A WAVELET-BASED ITERATIVE LEARNING CONTROL SCHEME FOR MOTION CONTROL SYSTEMS

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Abstract: A wavelet-based iterative learning control scheme is presented in this article. To improve the learning behaviour, wavelet transform is employed to extract the learnable dynamics from measured output signal before it can be used to update the control profile. The wavelet transform is adopted to decompose the original signal into many low-resolution signals that contain the learnable and unlearnable parts. The desired control profile is then compared with the leanable part of the transformed signal. Thus, the effect from unlearnable dynamics on the controlled system can be attenuated solely by a feedback controller design. Both the feedback and learning controllers are of proportional type to show the efficacy of this proposed scheme. Convergence analysis is also presented to provide theoretical background. A typical DC servo system is employed as the control target for experimental verification. Experimental results have shown a much-improved speed-tracking performance.

Keywords: Iterative Learning Control, Wavelet Transform, Servo Control.

1. INTRODUCTION

Iterative learning control (ILC) is a very effective control methodology that improves tracking control performance through repeated trials. Since the pioneering work from Arimoto (Arimoto et al. 1984) introduced iterative learning control (ILC), many researchers have been focused on this topic. A more detailed discussion on this control technique can be found in (Moore, et al., 1992, Mo0re, 1999). The ILC scheme proposed by Arimoto et al. is a feedforward only action soley depended upon the previous performance of an identical task. Therefore, the resulting control system is basically an open-loop system. Although the applicability of the feedforward only learning control scheme, also called previous cycle error (PCE) type ILC scheme (Bien and Xu 1998), has been theoretically proven stable, it still suffers from certain drawbacks. Firstly, this ILC system may encounter adverse effects if the open-loop system is unstable and it is not robust against disturbances that are not repeatable among iterations. In practical applications, several feedback-based ILC algorithms were proposed (Chen, et al., 1997, Moon, et al., 1998, Tayebi and Zaremba, 2003). These feedback-based ILC schemes are also called current cycle error (CCE) type ILC. In these schemes, a feedback controller is adopted to ensure the closed-loop stability and suppress the exogenous disturbances, and an iterative learning law is employed to provide improved tracking performance over a pre-specified trajectory utilizing previous control results. The CCE-type ILC scheme can achieve faster convergent rate than the PCE-type ILC. Most existing literatures are mainly focused on the derivation of sufficient conditions under which the ILC system converges. Although the CCE-type ILC has been applied to many nonlinear systems and provided satisfactory performance, it has certain limitations, still. Among those, the most stringent condition is that the system must be “learnable”, which requires that system dynamics must be “repeatable” including disturbances during iterative process. Unfortunately, many practical systems do not possess such properties and the disturbances are usually non-repeatable. For motion control system using ILC scheme, nonlinear disturbances such as dead-zone, backlash, friction or certain state dependent nonlinearities are always non-repeatable

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during iterative process. Alternatively, the unlearnable dynamics of a motion control system will corrupt the control profile and may cause instability during the iterative operation. In this paper, an enhanced iterative learning scheme with wavelet transform is presented, which is consisted of a feedforward and feedback controllers as most CCE-type scheme. To improve its learning behavior, wavelet transform is employed to extract the learnable dynamics from the output response before it can be used to update the control profile. The wavelet transform (Cohen and Ryan, 1995) can be adopted as a contraction mapping operator which decomposes the original signal into many low-resolution signals with the learnable and unlearnable parts. The desired control profile is then compared with the learnable part of the transformed signal only. By this way, the effects of unlearnable dynamics of the non-repeatable system can solely be attenuated by a feedback controller but the ILC.

2. PROBLEM STATEMENT

A CCE-type ILC scheme would perform very well while applying to repeatable and learnable systems. Considering a linear time-invariant system with non-repeatable disturbance is described as follows (Tao and Kokotovic, 1995, Linden and Lambrechts, 1993).

