PID-P CONTROLLER FOR TITO SYSTEMS

Padhy, P.K.[1], Majhi, S.[1] and Atherton, D.P.[2]

[1]Department of Electronics & Communication Engineering Indian Instutute of Technology Guwahati, Guwahati – 781039, Assam, India
[2]Department of Engineering and Design, University of Sussex, Falmer, Brighton, BN1 9QT, UK

Abstract: A simple closed loop identification method is proposed to design a PID-P controller for TITO systems. A pair of relays is simultaneously connected in parallel with the PID controllers. Based on the limit cycle data, a diagonal TITO transfer function model of the system dynamics are obtained. Then the PID parameters are estimated using the identified model and phase and gain margin design criteria. The effectiveness of the proposed control scheme is illustrated by a simulation study.

Keywords: PID-P controller, TITO system, Relay, Limit cycle, Identification.

1. INTRODUCTION

The main difficulties in controlling a multi input-multi-output (MIMO) system are the interactions between the loops and among various plant variables. To avoid these difficulties the control loops are often independently tuned as SISO systems. The commonly used well-known form of a MIMO system is a TITO (two-input two-output) system. Many methods on the design of PID controllers for TITO systems have been discussed in the literature (Maciehowski, 1989; Palmor, *et al.*, 1995; Zhuang and Atherton, 1994; Padhy and Majhi, 2005a, b).

The general approach often adopted is to find a model for the plant and design a controller based on this model. A decentralized relay test can be used for the identification of a plant model without prior knowledge of the plant dynamics. In this paper, a new identification technique for a TITO system with significant interaction is proposed. During identification, a PID-P type controller is used where the PID controller remains in the feed forward path and the P-controller in the inner feedback path. A pair of relays is used in parallel with the PID controllers for limit cycle experiments. The P controller in the inner feedback loop is used to make the system relatively more stable and to reduce the interaction between the loops. An advantage of the technique is that the inner P-controller can be used to stabilize an unstable plant transfer function (Majhi and Atherton, 2000). Technical difficulties associated with robust performance of the system are presented in section-2. Section-3 describes the identification technique to estimate the plant transfer function model parameters. A method to estimate the steady state gain of the plant model is given in section-4. Section-5 includes the PID controller design method given in (Padhy and Majhi, 2005b). A simulation study is considered in section-6. Finally, conclusions are given in section-7.

2. P-CONTROLLER FOR REDUCTION OF INTERACTION

Fig.1 shows the control structure of the TITO plant with PID-P controllers. Here a proportional controller is connected in the inner feedback path.

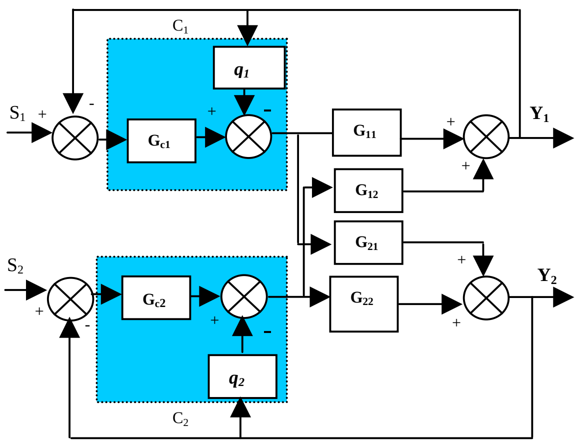

Fig. 1. PID-P control scheme

Fig.2 has two relays in parallel with the PID controllers for inducing a limit cycle. The limit cycle parameters (A_i and ω_u), the amplitudes of the limit cycle at the two relay inputs and the limit cycle frequency, are measured in the experiment. From the measured parameters a linearized model is identified and the controller parameters are calculated. The feedback proportional controllers make the system relatively more stable and can also be adjusted to reduce the interaction between the loops.

