INTEGRATED BATCH-TO-BATCH ITERATIVE LEARNING CONTROL AND WITHIN BATCH CONTROL OF PRODUCT QUALITY FOR BATCH PROCESSES

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Abstract: An integrated strategy for the tracking control of product qualities in batch processes is proposed by combing batch-to-batch iterative learning control (ILC) with on-line shrinking horizon model predictive control (SHMPC) within a batch. ILC is used in batch-to-batch control and the convergence of batch-wise tracking error under ILC is guaranteed. On-line SHMPC within a batch can reduce the effects of disturbances immediately and improve the performance of the current batch run. The integrated control strategy can complement both strategies to obtain good performance of tracking trajectories. The proposed strategy is illustrated on a simulated batch polymerization process. The results demonstrate that the performance of tracking product qualities can be improved quite well under the integrated control strategy than under the simple batch-to-batch ILC, especially when disturbances exist.

Keyword: Batch-to-batch control, Iterative learning control, Shrinking horizon model predictive control, Batch processes

1. INTRODUCTION

Batch-to-batch control exploits the repetitive nature of batch processes to refine the operating policy. Recently, ILC has been used to directly update input trajectory (Amann, *et al.*, 1998). ILC can update the control trajectory for the next batch run using the information from previous batch runs so that the output trajectory converges asymptotically to the desired reference trajectory (Lee, *et al*., 2000; Doyle, *et al*., 2003; Xiong and Zhang, 2003).

However, batch-to-batch control can only improve the performance of future batch runs and cannot improve the performance of the current batch run. Batch-wise control also cannot handle disturbances that change from batch to batch in a completely random fashion and may actually amplify their effects (Lee, *et al.*, 2002). On the other hand, if output variables can be measured or inferred accurately on-line, it is possible to implement on-line control that adjusts the control policy for the remaining batch period while the batch is going on (Lee and Lee, 2003). SHMPC (Russell, *et al*., 1998) is most suitable for on-line control of batch processes within the current batch. Because on-line SHMPC can respond to disturbances immediately and batch-to-batch ILC can correct any bias left uncorrected by the on-line controller, it is natural to explore the possibility of combining both methods to obtain good control performance. The integrated control strategy can combine the advantages of both methods (Flores-Cerrillo and MacGregor, 2003; Lee and Lee, 2003). If disturbances occur, the integrated control method is expected to diminish more rapidly the effect of disturbances than the results for only implementing ILC from batch to batch.

Based on a batch-wise linear time-varying (LTV) perturbation model, the ILC method can be implemented from batch to batch for tracking trajectories and the convergence of tracking error is guaranteed (Xiong and Zhang, 2003). But in order to implement SHMPC on-line, more accurate predictive model has to be proposed based on the current output values and remaining input moves. A predictive LTV perturbation model also has to be utilized in a manner similar to the batch-to-batch control formulation.

The rest of this paper is organized as follows: Section 2 presents batch-to-batch ILC based on a batch-wise LTV perturbation model. Section 3 proposes a within-batch predictive model and the SHMPC method is implemented based on the model. The integrated control strategy of combining both methods is outlined in Section 4. Application of this strategy to a simulated batch polymerization process is given in Section 5. Section 6 draws some concluding remarks.

2. BATCH-TO-BATCH CONTROL

We consider batch processes where the batch run length (t_f) is fixed and consists of N sampling intervals and all batches run from the same initial condition. The batch-to-batch control problem is to manipulate the whole control profile so that the product quality variables follow specific desired reference trajectories. It would be convenient to consider a batch-wise static function relating the control profile to the product quality profile over the whole batch duration. It can be written in matrix form as

$$\mathbf{Y}_k = \mathbf{F}(y_0, \mathbf{U}_k) + \mathbf{v}_k \tag{1}$$

where the subscript k denotes the batch index, $\mathbf{Y}_k = [y_k^T(1), y_k^T(2), \ldots, y_k^T(N)]^T$ ($y \in R^n$, $n \geq 1$) is a matrix of product quality variables and can be obtained on-line or off-line, $\mathbf{U}_k = [u_k(0), u_k(1), \ldots, u_k(N-1)]^T$ ($u \in R^m$, $m=1$ in this work) is manipulated variable, y_0 is the initial value, $\mathbf{F}(\cdot)$ represents a non-linear static function, and $\mathbf{v}_k$ is a matrix of measurement noises, respectively.