$$\begin{aligned} \dot{x}_k &= Ax_k + Bu_k + Bf(x_k, t) \\ z_k &= Cx_k \\ y_k &= DZ(z_k) \end{aligned} \tag{1}$$

where $x \in \Re^n$, $u \in \Re^m$, k is the number of iteration and $f(x_k,t)$ is the nonlinear and non-repeatable disturbance, such as noise, friction and other state-depenedent nonlinearities etc. The non-repeatable and asymmetrical dead-zone $DZ(.)$ with input $z_k(t)$ and the output $y_k(t)$ may be described by:

$$y_k(t) = \begin{cases} l_f(z_k(t) - \eta_k) & \text{for } z_k(t) > \eta_k \\ 0 & \text{for } \mu_k \le z_k(t) \le \eta_k \\ l_r(z_k(t) - \mu_k) & \text{for } z_k(t) < \mu_k \end{cases} \tag{2}$$

where $l_f > 0$, $l_r > 0$, $\mu_{\min} < \mu_k < 0$, $0 < \eta_k < \eta_{\max}$, the bounds $\mu_{\min}$ and $\eta_{\max}$ are unknown constants.

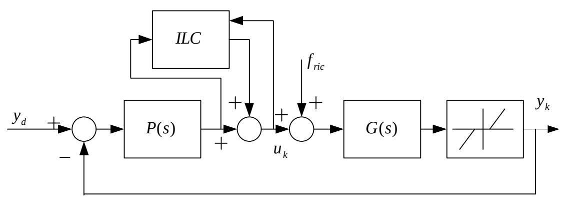


Fig. 1 A Typical CCE-Type ILC

Fig. 1 shows the block diagram of the control system described as above with a typical CCE-type ILC scheme (Owens 1992). $G(s)$ is the nominal transfer function of the system and $P(s)$ is the causal "learning" operator feeding back the current trial error. The physical meaning of this form of learning law is that current trial feedback obtained causally during the trial by normal feedback mechanisms for updating the control input. Now, consider the case when the input update law is a time-domain learning algorithm and set $P(s) = \overline{\beta}$ as the so-called P-type learning law (Owens, 1992) to yield

$$u_{k+1}(t) = \overline{\alpha} u_k(t) + \overline{\beta} e_{k+1}(t) \tag{3}$$

Obviously, the response due to non-repeatable dynamics will contaminate the learning control signal $u_k(t)$ through the feedback control process and the CCE-type ILC system may become diverge (Mezghan, *et al.*, 2001, Zheng and Alleyne, 2003). From the viewpoint of system dynamics, the nominal plant is corresponds to the learnable dynamics while the non-repeatable disturbances may excite the unlearnable dynamics. To ensure the stability of the controlled system, the unlearnable dynamics must be explicitly distinguished from the response of the system. In this way, the dynamics of the controlled system are decomposed into a learnable part and an unlearnable part by suitably choosing signal processing tools, e.g. wavelet transform. We then design the ILC scheme devoted to learn the learnable dynamics. Thus, the stability of the ILC scheme would not be destroyed by the unlearnable part.

3. AN ENHANCED ILC USING WAVELET TRANSFORM

3.1 The wavelet transform

The basic idea of the wavelet transform is to represent any arbitrary function $f(t)$ as a superposition of wavelets. Any such superposition decomposes $f(t)$ into different scale levels, where each level is then further decomposed with a resolution adapted to the level. The wavelet transform of a time function $f(t)$ with a mother wavelet $\psi_{a,b}(t)$, is defined as the inner product of $f(t)$ with $\psi_{a,b}(t)$ i.e.

$$Wf(a,b) = \langle f(t), \psi_{a,b} \rangle \tag{4}$$

where a is a scaling factor and b is a shift parameter. Using the wavelet transform, the signal $f(t)$ can be decomposed into multiple-level signals. Hence, the signal $f(t)$ is broken down into many lower-resolution components. This process is expressed in terms of the wavelet decomposition tree as shown in Fig 2.

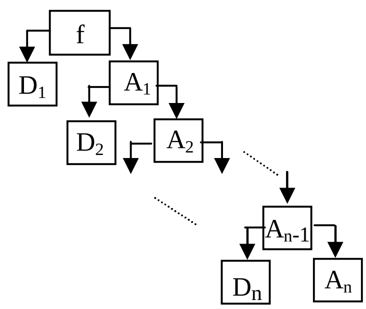


Fig. 2 The Decomposition Tree for *f(t)*

A_n and D_j, j=1,2,3,..,n are called the approximations (low-frequency components) and details (high-frequency components), respectively, and n is the level of decomposition. After the process of decomposition, the original signal $f(t)$ can also be reconstructed through the reconstruction filters as below

$$f(t) = W * f(t) = A_n + \sum_{j=1}^{n} D_j \tag{5}$$

In practical applications, some or all of the detail components can be excluded during the reconstruct process. In other words, the wavelet transform can be a contraction mapping on $f(t)$. To show this fact, let

$$f_1 = W * f \tag{6}$$

where the symbol $W*$ stands for the wavelet transform as a filtering operator. Then we have $\| f_1 \|_1 < \| f \|_1$. Denote $\| . \|_1$ as the one-norm of a time function. The following lemma will be useful in the sequel.

