#### INDUSTRIAL APPLICATION OF DATA RECONCILIATION FOR HYBRID SYSTEMS

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Abstract: For hybrid systems which incorporate both dynamical and discrete event models in process industries, the mass balance models will be changed because of discrete scheduling events. The redundancy degree of whole sensor network is time variant so that the conventional data reconciliation is very difficult to be implemented. In this paper a new approach of data reconciliation for hybrid system is proposed and its industrial application is discussed. The whole process includes 15 units and more than 100 tanks. Comparing with AspenTech Advisor, the application results demonstrate the efficiency and consistency of the proposed approach. *Copyright* © 2005 IFAC

Keywords: data handling systems, discrete-event systems, data reconciliation, data models

# 1. INTRODUCTION

Measurements in a chemical process are subject to errors, both random and systematic, so that the laws of conservation of mass and energy are not obeyed. In order to record the performance of the process, these measurements are adjusted in order that they conform to the conservation laws and any other constraints imposed upon them. This procedure is known as data reconciliation (Crowe, 1996).

For refinery, the most common technology is based on steady-state data reconciliation (Crowe, *et al.*, 1983). But the conventional steady-state linear data reconciliation based on mass balance is insufficient sometimes. For hybrid systems which incorporate both dynamical and discrete event models in process industries, the mass balance models will be changed because of discrete scheduling events. The redundancy degree of whole measurement network is time variant so that the traditional data reconciliation methods can hardly be applied in practical process. Some attempts to the problem have already been recently presented (Mandel, *et al.*, 1998; Maquin, *et al.*, 2000). The Aspen Advisor system includes a powerful Expert system used to calculate missing stream flows, identify and reconcile data discrepancies. But in the practical application, the reconciliation results are unsatisfactory.

In this paper a new approach of data reconciliation for hybrid systems is proposed. The new method network includes three parts: measurement reconstruction, gross error detection and reconciliation algorithm. Most importantly, the proposed approach is employed on a industrial plant. Comparing with AspenTech Advisor, the application results demonstrate the efficiency and consistency of the proposed approach.

This paper is organized as follows. It begins with a brief description of the process under consideration,

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followed by the proposed approach of data reconciliation for hybrid systems. The application results are discussed in Section 4 and Section 5 concludes the whole paper.

## 2. PROCESS DESCRIPTION AND MASS BALANCE MODEL

The plant includes 15 units and more than 100 tanks. Tanks containing the same material are treated as one virtual tank. Each unit or virtual tank is represented by one node. For each virtual tank, there is a virtual variable showing its capacity change over one day. Fig 1 shows the simplified measurement network from the refinery. Where R1~R15 are units, G1~G12 are virtual tanks, M1~M16 are manifolds that are used to represent the combination or splitting of flows in a processing plant. The steady-state linear model of the process is defined as follows

$$\mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} = 0 \tag{1}$$

Where **A** is matrix corresponding to measured variables, **B** is matrix corresponding to unmeasured variables, **x** is vector of measured variables, **u** is vector of unmeasured variables.

As shown in Fig 1, the whole process includes 43 steady-state mass balance equations, 108 measured and 17 unmeasured variables. There are discrete scheduling events on nodes M4, M5, M6, M7, M8, M12, M13. In a reconciliation period (24 hours), the plant operates at a steady-state. The flows of all streams are accumulative values of one day.

## 3. APPROACH OF DATA RECONCILIATION FOR HYBRID SYSTEMS

A scheduling event consists of some useful information and can be taken into account. For those nodes where the discrete scheduling events may happen, equations that consist of the information are established and added to the models. These equations are defined as random scheduling-equations. In this way, the redundancy degree of the model is improved. Then its optimal solution can be obtained by applying a reconciliation algorithm with uncertain models (Maquin, et al., 2000). In order to guarantee that the measurement errors are normally distributed with zero mean and known covariance matrix V, the gross errors are identified and eliminated with the aid of an adaptation of the NT (Nodal test) (Mah, et al., 1976) algorithm. The details of the proposed approach are described as follows.