Subtracting the time-varying nominal trajectories from the process operation trajectories removes the majority of the process non-linearity and allows linear modeling methods to perform well on the resulting perturbation variables (Russell, *et al.*, 1998). An LTV perturbation model can be obtained by linearizing a non-linear model with respect to the nominal (mean or reference) trajectories. Linearizing the non-linear batch process model described by Eq(1) with respect to control sequence around the nominal trajectories, the following can be obtained

$$\mathbf{Y}_k = \mathbf{Y}_s + \left.\frac{\partial \mathbf{F}(y_0, \mathbf{U}_k)}{\partial \mathbf{U}_k}\right|_{\mathbf{U}_s} (\mathbf{U}_k - \mathbf{U}_s) + \mathbf{w}_k + \mathbf{v}_k \tag{2}$$

where $\mathbf{U}_s$ is the nominal control trajectory, $\mathbf{Y}_s$ is the nominal product quality trajectory and $y_s(0) = y_0$, and $\mathbf{w}_k$ is a sequence of model errors due to the linearization (i.e., due to neglecting the higher oder terms). Then a batch-wise LTV *perturbation* model can be obtained as

$$\overline{\mathbf{Y}}_k = \mathbf{G}_s \overline{\mathbf{U}}_k + \mathbf{d}_k \tag{3}$$

where $\mathbf{G}_s = (\partial \mathbf{F}(y_0, \mathbf{U})/\partial \mathbf{U})|_{(\mathbf{U}_k = \mathbf{U}_s)}$, $\overline{\mathbf{U}}_k = \mathbf{U}_k - \mathbf{U}_s$ and $\overline{\mathbf{Y}}_k = \mathbf{Y}_k - \mathbf{Y}_s$, respectively, are *perturbation variables* of control and product quality variables and $\bar{y}_k(0) = 0$, and $\mathbf{d}_k = \mathbf{w}_k + \mathbf{v}_k$ is the model disturbance sequence. $\mathbf{G}_s$ is batch-wise linear time-varying in the sense that it varies with $\mathbf{U}_s$, which usually varies from batch to batch. Due to the causality, the structure of $\mathbf{G}_s$ is of the following lower-block-triangular form:

$$\mathbf{G}_s = \begin{bmatrix} g_{10} & 0 & \cdots & 0 \\ g_{20} & g_{21} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ g_{N0} & g_{N1} & \cdots & g_{NN-1} \end{bmatrix} \tag{4}$$

where $g_{ij} \in R^n$. The batch-wise LTV model $\mathbf{G}_s$ can be found by linearizing a non-linear model along the nominal trajectories or through direct identification from process operational data. Available methods for identifying $\mathbf{G}_s$ range from simple static linear regression, such as the least squares and its variants (Srinivasan, *et al*., 2003; Xiong and Zhang, 2003), to more elaborate optimal dynamic estimation methods like the Kalman filtering (Lee, *et al*., 2002).

In this study, we utilize the model errors of the immediate previous batch run to modify predictions of the perturbation model. The *model prediction* and *modified model prediction* in the $(k+1)^{\text{th}}$ batch run are obtained respectively as

$$\hat{\overline{\mathbf{Y}}}_{k+1}^{ILC} = \hat{\mathbf{G}}_s \overline{\mathbf{U}}_{k+1}^{ILC} \tag{5}$$

$$\tilde{\overline{\mathbf{Y}}}_{k+1}^{ILC} = \hat{\overline{\mathbf{Y}}}_{k+1}^{ILC} + \hat{\boldsymbol{\varepsilon}}_k^{ILC} \tag{6}$$

where $\hat{\boldsymbol{\varepsilon}}_k^{ILC} = \overline{\mathbf{Y}}_k - \hat{\overline{\mathbf{Y}}}_k^{ILC}$ and the superscript *ILC* represents batch-to-batch *iterative learning control*. Considering that the objective of ILC is to track the desired reference trajectories of product quality, *tracking errors of process* and *modified model prediction* are defined respectively as

$$\mathbf{e}_k^{ILC} = \overline{\mathbf{Y}}_d - \overline{\mathbf{Y}}_k^{ILC} \tag{7}$$

$$\tilde{\mathbf{e}}_k^{ILC} = \overline{\mathbf{Y}}_d - \tilde{\overline{\mathbf{Y}}}_k^{ILC} \tag{8}$$

where $\overline{\mathbf{Y}}_d = \mathbf{Y}_d - \mathbf{Y}_s$, and $\mathbf{Y}_d$ is the specified reference trajectory and assumed here to be set reasonably. Then an iterative relationship for $\tilde{\mathbf{e}}_k^{ILC}$ along the batch index k can be obtained as (Xiong and Zhang, 2003)

