that the (unknown) solution for the LTI controller exists. Given the plant (1)-(2), one implements the controller

$$u_p(t) = K_e(t) e_p(t) + K_x(t) x_m(t) + K_u(t) u_m(t) \quad (63)$$

or

$$u_p(t) = K(t) r(t) \quad (64)$$

Here

$$K(t) = \begin{bmatrix} K_e(t) & K_x(t) & K_u(t) \end{bmatrix} \quad (65)$$

$$r(t) = \begin{bmatrix} e_y(t) \\ x_m(t) \\ u_m(t) \end{bmatrix} \qquad \Gamma = \begin{bmatrix} \Gamma_e & 0 & 0 \\ 0 & \Gamma_x & 0 \\ 0 & 0 & \Gamma_u \end{bmatrix} \quad (66)$$

For convenience, we also define the fixed gain

$$K = \begin{bmatrix} K_e & K_x & K_u \end{bmatrix} \quad (67)$$

The adaptive gain are given by the algorithm

$$K(t) = K(t-1) + e_y(t) r^T(t) \Gamma \quad (68)$$

Define the state error

$$e_x(t) = x_p^*(t) - x_p(t) \quad (69)$$

that gives after the appropriate algebra

$$e_x(t+1) = A_{pc} e_x(t) - B_{pc}\left(K(t) - K\right) r(t) + F A_m^t \delta_0 \quad (70)$$

where

$$F = B_p K_{ec}\left(C_p X_x + D_p K_x - C_m\right) - \left(A_p X_x + B_p K_x - X_x A_m\right) \quad (71)$$

The adaptive control must deal with stability of both the state and the gains, or in other words with the combined system (68) and (70). Therefore, one selects the quadratic Lyapunov equation

$$V(t) = e_x^T(t)Pe_x(t) + tr\left[\left(K(t)-K\right)\Gamma^{-1}\left(K(t)-K\right)\right] \quad (72)$$

Using the ASPR conditions (12)-(14) and following the lines of Barkana (1989) finally gives

$$\begin{aligned}\Delta V(t) &= -e_x^T(t)Qe_x(t) \\ &-r^T(t)\left(K(t)-K\right)^T Q_0\left(K(t)-K\right)r(t) \\ &-\left[e_x^T(t)L^T + r^T(t)\left(K(t)-K\right)^T W^T\right] \\ &\bullet\left[Le_x(t)+W\left(K(t)-K\right)r(t)\right] \\ &-e_y^T(t)e_y(t)r^T(t)\Gamma r(t) \\ &-2e_x^T(t+1)PFA_m^t\delta_0 \\ &-2r^T(t)\left(K(t)-K\right)^T\left(I+D_pK_e\right)^{-1} \\ &\bullet\left(C_pX_x+D_pK_x-C_m\right)A_m^t\delta_0 \\ &-\delta_0^T A_m F^T PFA_m^t\delta_0\end{aligned} \quad (73)$$

$\Delta V(t)$ is not positive definite or even positive semidefinite, because of the last transient terms in (73). Although the transient terms may keep $\Delta V(t)$ positive and their cumulative effect could lead to divergence. However, if either $e_x(t)$, $e_y(t)$, $K(t)-K$, or $\left(K(t)-K\right)r(t)$ becomes large, the negative definite terms in (73) become dominant and $\Delta V(t)$ becomes negative, thus guaranteeing that all adaptation variables are bounded. Therefore, the transient terms in (73) indeed vanish in time and according to the modified LaSalle's Invariant Principle (Barkana, 1983; Kaufman et al, 1998) the system ends in that domain of the state space where $\Delta V(t) \equiv 0$, which implies $e_x(t) \equiv 0$ and $e_y(t) \equiv 0$. Therefore, the system ultimately performs perfect tracking, the adaptation ends, and the adaptive gains reach a steady state set of values that allow perfect following. Moreover, LaSalle's invariance principle allows reaching more detailed conclusion on the stability of the adaptive systems, as it does not only imply that the errors vanish in time, but also that all values involved in adaptation reach the domain of perfect tracking. This is important in particular with respect to the ultimate gain values. The mere fact that the adaptation ultimately stops and the gain difference tends to vanish is not by itself sufficient to guarantee that the adaptive gains ultimately reach constant values, as the counterexample $k(t) = \sin(\log t)$ seems to illustrate. It is easy to show that the gain difference $\Delta k(t) = \sin\left(\log(t+1)\right) - \sin(\log t)$ approaches zero as time tends to infinity, yet the gain itself continues changing and has no limit at all. The example seems to suggest that although the assumption on the existence of some constant ideal gains was convenient for the proof of stability, more general ideal control are possible, and the ultimate gains may continue varying although the errors are zero. Therefore, one must consider the most general representation of ideal trajectory and ideal control.

$$u_p^*(t) = K_x(t)x_m(t) + K_u(t)u_m(t) \quad (74)$$

$$x_p^*(t) = X_x(t)x_m(t) + X_u(t)u_m(t) \quad (75)$$

Thus, in spite of successful applications, the fate of the adaptive gains seems to be an issue that has remained open in adaptive control for more than 25 years and only very recently has the issue been closed for the continuous-time version of SAC (Barkana, 2005a) and it will be closed here for the discrete SAC. With varying gains, conditions (51)-(52) are replaced by the new conditions

$$A_pX_{xu}(t) + B_pK_{xu}(t) = X_{xu}(t+1)A_u \quad (76)$$

$$C_pX_{xu}(t) + D_pK_{xu}(t) = C_D \quad (77)$$

Because D_p is not singular, one gets from (77)

$$K_{xu}(t) = D_p^{-1}\left(C_D - C_pX_{xu}(t)\right) \quad (78)$$

and then from (76)

$$X_{xu}(t+1)A_u - A_pX_{xu}(t) + B_pD_p^{-1}C_pX_{xu}(t) = B_pD_p^{-1}C_D \quad (79)$$

Equation (79) is a linear time-invariant difference equation with all solutions of the form

$$x(t) = x_0 + \sum_l\sum_m x_{lm}t^l\beta^t \quad (80)$$

Similarly, from (78) the control gains have the form

$$k(t) = k_0 + \sum_l\sum_m k_{lm}t^l\beta_m^t \quad (81)$$

and the gain difference has the form

$$\Delta k(t) = k(t) - k(t-1) = \sum_i\sum_j g_{ig}t^i\beta_j^t \quad (82)$$

First, observe that the so called "counterexample" is only apparent, because it cannot be a solution of (81). Along with the constant term k_0, the gain (81) and the difference (82) contain generalized exponential terms that could be convergent, divergent, or lead to steady sinusoidal. As the proof of stability implies that all terms are ultimately bounded, and the gain difference ultimately vanishes, the diverging and the steady terms are obviated. Therefore, all transient terms in (81) vanish in time, ultimately leaving the only possible solution

$$k(t) \to k_0 \quad as \quad t \to \infty \quad (83)$$

The final ideal gains are therefore constant, and as such they belong to the set of solutions of (55)-(56).

5. CONCLUSIONS

This paper extends the feasibility results of simple adaptive control to discrete systems. It showed that the simple adaptive controller can perform asymptotically perfect tracking of realistic signals. As basic stabilizability properties of systems and parallel feedforward can be used to implement the desired ASPR configurations, this extends the feasibility of adaptive control to real-world systems.

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