#### 3.1 Measurement network reconstruction

As shown in Fig 1, the redundancy degree of the model is too low to use matrix projection method (Crowe, *et al.*, 1983). The mass balance model

should be reconstructed. Assume there are s scheduling events on node M, as shown in Fig 2.  $x_m$  is measured,  $u_1 \sim u_s$  are all unmeasured and the reconciliation period is T (24 hours).



Fig 2 Sketch map of scheduling

Suppose

- 1. When one scheduling event is performed, only one branch way has mass flow, and the other s-1 branch ways have no flows.
- 2. The plant operates at a steady-state, the flow of  $x_m$  is consistent.

If the *i*th scheduling event execution time is  $\Delta t_i$ , then

$$\mathbf{u}_{i} = \frac{\Delta t_{i}}{T} \mathbf{x}_{m} \quad (i = 1, 2, \cdots, s)$$
(2)

$$\sum_{i=1}^{s} \Delta t_i = T$$
 (3)

Let

$$\theta_{i} = \frac{\Delta t_{i}}{T}$$
(4)

then

$$\mathbf{u}_{i} = \boldsymbol{\theta}_{i} \mathbf{x}_{m} \quad (i = 1, 2, \cdots, s) \tag{5}$$

$$\sum_{i=1}^{s} \theta_i = 1 \tag{6}$$

Although the execution time of scheduling events is definitely known. As other measurements, the actually execution time is polluted by errors. Suppose in the absent of gross errors,  $\Delta t_i$  are measured variables whose covariance matrix is known. Using this method, the random scheduling-equations of node M4, M5, M6, M7, M8, M12, M13 are

$$\begin{aligned} \mathbf{u}_{1} &= \theta_{1}\mathbf{x}_{5} & \mathbf{u}_{7} = \theta_{7}\mathbf{x}_{6} & \mathbf{u}_{13} = \theta_{13}\mathbf{x}_{19} \\ \mathbf{u}_{2} &= \theta_{2}\mathbf{x}_{5} & \mathbf{u}_{8} = \theta_{8}\mathbf{x}_{15} & \mathbf{u}_{14} = \theta_{14}\mathbf{x}_{19} \\ \mathbf{u}_{3} &= \theta_{3}\mathbf{x}_{5} & \mathbf{u}_{9} = \theta_{9}\mathbf{x}_{15} & \mathbf{u}_{15} = \theta_{15}\mathbf{x}_{35} \\ \mathbf{u}_{4} &= \theta_{4}\mathbf{x}_{6} & \mathbf{u}_{10} = \theta_{10}\mathbf{x}_{16} & \mathbf{u}_{16} = \theta_{16}\mathbf{x}_{35} \\ \mathbf{u}_{5} &= \theta_{5}\mathbf{x}_{6} & \mathbf{u}_{11} = \theta_{11}\mathbf{x}_{16} & \mathbf{u}_{17} = \theta_{17}\mathbf{x}_{12} \\ \mathbf{u}_{6} &= \theta_{6}\mathbf{x}_{6} & \mathbf{u}_{12} = \theta_{12}\mathbf{x}_{19} \end{aligned}$$
(7)

Combining Equation (7) and (1), we can get the new steady-state linear model of the process. It is defined as follows

$$\mathbf{M}(\boldsymbol{\theta})\mathbf{x} = 0 \tag{8}$$

Where  $\mathbf{M}(\boldsymbol{\theta})$  is a matrix depending on a parameter vector  $\boldsymbol{\theta}$  for which the mathematical expectation  $\theta_0$  and the diagonal variance matrix  $\mathbf{W}$  are assumed to be known. Obviously Equation (8) consists of measured variables and parameters of random scheduling-equations, the unmeasured variables are eliminated.