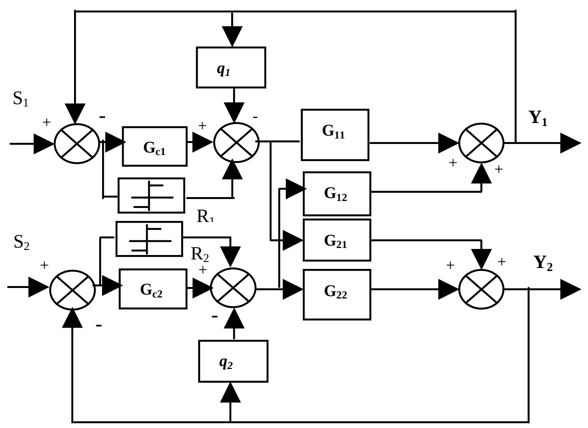

Fig. 2. Redrawing of Fig.1.

Fig.2 can be redrawn as in Fig.3 for a better understanding of the advantages of the proposed identification method.

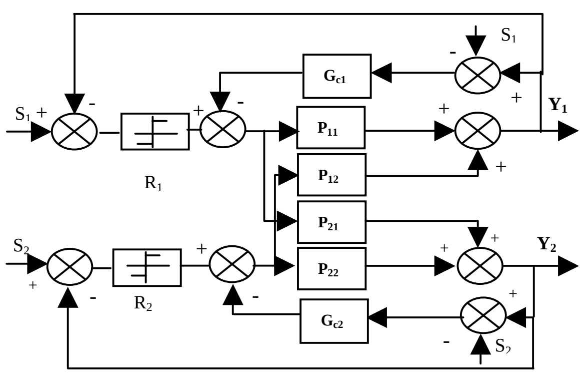

Fig. 3. Redrawing of Fig.2

From Figs.2 and 3, one obtains

$$P(s) = \frac{G(s)}{I + G(s)Q} \qquad \text{.......... ....(1)}$$

$$= \frac{\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}\begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix}}$$

$$= \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}\begin{bmatrix} 1+G_{11}q_1 & G_{12}q_2 \\ G_{21}q_1 & 1+G_{22}q_2 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$$

where $G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}$ is the original plant model and $Q = \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix}$. Then,

$$P_{11} = \frac{G_{11}(1+G_{22}q_2) - G_{12}G_{21}q_1}{(1+G_{11}q_1)(1+G_{22}q_2) - G_{12}G_{21}q_1q_2}$$

$$P_{12} = \frac{-G_{11}G_{12}q_2 + G_{12}(1+G_{11}q_1)}{(1+G_{11}q_1)(1+G_{22}q_2) - G_{12}G_{21}q_1q_2}$$

$$P_{21} = \frac{G_{21}(1+G_{22}q_2) - G_{22}G_{21}q_1}{(1+G_{11}q_1)(1+G_{22}q_2) - G_{12}G_{21}q_1q_2}$$

$$P_{22} = \frac{-G_{21}G_{12}q_2 + G_{22}(1+G_{11}q_1)}{(1+G_{11}q_1)(1+G_{22}q_2) - G_{12}G_{21}q_1q_2}.$$

In the above expressions the complex variable‘s’ has been omitted for ease of presentation. Assuming $q_1 = q_2$, at steady state condition (from Fig.3) we have

$$P_{11}(0) = \frac{K_{11}(1+K_{22}q_2) - K_{12}K_{21}q_1}{(1+K_{11}q_1)(1+K_{22}q_2) - K_{12}K_{21}q_1q_2}$$

$$P_{12}(0) = \frac{K_{12}}{(1+K_{11}q_1)(1+K_{22}q_2) - K_{12}K_{21}q_1q_2}$$

$$P_{21}(0) = \frac{K_{21}}{(1+K_{11}q_1)(1+K_{22}q_2) - K_{12}K_{21}q_1q_2}$$

$$P_{22}(0) = \frac{-K_{21}K_{12}q_2 + K_{22}(1+K_{11}q_1)}{(1+K_{11}q_1)(1+K_{22}q_2) - K_{12}K_{21}q_1q_2}.$$

where $\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$ are the steady state values of the plant ‘G’.