***Lemma**: Suppose that a signal f(t) can be decomposed as (5) with $\|A_n\|_1 > \gamma \sum_{j=1}^{n} \|D_j\|_1$, where $\gamma > 2$, then there exists a positive real number a, and $\frac{2}{\gamma} < \alpha < 2 - \frac{2}{\gamma}$, such that wavelet operator $(1-\alpha W*)$ is a contraction mapping on $f(t)$, i.e.*

$$\| (1-\alpha W*)^k f(t) \|_1 \le \rho^k \| f(t) \|_1 \to 0 \text{ as } k \to \infty$$
$$\text{for } 0 < \rho < 1$$

<u>*Proof*</u>:
From (5), without losses of generality, let $n=1$. The signal f can be decomposed into

$$f = A_1 + D_1 \tag{7}$$
$$\text{with } \|A_1\|_1 > \gamma \|D_1\|_1 \text{ for } \gamma > 2 \tag{8}$$

The detail components can be eliminated during the reconstructed process as described in (6) such that

$$\| f - \alpha W * f \|_1 = \| (1-\alpha W*) f \|_1$$
$$= \| A_1 + D_1 - \alpha A_1 \|_1 = \| (1-\alpha) A_1 + D_1 \|_1 \tag{10}$$

If we choose a constant δ, such that $\delta > \frac{1}{\gamma - 1}$ and applying (8) , we have

$$\delta \| A_1 \|_1 > (1+\delta) \| D_1 \|_1 \tag{11}$$

Continuing from (10)

$$\| (1-\alpha) A_1 + D_1 \|_1 < (|1-\alpha| + \frac{1}{\gamma}) \|A_1\|_1$$
$$< (|1-\alpha| + \frac{1}{\gamma}) \|A_1\|_1 + (|1-\alpha| + \frac{1}{\gamma}) [\delta \|A_1\|_1 - (1+\delta) \|D_1\|_1]$$
$$< (1+\delta)(|1-\alpha| + \frac{1}{\gamma}) [\|A_1\|_1 - \|D_1\|_1]$$
$$< \rho \|A_1 + D_1\|_1 = \rho \|f\| \tag{12}$$

where $\rho \equiv (1+\delta)(|1-\alpha| + \frac{1}{\gamma})$, with $\frac{2}{\gamma} < \alpha < 2 - \frac{2}{\gamma}$ and $\delta > \frac{1}{\gamma - 1}$, $\gamma > 2$

We conclude that $\rho < 1$

Therefore,

$$\| (1-\alpha W*) f \|_1 < \rho \| f \|_1$$
$$\Rightarrow \| (1-\alpha W*)^k f \|_1 < \rho^k \| f \|_1 \to 0 \quad \text{as } k \to \infty$$

Q.E.D.

3.2 The Proposed Controller Design

For the dynamic system of interest which has the unlearnable dynamics, the CCE-type ILC scheme fails to work properly over the iterative process. In such applications, the ILC controller updates the profile of control effort by using tracking error which is contaminated by non-repeatable disturbance, dead-zone and measured noise etc., the error will eventually grow up and the learning behaviour will fail. In other words, the iterative learning controller will induce the system instability due to the unlearnable dynamics inherent in the non-repeatable system. A block-diagram of the overall control system is shown in Fig. 3, where y_d is the desired output trajectory and y_k is the system output of the k_{th} iteration.