$$\tilde{\mathbf{e}}_{k+1}^{ILC} = \mathbf{e}_k^{ILC} - \hat{\mathbf{G}}_s \Delta \overline{\mathbf{U}}_{k+1}^{ILC} \tag{9}$$

where $\Delta \overline{\mathbf{U}}_{k+1}^{ILC} = \overline{\mathbf{U}}_{k+1}^{ILC} - \overline{\mathbf{U}}_k^{ILC}$ represents the input change between two adjacent batch runs. Given the above batch-wise error transition model, the objective of ILC is to design a learning algorithm to manipulate the control policy so that the product qualities follow the specific desired reference trajectories from batch to batch. We consider solving the following quadratic objective function based on the modified prediction errors upon the completion of the k^{th} batch run to update the input trajectory for the $(k+1)^{th}$ batch run

$$\min_{\Delta \overline{\mathbf{U}}_{k+1}^{ILC}} J_{k+1}^{ILC} = \frac{1}{2}[\tilde{\mathbf{e}}_{k+1}^{ILC^T} \mathbf{Q}_s \tilde{\mathbf{e}}_{k+1}^{ILC} + \Delta \overline{\mathbf{U}}_{k+1}^{ILC^T} \mathbf{R}_s \Delta \overline{\mathbf{U}}_{k+1}^{ILC}] \quad (10)$$

where $\mathbf{Q}_s$ and $\mathbf{R}_s$ are positive definitive matrices and selected here as $\mathbf{Q}_s = \lambda_q \cdot \mathbf{I}_N$ and $\mathbf{R}_s = \lambda_r \cdot \mathbf{I}_N$. Through straightforward manipulation, the following ILC law can be obtained

$$\overline{\mathbf{U}}_{k+1}^{ILC} = \overline{\mathbf{U}}_{k}^{ILC} + \hat{\mathbf{K}}^{ILC} \mathbf{e}_k^{ILC} \quad (11)$$

where $\hat{\mathbf{K}}^{ILC} = [\hat{\mathbf{G}}_s^T \mathbf{Q}_s \hat{\mathbf{G}}_s + \mathbf{R}_s]^{-1} \hat{\mathbf{G}}_s^T \mathbf{Q}_s$.

The convergence of ILC alogrithm can be obtained and its proof can directly be derived from the convergence theorems in the literature. It can be proved that $\mathbf{e}_k^{ILC}$ will nominally converge as $k \to \infty$ if $\mathbf{I} - \hat{\mathbf{G}}_s \hat{\mathbf{K}}^{ILC}$ has all its eigenvalues inside the unit circle, i.e. $\| \mathbf{I} - \hat{\mathbf{G}}_s \hat{\mathbf{K}}^{ILC} \| < 1$ (Xiong and Zhang, 2003).

3. ON-LINE CONTROL WITHIN BATCH

Batch-to-batch ILC strategy can only improve the performance of future batch runs and cannot improve the performance of the current batch run. In addition, it cannot handle disturbances that change from batch to batch in a completely random fashion and may actually amplify their effects (Lee, *et al.*, 2002). However, if on-line measurement of output variables can be made on a reliable basis, one can explore the possibility of implementing on-line control that adjusts the future input policy while the batch is going on (Lee and Lee, 2003). On-line batch control can be established in a manner similar to the batch-to-batch control formulation.

SHMPC (Russell, *et al.*, 1998) can be utilized within the current batch. In SHMPC, the horizon of model prediction p is equal to the control horizon m and the both are shrinking with time t as the batch progresses, i.e. $m=p=N-t$. During on-line SHMPC, it is quite useful to update the future control profile based on the calculated batch-to-batch ILC profile $\overline{\mathbf{U}}_{k+1}^{ILC}$, instead of directly calculating future control actions. It can be represented by

$$\overline{\mathbf{U}}_{k+1}^{OLC}(t+m) = \delta \overline{\mathbf{U}}_{k+1}^{OLC}(t+m) + \overline{\mathbf{U}}_{k+1}^{ILC}(t+m) \quad (12)$$

where $\overline{\mathbf{U}}_{k+1}^{OLC}(t+m) = [\bar{u}_{k+1}^{OLC}(t), ..., \bar{u}_{k+1}^{OLC}(t+m-1)]^T$ is a vector of remaining m ($m=N-t$) control actions to be obtained, $\overline{\mathbf{U}}_{k+1}^{ILC}(t+m) = [\bar{u}_{k+1}^{ILC}(t), ..., \bar{u}_{k+1}^{ILC}(t+m-1)]^T$ is a vector of control values in the same m horizons that have been calculated by ILC, $\delta \overline{\mathbf{U}}_{k+1}^{OLC}$ is the deviation, and the superscript *OLC* represents the *on-line control*, respectively.