#### 3.2 Gross error detection

The measurement data is in fact subject to random and possibly gross errors. The presence of gross errors will corrupt the reconciliation calculation and spread the errors over all relatively correct data. So detection of the gross errors is of great importance. In this paper, the gross errors are identified and eliminated with the aid of an adaptation of the NT (Nodal test) (Mah, *et al.*, 1976) algorithm.

Assume the mass balance constrains is  $\mathbf{M}(\boldsymbol{\theta})\mathbf{x} = 0$ , then the residual around i node is

$$Z_{r}(i) = \frac{\left|r_{i}\right|}{\sqrt{H_{r}(i,i)}} \tag{9}$$

$$\mathbf{r} = \mathbf{M}(\theta_0)\mathbf{x} \tag{10}$$

$$\mathbf{H}_{\mathbf{r}} = \mathbf{M}(\theta_0) \mathbf{V} \mathbf{M}^{\mathrm{T}}(\theta_0) + \mathbf{G}(\mathbf{x}) \mathbf{W} \mathbf{G}^{\mathrm{T}}(\mathbf{x})$$
(11)

Where  $Z_r$  is statistic criterion based on residuals. **r** is the residuals of balance equations. **H**<sub>r</sub> is the covariance matrix of **r**. The definition of **M**( $\theta$ ) and **G**(**x**) is

$$\mathbf{M}(\boldsymbol{\theta}) = \frac{\partial (\mathbf{M}(\boldsymbol{\theta})\mathbf{x})}{\partial \mathbf{x}^{\mathrm{T}}}$$
(12)

$$\mathbf{G}(\mathbf{x}) = \frac{\partial (\mathbf{M}(\boldsymbol{\theta})\mathbf{x})}{\partial \boldsymbol{\theta}^{\mathrm{T}}}$$
(13)

At 95% confidence level, there is a critical value  $Z_c$ , if  $Z_r$  (i)>  $Z_c$ , it means there could be some gross errors on i node. Based on the structure of measurement network, it can point out which stream data has the gross errors.

## 3.3 Reconciliation algorithm with uncertain models

For mass balance model  $\mathbf{M}(\boldsymbol{\theta})\mathbf{x} = 0$ , its optimal solution can be obtained by applying a reconciliation algorithm with uncertain models (Maquin, *et al.*,

2000). We conclude the algorithm as the five following steps.

Step 1: let us define a vector of a posteriori residuals  ${}^{\Lambda}_{\mathbf{R}}$  whose components are

$$\mathbf{\hat{r}}_{i} = \mathbf{m}_{i}(\theta) \mathbf{\hat{x}}$$
(14)

Then the following deduction factor  $\beta_i$  is introduced.

$$\mathbf{\hat{r}}_{i} = \beta_{i} \mathbf{r}_{i} = \beta_{i} \mathbf{m}_{i}(\theta) \mathbf{x}$$
(15)

$$\beta_{\mathbf{i}} = 1 - \frac{\left(\mathbf{m}_{\mathbf{i}}(\theta_0) \mathbf{V} \mathbf{m}_{\mathbf{i}}^{\mathrm{T}}(\theta_0)\right)^{1/2}}{\left(\mathbf{m}_{\mathbf{i}}(\theta_0) \mathbf{V} \mathbf{m}_{\mathbf{i}}^{\mathrm{T}}(\theta_0) + \mathbf{g}_{\mathbf{i}}(\mathbf{X}) \mathbf{W} \mathbf{g}_{\mathbf{i}}^{\mathrm{T}}(\mathbf{X})\right)^{1/2}} \quad (16)$$

Step 2: let us define Weighting factor K and initialise.

$$\mathbf{K} = \begin{pmatrix} \mathbf{k}_0 & 0 \\ & \ddots & \\ 0 & & \mathbf{k}_0 \end{pmatrix}$$
(17)

This constant  $k_0$  will be chosen respectively close to zero or trending towards infinity.