Here same values of q_1 and q_2 are chosen to reduce the values of off-diagonal elements of P. As the values of off-diagonal elements are decreased, the interaction is reduced. With higher values of q_1 and q_2, the values of off-diagonal elements can further be reduced. Let the input signal X be sinusoidal and the relay elements be described by the describing function matrix $N = \begin{bmatrix} N_1 & 0 \\ 0 & N_2 \end{bmatrix}$.

Then, for a limit cycle

$$G(j\omega)NX = -X$$
$$\Rightarrow \left[I + G(j\omega)N\right]X = 0$$
$$\Rightarrow \det\left[I + G(j\omega)N\right] = 0 .$$

Assuming the limit cycles of the loops are at the same frequency (ω), the input signal vector elements can be expressed as

$$-X_1 = G_{11}(j\omega)N_1X_1 + G_{12}(j\omega)N_2X_2 \text{............}(2)$$

$$-X_2 = G_{21}(j\omega)N_1X_1 + G_{22}(j\omega)N_2X_2 \text{......}(3)$$

Setting $X = \left[A_1\cos(\omega t) \quad A_2\cos(\omega t + \psi)\right]$, the values of A_1, A_2, ω, and Ψ can be calculated from equations (2) and (3). Also choosing a suitable suitable h_1/h_2 ratio one can find the approximate relationship between K_{1cr} and K_{2cr} as

$$\frac{K_{1cr}}{K_{2cr}} = \frac{P_{21}(0)\dfrac{h_1}{h_2} + P_{22}(0)}{P_{11}(0) + P_{12}(0)\dfrac{h_2}{h_1}} \text{...}(4)$$

where K_{1cr} and K_{2cr} are the critical gains and h_1 and h_2 are the relay amplitudes of loop-1 and loop-2, respectively.

As the values of P_{12} and P_{21} are reduced for certain q_1 and q_2, P_{12} and P_{21} can be brought near to 'zero'. So from equation (4), we have

$$\frac{K_{1cr}}{K_{2cr}} \approx \frac{P_{22}(0)}{P_{11}(0)}. \qquad \text{....................}(5)$$

For TITO systems two critical gains (K_{1cr}, K_{2cr}) are required for stability study. Three typical cases of stability limit are shown in fig.4.

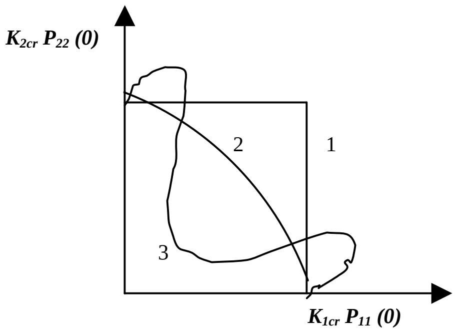

Fig. 5(a). Typical cases of stability limit

Since the significant performance parameter is loop gain, the axes are K_{icr} P_{ii} (0). Each point on the curves corresponds to a pair of gains (K_{1cr}, K_{2cr}) and a critical frequency (ω_{cr}). If the system has less interaction i.e. either $P_{12}(s)$ or $P_{21}(s)$ or both zero, the stability limit takes the rectangular form (1 in Fig.4). In this case the two critical gains are independent of each other and the system will be unstable if one of the gains exceeds its critical value. The other two cases (2 and 3 in Fig.4) represent a system with interactions.