In the batch-wise LTV perturbation model, output $\bar{y}_{k+1}^{ILC}(j)$ at time j is related to all input values up to time j, $\overline{\mathbf{U}}_{k+1}^{ILC}(j) = [\bar{u}_{k+1}^{ILC}(0), \ldots, \bar{u}_{k+1}^{ILC}(j-1)]^T$, i.e. $\bar{y}_{k+1}(j) = f_j(\bar{y}_k^{ILC}(0), \overline{\mathbf{U}}_{k+1}^{ILC}(j))$ ($j=1,2,\ldots,N$). From Eq(3) and Eq(4) and due to $\bar{y}_k^{ILC}(0)=0$, the prediction of the batch-wise LTV model can be represented by $\hat{\bar{y}}_{k+1}^{ILC}(j) = \hat{g}_j^T \overline{\mathbf{U}}_{k+1}^{ILC}(j)$, where g_j is the jth row of $\mathbf{G}_s$ in Eq(4). However, to utilize SHMPC within a batch based on the current value $\bar{y}_{k+1}(t)$, future values $\bar{y}_{k+1}(t+i)$ have to be predicted using future input sequence $\overline{\mathbf{U}}_{k+1}^{OLC}(t+i) = [\bar{u}_{k+1}(t), ..., \bar{u}_{k+1}(t+i-1)]^T$ ($i=1, 2, \ldots, m$). Therefore, we cannot directly use the batch-wise model to predict the future values $\bar{y}_{k+1}(t+i)$ at time t within a batch. In this study, using current output value $\bar{y}_{k+1}(t)$ and future control values $\overline{\mathbf{U}}_{k+1}^{OLC}(t+i)$, a new predictive LTV perturbation model within a batch is proposed:

$$\hat{\bar{y}}_{k+1}^{OLC}(t+i|t) = \bar{g}_{t0} \bar{y}_{k+1}(t) + \bar{g}_{ti} \overline{\mathbf{U}}_{k+1}^{OLC}(t+i) \quad (13)$$

where $\bar{g}_{t0} = \text{diag}\{g_{t,01}, \ldots, g_{t,0n}\}$ and $\bar{g}_{ti} = [g_{t,i0}, g_{t,i1}, \ldots, g_{t,ii-1}]^T$ ($i=1,2,\ldots,m$). Eq(13) can be written in the the following matrix form

$$\hat{\overline{\mathbf{Y}}}_{k+1}^{OLC}(t+m|t) = \mathbf{G}_{t0} \bar{y}_{k+1}(t) + \mathbf{G}_{tm} \overline{\mathbf{U}}_{k+1}^{OLC}(t+m) \quad (14)$$

where $\hat{\overline{\mathbf{Y}}}_{k+1}^{OLC}(t+m|t) = [\hat{\bar{y}}_{k+1}^{OLC}(t+1|t), \ldots, \hat{\bar{y}}_{k+1}^{OLC}(t+m|t)]^T$,

$$\mathbf{G}_{t0} = [\underbrace{\bar{g}_{t0}, \cdots, \bar{g}_{t0}}_{m}]^T, \quad \mathbf{G}_{tm} = \begin{bmatrix} g_{t,10} & 0 & \cdots & 0 \\ g_{t,20} & g_{t,21} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ g_{t,m0} & g_{t,m1} & \cdots & g_{t,mm-1} \end{bmatrix}.$$

In order to identify the parameters, $\mathbf{G}_{tm}$ is partitioned according to the time index of output trajectory as a block column matrix $\mathbf{G}_{tm} = [g_{t1}^T, g_{t2}^T, \ldots, g_{tm}^T]^T$, where $g_{ti} = [g_{t,i0}, g_{t,i1}, \ldots, g_{t,ii-1}, \underbrace{0, \cdots, 0}_{m-i}]^T$ ($i=1,2,\ldots,m$). Then model parameters to be estimated are reformed as a vector $\Theta_t^i = [g_{t,01}, g_{t,02}, \ldots, g_{t,0n}, g_{t,i0}, g_{t,i1}, \ldots, g_{t,ii-1}]^T$ and can also be identified by the least-square regression method based on the historical process operation data set as presented in Xiong and Zhang (2003).