Step 3: for i=1 to n (n is rows of  $\mathbf{M}(\boldsymbol{\theta})$ ), the calculus of the *i*th weighting factor. Auxiliary matrices:

$$\mathbf{r}_{\mathbf{i}} = \mathbf{m}_{\mathbf{i}} \mathbf{x} \tag{18}$$

$$\mathbf{D}_{i} = \mathbf{m}_{i} \mathbf{V} \mathbf{M}_{(-i)}^{T} \left( \mathbf{K}_{(-i)}^{-2} + \mathbf{M}_{-i} \mathbf{V} \mathbf{M}_{(-i)}^{T} \right)^{-1} \mathbf{M}_{(-i)} \mathbf{x} \quad (19)$$
$$\mathbf{Y}_{(-i)} = \left( \mathbf{I} - \mathbf{V} \mathbf{M}_{(-i)}^{T} \left( \mathbf{K}_{(-i)}^{-2} + \mathbf{M}_{(-i)} \mathbf{V} \mathbf{M}_{(-i)}^{T} \right)^{-1} \mathbf{M}_{(-i)} \right) \mathbf{V} \quad (20)$$
$$\mathbf{A}_{i} = \mathbf{m}_{i} \mathbf{Y}_{(-i)} \mathbf{m}_{i}^{T} \qquad (21)$$

Weighting factor:

$$\mathbf{k}_{i}^{2} = \frac{(1 - \beta_{i})\mathbf{r}_{i} - \mathbf{D}_{i}}{\beta_{i}\mathbf{r}_{i}\mathbf{A}_{i}}$$
(22)

Where  $\mathbf{M}_{(-i)}$  is equal to the matrix of constraints  $\mathbf{M}$  from which the *i*th row m<sub>i</sub> has been removed and  $\mathbf{K}_{(-i)}$  is the matrix of the weights from which the *i*th row and column have been removed.

Step 4: convergence analysis. If the relative variations of all the weighting factors between two consecutive estimations are greater than a fixed threshold then return to step 3, otherwise stop the algorithm.

Step 5: the estimation  $\mathbf{x}$  is

$$\overset{\Lambda}{\mathbf{x}_{i+1}} = \left(\mathbf{I} - \mathbf{y}_{i}\mathbf{m}_{i+1}^{T} \left(\mathbf{k}_{i+1}^{-2} + \mathbf{m}_{i+1}\mathbf{y}_{i}\mathbf{m}_{i+1}^{T}\right)^{-1} \mathbf{m}_{i+1}\right)^{\Lambda} \mathbf{x}_{i}$$

$$i = 0, \cdots, n-1$$
 (23)

$$\mathbf{y}_{i+1} = \left(\mathbf{I} - \mathbf{y}_{i}\mathbf{m}_{i+1}^{\mathrm{T}} \left(\mathbf{k}_{i+1}^{-2} + \mathbf{m}_{i+1}\mathbf{y}_{i}\mathbf{m}_{i+1}^{\mathrm{T}}\right)^{-1} \mathbf{m}_{i+1}\right) \mathbf{y}_{i}$$
  
i = 0,..., n - 2 (24)

$$\mathbf{x}_0 = \mathbf{x} \text{ and } \mathbf{y}_0 = \mathbf{V}$$
 (25)

## 4. APPLICATION RESULTS AND DISCUSSION

The process data of December 10, 2001 are chosen to be examined. The measured values and known covariance matrix V are given in Table 5. The information about the discrete scheduling events on this day is shown in Table 1. According to the information, we can get the mathematical expectation  $\theta_0$  and the diagonal variance matrix W of parameter vector  $\boldsymbol{\theta}$ , as shown in Table 2.