The choice of DCP (desired critical point i.e. K_{1cr}, K_{2cr}, ω_{cr}) depends on the relative importance of the two loops, which is expressed by weighting factor

$$\tan\phi = \frac{C}{1} = \frac{K_{2cr}P_{22}(0)}{K_{1cr}P_{11}(0)} \text{..........} \quad (6)$$

where C is called a weighting factor of second loop with respect to the first loop. $C > 1$ means loop 2 requires tighter control relative to first loop and vice versa. For satisfactory performance C_d (desired weighting factor) should be 1 i.e. $\phi_d = 45°$. From equation (5) and (6)

$$\tan\phi = \frac{C}{1} = 1 \Rightarrow \phi = 45^0$$

Therefore, error = $\left|\phi - \phi_d\right| = 0 \le \varepsilon$

So it is observed that the interaction between the loops is reduced due to inner P-controller.

3. IDENTIFICATION OF PLANT MODEL

The second order transfer function model with time delay has been found to be widely adequate for many industrial applications encountered. This choice of model also greatly facilitates the use of automatic control tuning approaches.

Let $G_p(s) = \begin{bmatrix} G_{p1}(s) & 0 \\ 0 & G_{p2}(s) \end{bmatrix}$

be the plant model with diagonal elements

$$G_{pi}(s) = \frac{K_{pi}}{(1 + sT_i)^2}e^{-\theta_i s} \text{....}(7)$$

and $G_c(s) = \begin{bmatrix} G_{c1}(s) & 0 \\ 0 & G_{c2}(s) \end{bmatrix}$

be the PID controller with elements

$$G_{ci}(s) = K_{ci}\left(1 + \frac{1}{T_{Ii}s}\right)\left(1 + T_{di}s\right). \text{.....}(8)$$

where i =1, 2. Using the methods given by Padhy and Majhi (2005a), expressions for the parameters of the plant models can be written as

$$T_i = \frac{\sqrt{K_{pi}\sqrt{a_i^2 + b_i^2} - 1}}{\omega_{cr}}$$

$$\theta_i = \frac{\pi + \tan^{-1}\left(\dfrac{b_i}{a_i}\right) - 2\tan^{-1}\left(\omega_{cr}T_i\right)}{\omega_{cr}}$$

where

$$a_i = \frac{4h_i}{\pi A_i} + K_{ci}\left(1 + \frac{T_{di}}{T_{Ii}}\right)$$

$$b_i = K_{ci}\left(\omega_{cr}T_{di} - \frac{1}{\omega_{cr}T_{Ii}}\right)$$

4. STEADY STATE GAINS

Figs. 5(a) and 5(b) are the redrawing of Fig.1 to estimate steady state gains (K_{pi}).

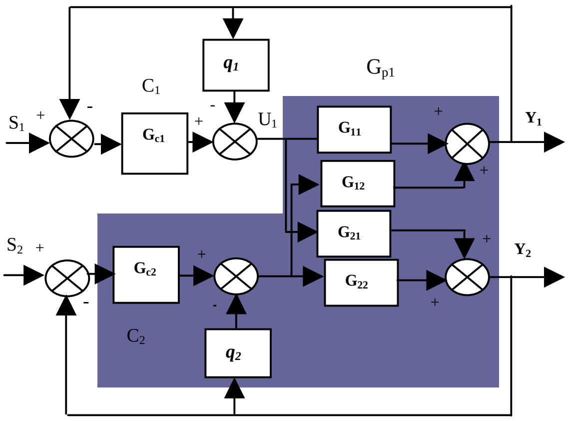

Fig. 5(a). Scheme to obtain steady state gain of G_{p1}

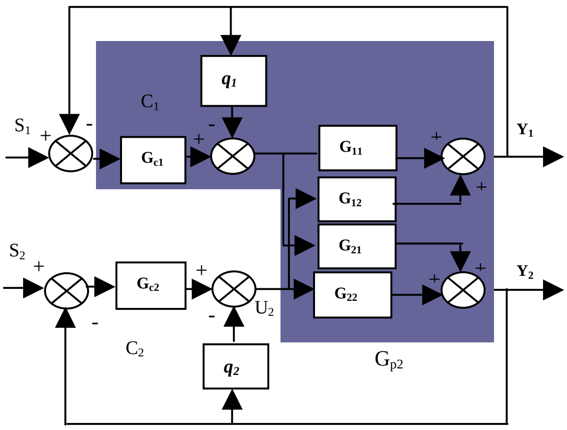

Fig. 5(b). Scheme to obtain steady state gain of G_{p2}

Here C_1 (s) and $C_2(s)$ control $G_{p1}(s)$ and G_{p2} (s) instead of $G_{11}(s)$ and $G_{22}(s)$ for good loop performance. The expression of $G_{p1}(s)$ and G_{p2} (s) can be written as