If the predictive errors calculated from the batch-to-batch controller are not added to the predictive model within a batch, on-line control calculation ends up 'undoing' the correction made by the batch-to-batch controller (Lee, *et al.*,2002). To improve the accuracy of the predictive model, predictive errors of the immediate previous batch run are utilized to modify predictions of the predictive model in the current batch run, which is defined as

$$\tilde{\overline{\mathbf{Y}}}_{k+1}^{OLC}(t+m|t) = \hat{\overline{\mathbf{Y}}}_{k+1}^{OLC}(t+m|t) + \hat{\boldsymbol{\varepsilon}}_k^{OLC}(t+m|t) \quad (15)$$

where $\hat{\boldsymbol{\varepsilon}}_k^{OLC}(t+m|t) = \overline{\mathbf{Y}}_k(t+m) - \hat{\overline{\mathbf{Y}}}_{k+1}^{OLC}(t+m|t)$. Substitute

Eq(12) and Eq(14) to Eq(15), then $\tilde{\tilde{\mathbf{Y}}}_{k+1}^{OLC}(t+m|t)$ can be rewritten further as

$$\tilde{\tilde{\mathbf{Y}}}_{k+1}^{OLC}(t+m|t)=\hat{\mathbf{G}}_{t0}\bar{y}_{k+1}(t)+\mathbf{G}_{tm}\overline{\mathbf{U}}_{k+1}^{ILC}(t+m) + \mathbf{G}_{tm}\delta\overline{\mathbf{U}}_{k+1}^{OLC}(t+m)+\hat{\boldsymbol{\varepsilon}}_{k}^{OLC}(t+m|t) \quad (16)$$

The *tracking error* of the modified predictive model for the remain trajectory is also defined as

$$\tilde{\mathbf{e}}_{k}^{OLC}(t+m|t)=\overline{\mathbf{Y}}_{d}(t+m)-\tilde{\tilde{\mathbf{Y}}}_{k+1}^{OLC}(t+m|t) \quad (17)$$

where $\overline{\mathbf{Y}}_{d}(t+m)=[\,\bar{y}_{d}(t+1),...,\bar{y}_{d}(t+m)\,]^{T}$. The objective function of the SHMPC is defined as

$$\min_{\delta\overline{\mathbf{U}}_{k+1}(t+m)} J_{k+1}^{OLC}(t)=\frac{1}{2}[\tilde{\mathbf{e}}_{k+1}^{OLC^{T}}(t+m|t)\mathbf{Q}_{t}\tilde{\mathbf{e}}_{k+1}^{OLC}(t+m|t) + \delta\overline{\mathbf{U}}_{k+1}^{OLC^{T}}(t+m)\mathbf{R}_{t}\delta\overline{\mathbf{U}}_{k+1}^{OLC}(t+m)] \quad (18)$$

where $\mathbf{Q}_t$ and $\mathbf{R}_t$ are positive definitive weighting matrices with appropriate dimensions. Through straightforward manipulation, the following on-line controller within batch can be obtained

$$\delta\overline{\mathbf{U}}_{k+1}^{OLC}(t+m)=\hat{\mathbf{K}}_{tm}^{OLC}\boldsymbol{\eta}(t+m) \quad (19)$$

where $\boldsymbol{\eta}(t+m)=\overline{\mathbf{Y}}_{d}(t+m)-\hat{\mathbf{G}}_{t0}\bar{y}_{k+1}(t)-\mathbf{G}_{tm}\overline{\mathbf{U}}_{k+1}^{ILC}(t+m)-\hat{\boldsymbol{\varepsilon}}_{k}^{OLC}(t+m|t)$, and $\hat{\mathbf{K}}_{tm}^{OLC}=[\hat{\mathbf{G}}_{tm}^{T}\mathbf{Q}_{t}\hat{\mathbf{G}}_{tm}+\mathbf{R}_{t}]^{-1}\hat{\mathbf{G}}_{tm}^{T}\mathbf{Q}_{t}$.

According to Eq(12), the SHMPC law can be obtained

$$\overline{\mathbf{U}}_{k+1}^{OLC}(t+m)=\overline{\mathbf{U}}_{k+1}^{ILC}(t+m)+\hat{\mathbf{K}}_{tm}^{OLC}\boldsymbol{\eta}(t+m) \quad (20)$$

Only the first element of $\overline{\mathbf{U}}_{k+1}^{OLC}(t+m)$ is applied to the process and the same procedure is repeated with time t increased by 1 but control horizon m shrinked by 1. The SHMPC law is similar to that in Lee *et al*. (2002) except for the term $\boldsymbol{\eta}(t+m)$ instead of $\mathbf{e}_k(t|t)$.