| Table | 1 Information about the discrete scheduling                             |
|-------|---|
|       | events  |
| Node  | Scheduling events   |
| 244   | Execution time of 3 scheduling events is:                               |
| M4    | $\Delta t_1 = 8$ hours, $\Delta t_2 = 8$ hours, $\Delta t_3 = 8$ hours. |
|       | Execution time of 4 scheduling events is:                               |

|       | $\Delta t_1 = 8$ nours, $\Delta t_2 = 8$ nours, $\Delta t_3 = 8$ nours.          |
|-------|--|
|       | Execution time of 4 scheduling events is:  |
| M5    | $\Delta t_4 = 24$ hours, $\Delta t_5 = 0$ hours, $\Delta t_6 = 0$ hours,         |
|       | $\Delta t_7 = 0$ hours.  |
| МС    | Execution time of 2 scheduling events is:  |
| MO    | $\Delta t_8 = 0$ hours, $\Delta t_9 = 24$ hours.                                 |
| M7    | Execution time of 2 scheduling events is:  |
| IVI / | $\Delta t_{10} = 0$ hours, $\Delta t_{11} = 24$ hours.                           |
| MO    | Execution time of 3 scheduling events is:  |
| M8    | $\Delta t_{12} = 8$ hours, $\Delta t_{13} = 8$ hours, $\Delta t_{14} = 8$ hours. |
| N/10  | Execution time of 2 scheduling events is:  |
| NI12  | $\Delta t_{15} = 24$ hours, $\Delta t_{16} = 0$ hours.                           |
| M13   | The flows of $u_{17}$ is approximately equal to                                  |
| 1113  | the flows of $x_{12}$  |

| Table 2 | <b>Mathematical</b> | expec | tation | and t  | he diagon | al |
|---------|---------------------|-------|--------|--------|-----------|----|
| ,       | variance matrix     | Wof   | param  | eter v | ector     |    |

| θ             | $	heta_0$ | Diagonal elements of W |
|---------------|-----------|------------------------|
| $	heta_1$     | 1/3       | 0.01                   |
| $\theta_2$    | 1/3       | 0.01                   |
| $\theta_3$    | 1/3       | 0.01                   |
| $\theta_4$    | 1         | 0.02                   |
| $\theta_5$    | 0         | 0.02                   |
| $\theta_{6}$  | 0         | 0.02                   |
| $\theta_7$    | 0         | 0.02                   |
| $\theta_8$    | 0         | 0.013                  |
| $	heta_9$     | 1         | 0.015                  |
| $	heta_{10}$  | 0         | 0.01                   |
| $\theta_{11}$ | 1         | 0.01                   |
|               |           |                        |

| $\theta_{12}$ | 1/3 | 0.01 |  |
|---------------|-----|------|--|
| $\theta_{13}$ | 1/3 | 0.01 |  |
| $\theta_{14}$ | 1/3 | 0.01 |  |
| $\theta_{15}$ | 1   | 0.01 |  |
| $\theta_{16}$ | 0   | 0.01 |  |
| $\theta_{17}$ | 1   | 1.1  |  |

According to Equations  $(9)\sim(13)$ , the statistic criterion  $Z_r$  are got, as shown in Table 3. At 95% confidence level, assume the critical value  $Z_c$  is equal to 30. We can find that the  $Z_r(i)$  of nodes R2, M1 are greater than 30, it means there are gross errors on nodes R2, M1. According to the structure of measurement network, the stream  $x_{11}$  is the only pipe that connects the node R2 and M1. It can be concluded that the stream data  $x_{11}$  has the gross errors. Based on mass balance equations, the measured values of  $x_{11}$  is adjusted from 315.51 to 115.51.