$$G_{p1}(s) = G_{11}(s) - \frac{G_{12}(s)G_{21}(s)}{C_2^{-1}(s) + G_{22}(s)}$$

$$G_{p2}(s) = G_{22}(s) - \frac{G_{12}(s)G_{21}(s)}{C_1^{-1}(s) + G_{11}(s)} .$$

The steady state gains can be obtained from the ratio $\frac{Y_i(0)}{U_i(0)} \approx G_{pi}(0)$.

As the controllers possess integrators, at steady state conditions, $C_1^{-1}(s)=(q_1+G_{c1}(s))^{-1}$ and $C_2^{-}(s)^{1}=(q_2+G_{c2}(s))^{-1}$ will be zero. Therefore

$$K_{p1} = G_{p1}(0) = G_{11}(0) - \frac{G_{12}(0)G_{21}(0)}{G_{22}(0)} ,$$

$$K_{p2} = G_{p2}(0) = G_{22}(0) - \frac{G_{12}(0)G_{21}(0)}{G_{11}(0)}$$

It is observed from the above expressions that the values of steady state gains are independent of the controller parameters.

5. CONTROLLER DESIGN

The inner feedback controllers, q_i, are designed before the forward path PID controller. After setting the values of q_1 and q_2, the parameter of PID controller are tuned using the phase and gain margin based design criteria.

5.1. Design of inner feedback P-controller

To assist in seeing the design procedure for the inner feed back P-controller q_i, Fig.1 has been redrawn and shown in Fig.6.

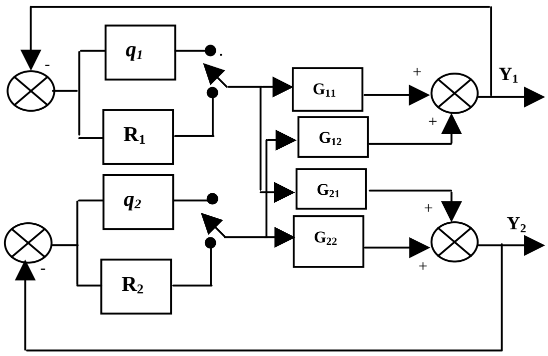

Fig. 6. Scheme to obtain q_1 and q_2

The P-controller is replaced by two symmetrical relays to calculate two critical gains (K'_{1cr}, K'_{2cr}). Using generalized Ziegler-Nichols tuning rule for P-controller, the proportional gains become

$$K_{ci} = a_i K'_{icr} , \qquad \text{..............(9)}$$

where $0.5 \le a_i \le \sqrt{0.5}$.

The critical gains K'_{icr} can be related with q_i as

$$q_i K_p = a_i K'_{icr} \qquad \text{...............(10)},$$

where $K_p = K_{p1} + K_{p2}$ and $i = 1, 2$.

5.2. Design of PID controller

A set of PID controller tuning formulae have been derived by Padhy and Majhi (2005b) to achieve user-defined phase and gain margins. They give analytical relations between controller and identified plant parameters. As given by Padhy and Majhi (2005b), the expressions for the controller parameters are

$$K_{ci} = \left(\frac{C_{1i}}{K_{pi}}\right)\left(\frac{T_i}{\theta_i}\right)$$

$$T_{di} = T_i$$

$$T_{Ii} = \frac{T_i}{1 + \left(C_{2i}\left(\frac{T_i}{\theta_i}\right)\right)}$$

where

$$C_{1i} = \frac{2\phi_{mi} + \pi(g_{mi} - 1)}{2(g_{mi}^2 - 1)};$$

$$C_{2i} = 2g_{mi}C_{1i}\left(1 - \frac{2g_{mi}C_{1i}}{\pi}\right)$$

and ϕ_{mi} and g_{mi} are the phase margin and gain margin of i^{th} loop.