4. INTEGRATED CONTROL

Because on-line SHMPC can respond to disturbances immediately and batch-to-batch ILC can correct any bias left uncorrected by the on-line controller, it is natural to explore the possibility of combining them to obtain good performance of tracking trajectories. The integrated control strategy can combine the benefits of the both methods. The procedure of integrated control by combing batch-to-batch ILC with on-line SHMPC within batch is outlined as follows:

Step 1. Based on the historical process operation data set, select the nominal trajectories ($\mathbf{U}_s$,$\mathbf{Y}_s$). Initially set k=0 and $\overline{\mathbf{U}}_{k}^{ILC}=\overline{\mathbf{U}}_{k}^{OLC}=\mathbf{U}_s$.

Step 2. Based on $\overline{\mathbf{U}}_{k}^{ILC}$, use batch-to-batch ILC to calculate the whole control trajectory $\overline{\mathbf{U}}_{k+1}^{ILC}$ of the (k+1)th batch run. The model prediction errors $\hat{\varepsilon}_{k}^{ILC}$ are calculated and used to correct the batch-wise model predictions. Then based on the modified predictions $\tilde{\tilde{\mathbf{Y}}}_{k+1}^{ILC}$, a new control policy $\overline{\mathbf{U}}_{k+1}^{ILC}$ for the next batch is calculated by using the ILC law Eq(11).

Step 3. During the (k+1)th batch, at time t (t=1,…,N), based on the calculated $\overline{\mathbf{U}}_{k+1}^{ILC}(t+m)$ by ILC and the errors $\hat{\boldsymbol{\varepsilon}}_{k}^{OLC}(t+m|t)$ calculated in the previous batch, the remaining control policy $\overline{\mathbf{U}}_{k+1}^{OLC}(t+m)$ is obtained by using SHMPC method Eq(20). Then its first element is applied to the process. Repeat this procedure in the current batch run until time t reaches the end of this batch. After completion of the (k+1)th batch run, both the output profile $\overline{\mathbf{Y}}_{k+1}$ and the whole on-line control policy $\overline{\mathbf{U}}_{k+1}^{OLC}$ are obtained.

Step 4. Set $\overline{\mathbf{U}}_{k+1}^{ILC}=\overline{\mathbf{U}}_{k+1}^{OLC}$ and k=k+1, return to step 2.

It has been shown that convergence holds under some reasonable assumption in batch-to-batch ILC (Xiong and Zhang, 2003). SHMPC can further improve the control performance within a batch. Robustness of integrated control has not been investigated rigorously (Lee, *et al*., 2002). However, experience from extensive numerical studies demonstrates that integrated control has good performance.

5. APPLICATION TO A SIMULATED BATCH POLYMERIZATION REACTOR

This example involves a thermally initiated bulk polymerization of styrene in a batch reactor. The differential equations describing the polymerization process are given by Kwon and Evans (1975) through reaction mechanism analysis and laboratory testing. Gattu and Zafiriou (1999) report the parameter values of the first principle model. The product quality variables include the conversion (y_1), the dimensionless number-average chain lengths (NACL, y_2) and dimensionless weight-average chain lengths (WACL, y_3). The input variable is $u=T/T_{ref}$, where T is the absolute temperature of the reactor and T_{ref} is the reference value. In this study, the final time t_f is fixed to be 313 minutes and the batch length is divided into N equal stages, and initial values of the outputs are $y_1(0)$=0, $y_2(0)$=1, and $y_3(0)$ =1. The desired product reference trajectory $\mathbf{Y}_d$ was taken from Gattu and Zafiriou (1999).

Thirteen batches of process operation under different temperature profiles were simulated from the mechanistic model and used as the historical process data sets for building relationship between u and $\boldsymbol{y}$= $[y_1, y_2, y_3]^T$. A batch-wise LTV perturbation model $\mathbf{G}_s$ is utilized to build the relationship between u and $\boldsymbol{y}$. A predictive model ($\mathbf{G}_{t0}$, $\mathbf{G}_{tm}$) within a batch is built to represent the dynamic model between u and $\boldsymbol{y}$, and parameters of the model are estimated by using the least-square regression method.