| Table 3 | Results | of the | statistic | criterion | $Z_r o$ | f each |
|---------|---------|--------|-----------|-----------|---------|--------|
|         |         |        | node      |           | -       |        |

| Node | Z <sub>r</sub> (i) | Node | Z <sub>r</sub> (i) |
|------|--------------------|------|--------------------|
| G1   | 0                  | R12  | 6.9                |
| R1   | 25.83              | R13  | 10                 |
| R2   | 86.3               | R14  | 1.1                |
| M9   | 8.0                | R15  | 5.2                |
| R3   | 7.4                | M1   | 33                 |
| M10  | 10.4               | M2   | 1.1                |
| M11  | 0.34               | M3   | 0.1                |
| R4   | 6.2                | G2   | 0.8                |
| M13  | 0.01               | G3   | 0.6                |
| R5   | 12.9               | G4   | 1.4                |
| M14  | 9.5                | G5   | 0.4                |
| R6   | 10.7               | G6   | 3.5                |
| R7   | 2.2                | G7   | 14.9               |
| R8   | 1.5                | G8   | 4.1                |
| M15  | 0.15               | G9   | 0                  |
| M16  | 4.2                | G10  | 1.0                |
| R9   | 7.5                | G11  | 0.003              |
| R10  | 4.3                | G12  | 2.8                |
| R11  | 0.08               |      |                    |

The reconciliation results are shown in Table 5. In order to compare and analysis, the residuals of the model are calculated, as shown in Table 4. Where R<sub>1</sub> is the residuals of the model before reconciliation,  $\mathbf{R}_2$  is the residuals of the model after reconciliation using AspenTech Advisor, R<sub>3</sub> is the residuals of the model after reconciliation using the proposed approach. It can be indicated that the reconciliation results of the proposed approach are more reasonable.

## 5. CONCLUSIONS

The conventional steady-state linear data reconciliation is insufficient sometimes, especially when discrete scheduling events are performed in the process industry. In this paper a new approach of data reconciliation for hybrid system is proposed. The information about the scheduling events is taken into account. So the random scheduling-equations are established and added to the models. Then its optimal solution can be obtained by applying a reconciliation algorithm with uncertain models. Before reconciliation, the gross errors are identified and eliminated with the aid of an adaptation of the NT (Nodal test) algorithm. Comparisons between the proposed approach and the AspenTech Advisor are made in the industrial application. The application results demonstrate the efficiency and consistency of the proposed approach. However, the assumptions that the execution time of scheduling events is measured variable whose covariance matrix is known are the limitation of the proposed approach. This problem will be the focus of our future work.

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Fig 1 Simplified measurement network from the refinery

#### Table 4 Results of the residuals of the model

| Node | $R_1$ | $R_2$ | $R_3$ | Node | $R_1$ | $R_2$ | $R_3$ |  |
|------|-------|-------|-------|------|-------|-------|-------|--|
| G1   | 0     | 25.6  | 0     | R12  | 7.7   | 0     | 0     |  |
| R1   | 89.8  | 0     | 0.2   | R13  | 20    | 0     | 0     |  |
| R2   | 85.8  | 0     | 0     | R14  | 15.5  | 0     | 0     |  |
| M9   | 22.4  | 0     | 0     | R15  | 1.6   | 0     | 0     |  |
| R3   | 20.1  | 0     | 0     | M1   | 6.6   | 0.3   | 0     |  |
| M10  | 15.0  | 0.2   | 0     | M2   | 3.8   | 0     | 0     |  |

| M11 | 1.4   | 0 | 0   | M3  | 254.9  | 30.0 | 10.4 |
|-----|-------|---|-----|-----|--------|------|------|
| R4  | 22.2  | 0 | 0   | G2  | 1.2    | 1.2  | 0    |
| M13 | 286.0 | 0 | 6.2 | G3  | 969.8  | 38.7 | 21.6 |
| R5  | 43.8  | 0 | 0   | G4  | 577.2  | 1.2  | 0    |
| M14 | 2.0   | 0 | 0   | G5  | 841.7  | 9.2  | 1.0  |
| R6  | 21.2  | 0 | 0   | G6  | 540.2  | 0    | 0    |
| R7  | 3.8   | 0 | 0   | G7  | 90     | 14.6 | 0.2  |
| R8  | 3.1   | 0 | 0   | G8  | 677.2  | 52.5 | 14.1 |
| M15 | 0.3   | 0 | 0   | G9  | 0      | 0    | 0    |
| M16 | 90    | 0 | 0   | G10 | 5018.6 | 21.7 | 22.8 |
| R9  | 26.6  | 0 | 0   | G11 | 1165   | 1.7  | 0.1  |
| R10 | 9.1   | 0 | 0   | G12 | 2471.3 | 76.4 | 13.0 |
| R11 | 0.2   | 0 | 0   |     |        |      |      |