6. SIMULATION STUDY

Consider a fourth order plant with transfer function matrix (Zhuang and Atherton, 1994)

$$G(s) = \frac{1}{D(s)}\begin{bmatrix} 1 & \frac{-2.4}{1+0.5s} \\ \frac{0.5}{1+0.1s} & 1 \end{bmatrix}$$

where $D(s) = (1+0.1s)(1+0.2s)^2$.

Choosing $h_1 = h_2$=5 and q_1=q_2=1, the error is found to be $|\phi - \phi_d|$ =0.5 ^{0}. The limit cycle outputs of the loops are shown in Figs. 7(a) and 7(b).

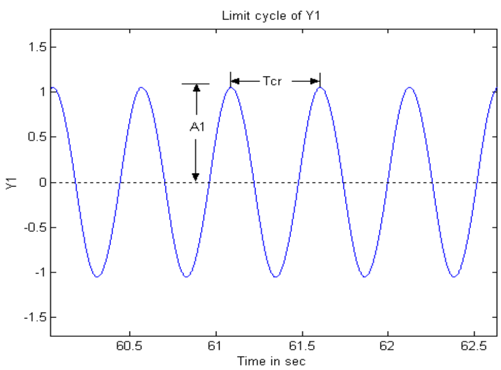

Fig.7 (a). Limit cycle output of loop-1

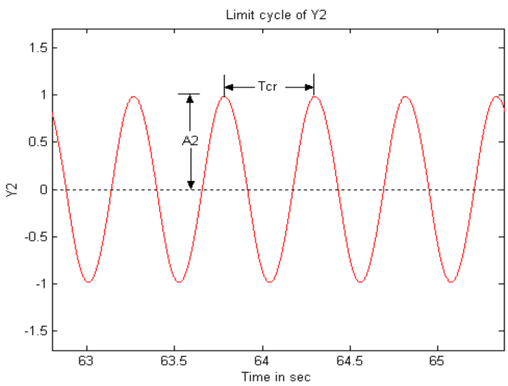

Fig.7 (b). Limit cycle output of loop-2

The values of critical gains and frequency for the chosen relay heights are tabulated below.

Table 1: Critical gains and critical frequency

h_1	h_2	K_{1cr}	K_{2cr}	ω_{cr}	$\lvert\phi-\phi_d\rvert$ in °
5	5	7.062	7. 471	12.146	1.6

Using the method given by Padhy and Majhi (2005a), the identified plant transfer function becomes

$$G_p(s) = \begin{bmatrix} \frac{2.2e^{-0.0412s}}{(1+0.3163s)^2} & 0 \\ 0 & \frac{2.2e^{-0.0404s}}{(1+0.3226s)^2} \end{bmatrix}.$$

Choosing ϕ_m=45^0 and g_m=2 for both the loops, the parameters of the controller G_{c1} were estimated as K_{c1} = 2.7389, T_{d1} = 0.3163, T_{i1} = 0.3163 and that of G_{c2} were K_{c2} = 2.8470, T_{d2} = 0.3226, T_{i2} = 0.3224. For comparison of results, the design values suggested by two other methods are considered here. Zhuang's CL method (Zhuang and Atherton, 1994) suggests K_{c1} = K_{c2} = 5.830, T_{d1} = T_{d2} = 0.561 and T_{i1} = T_{i2} = 0.140. Similarly K_{c1} = K_{c2} = 6.40, T_{d1} = T_{d2}= 0.364 and T_{i1} = T_{i2}= 0.091 were estimated by the use of Ziegler-Nichols method (Zhuang and Atherton, 1994). The unit step input responses of the loops are shown in Figs. 8(a) and 8(b).