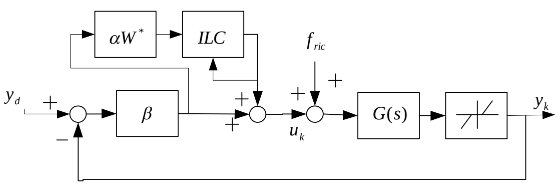


Fig. 3 The Proposed Control Structure

The control scheme is expressed as follows

$$u_k(t) = u_k^l(t) + u_k^f(t) \tag{13}$$
$$u_k^l(t) = u_{k-1}^l(t) + \alpha W * u_{k-1}^f \tag{14}$$
$$u_k^f(t) = \beta e_k(t) \tag{15}$$

where α is a positive and fixed learning gain, the u_k^f is a proportional feedback controller and u_k^l is the iterative learning controller. The feedback control signal u_k^f is processed by wavelet transform and then applied to update the learning profile. In practical applications, it is reasonable to assume that the control system with transfer function $G(s)$ satisfy the following properties.

(A1) The transfer function G(s) has a positive real part for $0 < \omega < \omega_c$, i.e., $\inf_{0<\omega<\omega_c} \mathrm{Re}(G(j\omega)) > 0$, where ω_c denotes the cut-off frequency of the system.

(A2) The control signal $u_k(t)$ can be decomposed into the learnable part $u_d^l(t)$ and the unlearnable part $u_k^{ul}(t)$, i.e.,

(A3) At the k -th iteration, the frequency spectral of the unlearnable part u_k^{ul} is of band-pass type and is bounded by a residual function, $\varepsilon(t)$. After wavelet transform, we have

$$\| W * u_k^{ul} \|_1 \leq \|\varepsilon\|_1 \qquad \forall\, k$$

Theorem: *Suppose that an ILC system satisfies assumption (A1)-(A3) and with the control laws described as (13)-(15). As* $\|\varepsilon\|_1 \to 0$, *there exists a real number* α, *as defines in Lemma, such that the iterative learning process will convergent, i.e.*

$$\lim_{k\to\infty} \| u_k^l \|_1 \to \| u_d^l \|_1$$

<u>Proof</u>:
From (13) and (A2), at the k-th iteration, the feedback signal is expressed as

$$u_k^f = u_k - u_k^l = u_d^l + u_k^{ul} - u_k^l \qquad (16)$$

Applying the wavelet operator and using (A3)

$$W * u_k^f \leq W * u_d^l - W * u_k^l + \varepsilon \qquad (17)$$

Substituting (17) into (14) to yield

$$\begin{aligned} \left\|u_{k+1}^l\right\|_1 &\leq \left\|u_k^l + \alpha(W * u_d^l - W * u_k^l + \varepsilon)\right\|_1 \\ &= \left\|(1-\alpha W*)u_k^l + \alpha(W * u_d^l + \varepsilon)\right\|_1 \end{aligned} \qquad (18)$$

After iterating k times on u_k^f in (18), the following inequality holds.

$$\begin{aligned} \left\|u_k^l\right\|_1 \leq \Big\|&(1-\alpha W*)^{k+1} u_0^l + [1-(1-\alpha W*)^{k+1}]u_d^l \\ &+ [1-(1-\alpha W*)^{k+1}(W*)^{-1}\varepsilon\Big\|_1 \end{aligned} \qquad (19)$$

Since the operator $(1-\alpha W*)$ is a contraction mapping, $\lim_{k\to\infty} \| 1-(\alpha W*)^{k+1} \|_1 = 0$ as described in the Lemma . Eq. (19) can be rewritten as

$$\lim_{k\to\infty} \| u_k^l \|_1 \leq \| u_d^l \| + \| \tilde{\varepsilon} \|_1 \qquad (20)$$

where $\tilde{\varepsilon} = (W^*)^{-1}\varepsilon$ and $(W*)^{-1}$ is an inverse wavelet transform or the reconstruction process.
Therefore, as $\|\varepsilon\|_1 \to 0$, so does $\|\tilde{\varepsilon}\|_1$, then

$$\lim_{k\to\infty} \left\|u_k^l\right\| = \left\|u_d^l\right\|_1 \qquad (21)$$

Q.E.D.