# Table 5 Measured values and reconciliation results

| Var                    | X       | V    | $\Lambda$<br>X | Var             | X      | V     | $\Lambda$<br>X | Var              | X       | V     | $\Lambda$<br>X |
|------------------------|---------|------|----------------|-----------------|--------|-------|----------------|------------------|---------|-------|----------------|
| <b>X</b> <sub>1</sub>  | 30928.8 | 0.5  | 30928          | X37             | 768.50 | 0.9   | 771.51         | X73              | 220.4   | 0.99  | 213.2          |
| <b>X</b> <sub>2</sub>  | 11678.0 | 0.5  | 11670          | X38             | 179.19 | 1.9   | 174.51         | X74              | 151.8   | 0.99  | 149.5          |
| <b>X</b> 3             | 19250.8 | 0.5  | 19258          | X39             | 60.53  | 2.1   | 59.75          | X75              | 277.77  | 0.99  | 275.39         |
| $\mathbf{X}_4$         | 71.868  | 3.0  | 120.35         | X40             | 83.65  | 0.5   | 77.12          | X76              | 507.96  | 0.99  | 505.58         |
| X5                     | 1809.0  | 0.2  | 1812.3         | X41             | 6036.3 | 0.5   | 6032.3         | X77              | 36.322  | 0.99  | 33.945         |
| X6                     | 994.0   | 0.5  | 993.48         | X42             | 1149.8 | 0.5   | 1146.8         | X78              | 610.13  | 0.5   | 607.64         |
| $X_7$                  | 3122.7  | 0.99 | 3121.6         | X43             | 263.64 | 2.6   | 276.19         | X79              | 73.78   | 0.2   | 75.27          |
| X8                     | 2693.8  | 1.2  | 2716.9         | X44             | 514.78 | 1.8   | 524.59         | X80              | 397.03  | 0.115 | 395.98         |
| <b>X</b> 9             | 2641.9  | 0.3  | 2647.2         | X45             | 2527.2 | 0.39  | 2528.4         | X81              | 18.24   | 0.09  | 18.23          |
| X10                    | 254.98  | 0.2  | 258.34         | X46             | 772.32 | 0.04  | 772.67         | X82              | 88.392  | 0.2   | 88.625         |
| x <sub>11</sub>        | 315.51  | 3.0  | 72.933         | X47             | 2682.6 | 0.09  | 2683.3         | X83              | 4.177   | 0.99  | 2.725          |
| x <sub>12</sub>        | 284.07  | 0.9  | 270.23         | X48             | 329.08 | 0.5   | 329.36         | X84              | 37.951  | 0.99  | 42.309         |
| x <sub>13</sub>        | 268.84  | 0.4  | 264.04         | X49             | 336.24 | 0.6   | 339.51         | X85              | 2471.3  | 0.5   | 2474.4         |
| X14                    | 2842.8  | 0.8  | 2830.1         | X50             | 197.28 | 0.05  | 197.06         | X86              | 9.000   | 0.9   | 11.369         |
| X15                    | 2591.5  | 0.03 | 2591           | X51             | 577.0  | 0.2   | 575.61         | X87              | 2216.4  | 0.08  | 2215.9         |
| X16                    | 5084.8  | 0.09 | 5083.8         | X52             | 2204.7 | 0.07  | 2204.9         | X88              | 276.94  | 1.7   | 264.04         |
| X17                    | 4368.3  | 0.12 | 4366.9         | X53             | 61.50  | 0.9   | 54.851         | X89              | 7.000   | 0.2   | 5.822          |