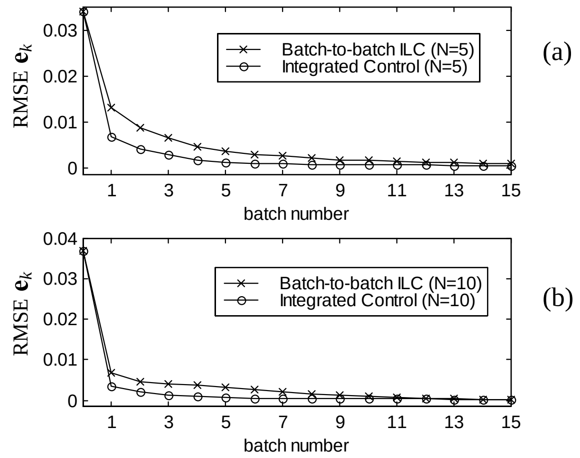

Fig. 1. Convergence of RMSE under two strategies in different time stages: (a) N=5 (b) N=10

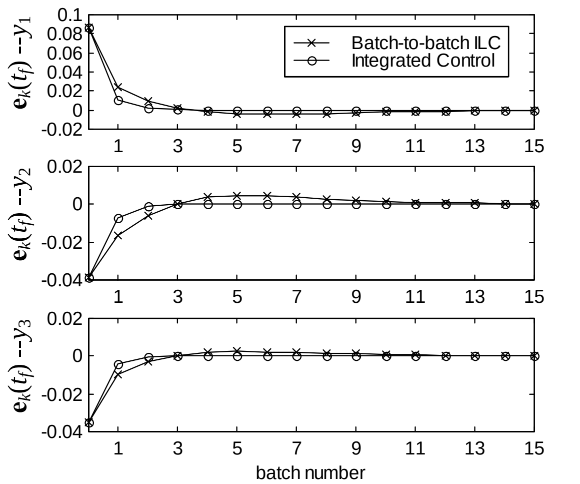

Fig. 2. Convergence of $\mathbf{e}_k(t_f)$ under two strategies (N=10)

To investigate the performance of the proposed integrated control strategy, it is compared with the simple batch-to-batch ILC scheme. The parameters in ILC were set as $\mathbf{Q}=\mathbf{I}_N$ and $\mathbf{R}=0.05\mathbf{I}_N$, while the parameters in integrated control were set as $\mathbf{Q}_t=\mathbf{I}_m$ and $\mathbf{R}_t=0.05\mathbf{I}_m$, where $m \le N$ and it is shrinking with time t during a batch. Here two values of time stage N (N = 10 and N = 5) are studied. The results under two control strategies are shown in Fig. 1 and Fig. 2. Although the number of parameters to be estimated when N=10 is more than those for N=5, the models are more accurate than those for N=5 and the results for N=10 are slightly better than those for N=5 under two strategies. Fig. 1 shows the RMSE of tracking error of product quality $\mathbf{e}_k$ under two strategies at different time stages. Since the final product quality is of the main interest in batch process operation, the tracking errors $\mathbf{e}_k(t_f)$ at the batch end-point from these two strategies are also compared, as shown in Fig. 2. Fig 1 and Fig 2 show that $\mathbf{e}_k(t_f)$ is also improved gradually while the whole trajectory converges asymptotically to the desired trajectory. It can also be seen that when N=10, both the RMSE of $\mathbf{e}_k$ and $\mathbf{e}_k(t_f)$ have almost converged after about 3 batch runs under integrated control strategy, but they converge after 8 batch runs under the simple batch-to-batch ILC scheme. Fig. 3 and Fig. 4 show, respectively, the product quality profiles $\mathbf{Y}_k$ and control profile $\mathbf{U}_k$ of the 1st, 2nd, 5th and 15th batch runs when N=10. It can be seen that $\mathbf{Y}_k$ has already converged to the desired reference $\mathbf{Y}_d$ after 5 batch runs under the integrated control strategy. As can be seen, the integrated control strategy has the advantage of combined error correction within batch and the gradual reduction to the minimum error afforded by the batch-to-batch control.

We also consider how disturbances that occur just for a single batch affect the batch-to-batch behavior of the product qualities under the two control strategies. The scenario here is that the kinetic parameter A_m (the frequency factor of the overall monomer reaction) changes from nominal value A_{m0} to $1.2A_{m0}$ in the 10th batch and switches back to the original value at the 11th batch. Fig. 5 shows the RMSE of the product qualities under the two strategies.