The unlearnable part of the system response is filtered out by using the wavelet transform. Hence, the learning behaviour is improved while the feedback controller improves the transition response due to the unlearnable dynamics or non-repetitive disturbance. The convergent rate depends on the value of α and β. While α =1, and according to (A1) the convergent rate becomes

$$\frac{\| e_k \|}{\| e_{k-1} \|} = \frac{1}{\| 1 + \beta G(jw) \|} < 1 \text{ ; for } \omega < \omega_c \qquad (22)$$

Remark : Since the system loop-gain is usually greater than one below cut-off frequency, when α is fixed, larger β implies smaller steady-state error at every iteration process and the convergent rate remains unchanged as a small β is adopted. We can design the maximum value of feedback gain β_{max} by using the Ziegler-Nichols method (Stefani, et al, 1994). Since β is designed between zero and β_{max} (i.e. $0 < \beta < \beta_{\max}$), the overall control signal is equal to the feedback part at the first iteration (i.e. $u_k = u_k^f$ for *k=1*). According to the theorem, when the feedback control signal u_k^f approaches ε and the learning control signal u_k^l approaches u_d^l as $k \to \infty$, the stability of the closed-loop control system is ensured. For a fixed β, the choice of α = *1* will achieve the fastest convergent rate.

4. EXPERIMENTAL VERIFICATION

4.1 The experimental system

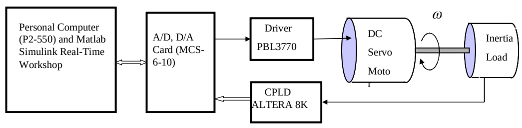


Fig. 4 The DC Servo System Under Study

The configuration of the overall control system is shown in Fig. 4. A DC servo motor equips with an inertia load as the control target. While the DC servo system is utilized for the purpose of speed-tracking control, there exists asymmetrical dead-zones in the forward and backward rotations. Since the shaft of DC motor starts with different angle in every iteration process, this yields different dead-zone for each iteration, which makes a typical non-repeatable system. In other words, the friction can act as a highly nonlinear disturbance on system. Especially, when the servo system is under heavy load, the friction will vary accordingly. Furthermore, a low frequency vibration will occur due to the variable inertia load. To model the non-repeatable term described above, a general dynamical description of the DC servo system can be expressed as shown in Eq. (1) and (2). To devise the experimental setup, a personal computer (P3-550) sends the speed commands to the PWM driver via an A/D card (MCS-6A-C-10), and the shaft position is detected by a photo encoder that delivering two-phase signals (A, B phase). The position data is then fed into a decoder circuit implemented on a CPLD (Complex Programmable Logic Device) board, which uses an over-sampling scheme (Su, 1998) to calculate the shaft speed in digital format. The controller is realized with the MATLAB, SIMULINK and RTW (Real-Time Workshop) toolboxes.

4. 2 Experimental results

By using the time domain system identification, an experimental transfer function (ETF)

$$G(s) = \frac{186.86}{s + 2.8983} \qquad (23)$$

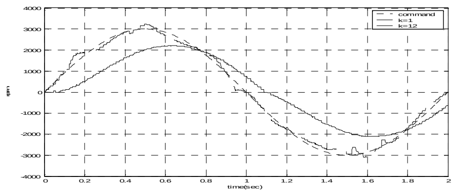
can be obtained. To verify the tracking performance, a sinusoidal speed trajectory is given as bellow, i.e.

$$y_d = 3000 \sin \pi t \ \text{rpm}$$

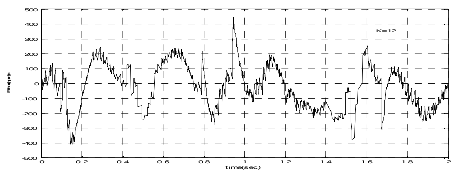
For the purpose of comparison, a CCE-type ILC scheme adopted from (Owens, 1992)

$$u_{k+1}(t) = \bar{\alpha} u_k(t) + \bar{\beta} e_{k+1}(t) \qquad (24)$$

is implemented, where $\bar{\beta}$ is the causal "learning" operator feeding back the current trial error. Here, we set $\bar{\alpha} = 1$, and $\bar{\beta} = 1$. As we will see in the experiment, this setting can achieve maximum feedback gain while inducing no vibration. Note that the CCE-type ILC (24) is identical to the proposed ILC scheme (13)-(15) during the first iteration. Using the CCE-type ILC scheme (24), the tracking response and the learning curve are shown in Fig. 5 and Fig. 6, respectively.