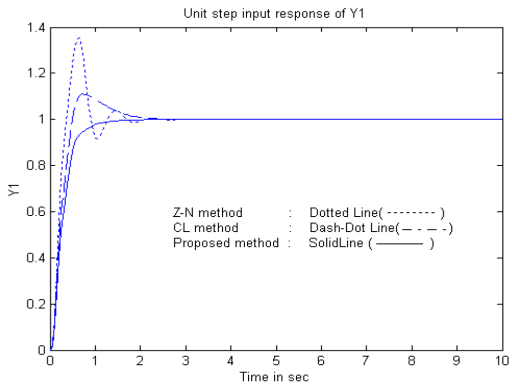

Fig. 8(a). Unit step input responses of loop-1

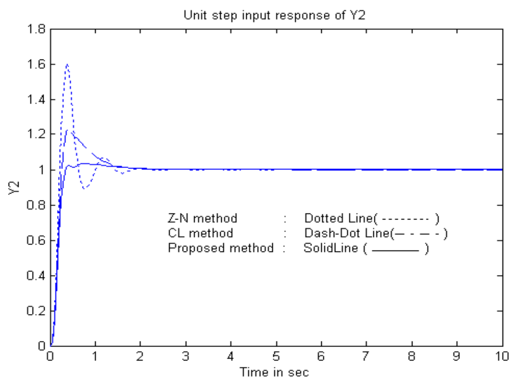

Fig. 8(b). Unit step input responses of loop-2

It is evident from the figures that, improved performances in terms of overshoot, speed of response and settling time are achievable by the proposed identification and control scheme.

In (Neiderlinski, 1971), for the above plant, the critical gains K_{1cr} and K_{2cr} are calculated as 8.25. So, using equation (10), the values of q_1 and q_2 will be 0.94. The unit step input responses for different values of q i.e. 0.5, 0.94 and 2 are shown in Figs. 9(a) and 9(b).

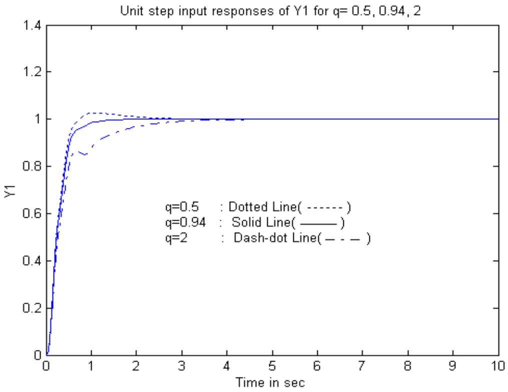

Fig. 9(a). Unit step input responses of loop-1 for *q* = 0.5, 0.94 and 2.

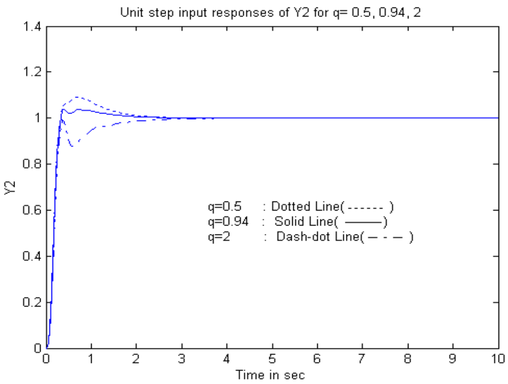

Fig. 9(b). Unit step input responses of loop-2 for *q* = 0.5, 0.94 and 2.

It is observed from the above figures 9(a) and 9(b) that the value of *q* = 0.94 results in improved performance.

7. CONCLUSION

A new identification method based on relay experiment is presented in this paper. Two SISO second order transfer function models with delay are identified. Based on the identified models, the PID controller parameters are tuned. It is observed from the simulation studies that the proposed control method results in satisfactory time domain performances such as overshoot, speed of response and settling time. This method can also be extended to design PID controller for MIMO systems.

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