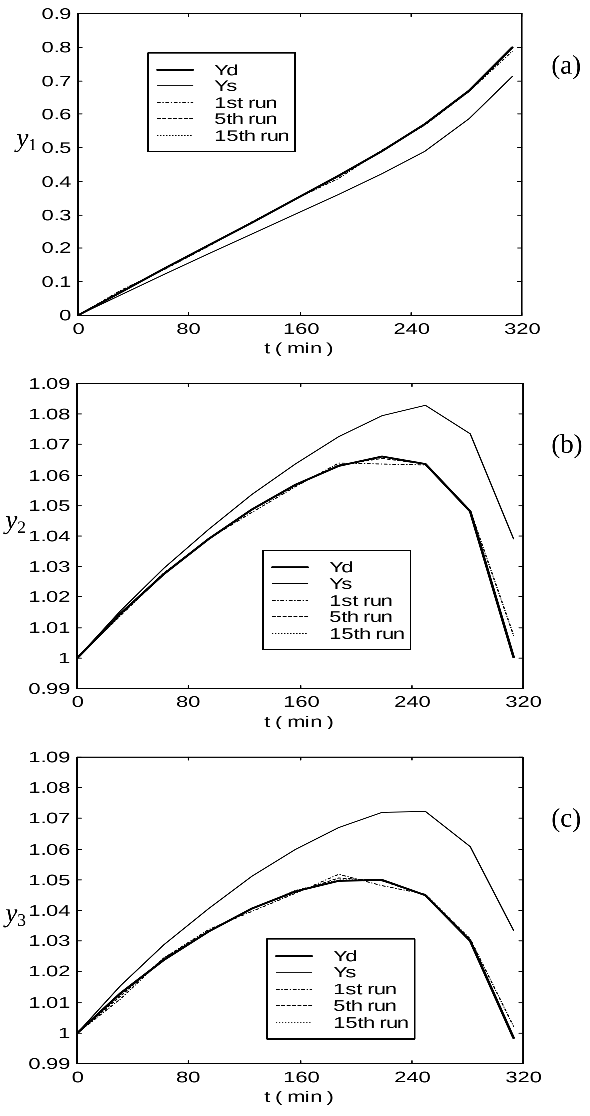

Fig. 3. Trajectories of quality product variables under integrated control (N=10): (a) y_1 (b) y_2 (c) y_3

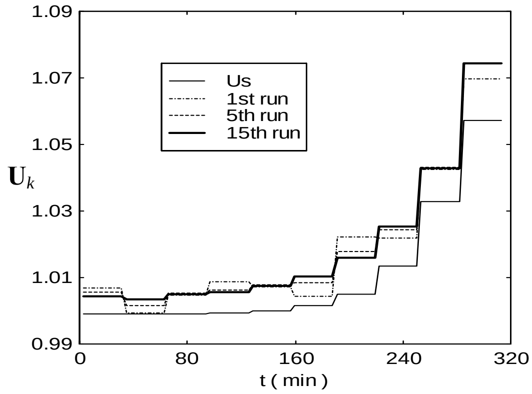

Fig. 4. Convergence of $\mathbf{U}_k$ under integrated control

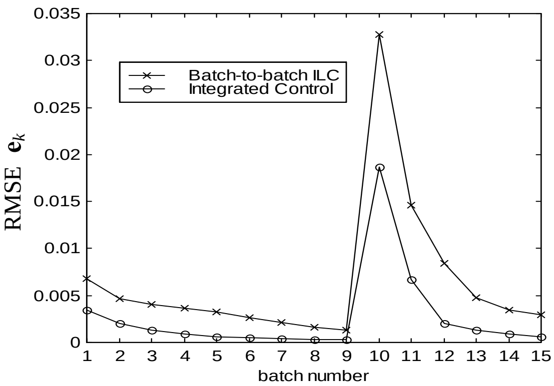

Fig. 5. Comparison of RMSE in the disturbance case (N=10)

In the case of the simple batch-to-batch ILC, the disturbance of the 10th batch resulted in a large error in that batch, and it caused incorrect control profile in subsequent batches. Errors of subsequent batches decreased by the batch-to-batch ILC action, but the error has more effect on the subsequent batches. In the case of integrated control, errors in the 10th batch were reduced significantly through the on-line SHMPC method. The effect of the disturbance does carry over to the next batch due to the batch-to-batch control but the effect is diminished more rapidly compared with the simple batch-to-batch ILC.

6. CONCLUSION

An integrated batch-to-batch ILC and on-line SHMPC strategy for the tracking control of product qualities in batch processes is proposed. On-line SHMPC within a batch can decrease the effects of disturbances immediately, while the batch-to-batch ILC can correct the bias left uncorrected by the on-line controller. The integrated strategy can complement each other to obtain good performance of tracking trajectories. The proposed method is illustrated on a simulated batch polymerization process. The results demonstrate that the performance of tracking product qualities can be improved quite well under the integrated control strategy than under the simple batch-to-batch ILC, especially when disturbances exist.

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