(a) The 1st and the 12th iteration



(b) Tracking Error at the 12th iteration

Fig. 5 Speed Tracking using CCE-ILC

From the response of the first iteration (*k=1*), it can be observed that there is a large lag between the command profile and the speed response due to the unlearnable dynamics (i.e. dead-zone, friction, noise of measurement and variation of load). After 12th iterations, the tracking error is still unacceptable and the learning behaviour is not robust. Using the proposal method, the 'Sym5' function (Wavelet Tool Box, 1997) is selected as the mother wavelet and six level details are decomposed. According to the Ziegler-Nichols method, when β_{max} =1.1, the resonant phenomenon occurs. After the 12th iteration, the speed response and tracking error with the proposed method are demonstrated in Fig.7.

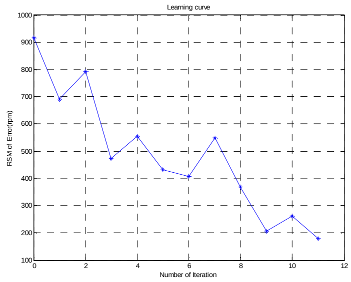


Fig. 6 The Learning Curve Using CCE-ILC

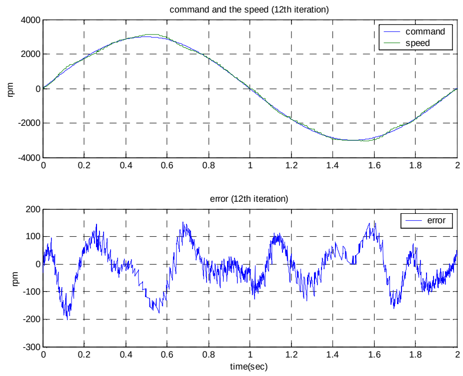


Fig. 7 Speed Tracking Using Proposed Scheme

The feedback control signal u_k^f is decomposed into six levels as shown in Fig. 8. The signal A_6 is the low frequency part, corresponding to the learnable dynamics of the DC servo system and the signals D_1~D_6 are the high frequency part due to the unlearnable dynamics. Although the resolution of the speed measurement is limited to 30 rpm by the over-sampling module (Su, 1998), the tracking performance is greatly improved by using the proposed scheme.

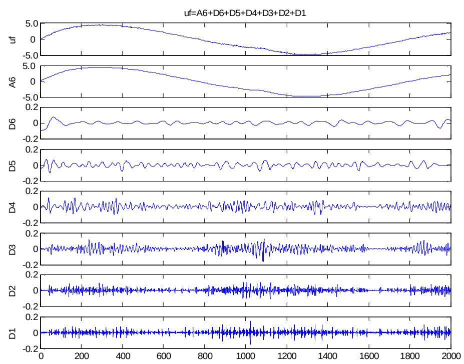


Fig. 8 The Decomposition of u_k^f

Fig. 9 shows the learning curve for the same β =1.0 but with $\alpha = 0.5$, 0.9, 1.0, 1.1. They clearly demonstrate that for $\alpha = 1.0$, the proposed ILC scheme can achieve the best convergent rate.

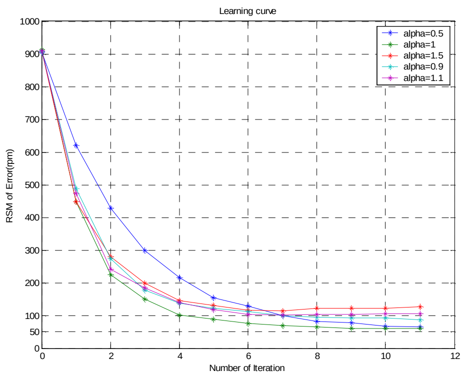


Fig.9 Learning Curve with Different Learning Gains($\alpha = 0.5$, 0.9, 1, 1.1, and 1.5 but Fixed Feedback Gain ($\beta = 1$)

5. CONCLUDING REMARKS

An iterative learning control scheme using wavelet transform is presented with experimental verification. The proposed iterative learning scheme is applicable to systems with unlearnable dynamics while maintain robust learning behavior. This paper also presents the relation between the learning gain, α , the convergent rate is explicitly exploited and confirmed via experiments. From experimental results, it is further verified that the proposed method can guarantee convergence of the learning process even under the influence of unlearnable dynamics. As a comparison, the experimental results of the CCE-type ILC are also given. It is seen that the CCE-type ILC is not applicable to a servo-system with unlearnable dynamics for the purpose of speed-tracking.

6. ACKNOWLEDGEMENT

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