| X18                    | 285.5   | 0.3  | 284.49         | X54             | 85.64  | 0.3   | 86.02          | X90              | 360.34  | 0.85  | 357.89         |
| X19                    | 3495.2  | 0.05 | 3494.6         | X55             | 111.00 | 0.12  | 110.42         | X91              | 75.635  | 0.5   | 75.781         |
| X20                    | 287.05  | 0.02 | 287.27         | X56             | 123.97 | 1.2   | 121.79         | X92              | 283.10  | 0.99  | 282.11         |
| x <sub>21</sub>        | 2715.0  | 0.07 | 2714.7         | X57             | 575.35 | 0.6   | 572.98         | X93              | 27.377  | 0.25  | 26.481         |
| X <sub>22</sub>        | 131.3   | 2.3  | 121.87         | X58             | 1470.3 | 0.014 | 1470.3         | X94              | 354.73  | 0.99  | 357.26         |
| X <sub>23</sub>        | 73.083  | 0.5  | 69.596         | X59             | 55.28  | 1.0   | 48.92          | X95              | 244.15  | 0.19  | 244.83         |
| X <sub>24</sub>        | 1241.0  | 0.1  | 1241.6         | X60             | 351.42 | 0.5   | 353.67         | X96              | 1527.5  | 0.78  | 1532.4         |
| X25                    | 896.92  | 0.22 | 898.28         | X <sub>61</sub> | 61.495 | 1.09  | 65.61          | X97              | 2728.0  | 0.025 | 2728.1         |
| X <sub>26</sub>        | 619.86  | 0.77 | 617.19         | X <sub>62</sub> | 121.1  | 0.99  | 123.3          | X98              | 64.00   | 0.03  | 63.95          |
| X27                    | 60.00   | 1.1  | 53.46          | X63             | 165.01 | 0.7   | 164.76         | X99              | 4100    | 0.01  | 4100           |
| X <sub>28</sub>        | 1943.0  | 0.4  | 1942.1         | X <sub>64</sub> | 1056.9 | 0.025 | 1056.9         | X100             | 7789.2  | 0.09  | 7789.9         |
| X29                    | 199.92  | 0.12 | 197.81         | X65             | 34.61  | 1.3   | 31.43          | X <sub>101</sub> | 13600.8 | 0.025 | 13601          |
| X30                    | 70.01   | 2.6  | 60.92          | X66             | 92.45  | 1.1   | 90.36          | X102             | 3058    | 0.025 | 3058           |
| <b>X</b> <sub>31</sub> | 2250.3  | 0.55 | 2248.4         | X67             | 13.99  | 0.99  | 13.37          | X <sub>103</sub> | 191.83  | 0.03  | 191.75         |
| X <sub>32</sub>        | 106.98  | 2.0  | 112.39         | X68             | 20.9   | 1.05  | 18.1           | X104             | 42.03   | 0.012 | 42.09          |
| X33                    | 39.84   | 0.39 | 39.77          | X69             | 87.07  | 1.76  | 90.77          | X105             | 2422.1  | 0.87  | 2430.2         |
| X34                    | 274.55  | 0.3  | 274.11         | X70             | 9808.0 | 0.15  | 9807.2         | x <sub>106</sub> | 235.00  | 0.03  | 234.98         |
| X35                    | 851.07  | 0.8  | 851.19         | X71             | 192.82 | 2.9   | 177.27         | X107             | 844.37  | 1.2   | 845.12         |
| X36                    | 257.15  | 0.45 | 260.39         | X72             | 9508.4 | 0.04  | 9507.5         | X108             | 18.56   | 1.0   | 15.93          |