OBSERVER-BASED ROBUST PREVIEW TRACKING CONTROL SCHEME FOR UNCERTAIN DISCRETE-TIME SYSTEMS

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Abstract: This paper deals with a design problem of an observer-based robust preview control system for uncertain discrete-time systems. In this approach, we adopt 2-stage design scheme and we derive an observer-based robust controller with integral and preview actions such that a given performance index is satisfactorily small for uncertainties. In this paper, we show that sufficient conditions for the existence of the observer-based robust preview controller are given in terms of linear matrix inequalities (LMIs).

Keywords: preview control, observer-based control, robust tracking performance, 2-stage design, LMIs

1. INTRODUCTION

One of the control problems is tracking control systems (i.e. servo systems), where output signal has to track reference signal or desired output without steady-state error. Therefore a great deal of interest has been directed to the design problem of servomechanisms for linear multivariable systems (Davison, 1972). Furthermore, design problems of robust servomechanisms have been extensively studied (Schmitendorf and B.R., 1986).

By the way, it is well known that when the future information about reference signals is available, the transient performance can be improved. This kind of control problem, in which information on future is utilized, is called the preview control problem (Tomizuka, 1975) and a large number of design method of preview control systems have been proposed (Katayama *et al.*, 1985; Fujisaki and T., 1997). Also, some robust preview controllers have been derived (Takaba, 1998).

On the other hand in the case that the full state of systems cannot be measured, some observer-based quadratic stabilizing controllers (Petersen, 1985) and robust output feedback control systems (Benton and D., 1999) have been presented. However, so far the design problem of the robust preview control system based on observer-based controllers for uncertain discrete-time systems has little been discussed as far as we know.

From this viewpoint, we present a design method of an observer-based robust preview controller for uncertain discrete-time systems. In order to derive a simple design method of the observer-based robust controller, we adopt a similar way to the design approach derived by Oya *et. al.* 2004, i.e. the proposed design method is separated into two parts. Firstly, an observer gain is designed and next, a control gain matrix is determined so that an upper bound on a given performance index is minimized. In this paper, we show that sufficient conditions for the existence of the observer-based robust preview controller are given in terms of linear matrix inequalities (LMIs).

2. PROBLEM FORMULATION

We consider a discrete-time system described by

$$\begin{aligned} x(t+1) &= A(\theta)x(t) + B(\theta)u(t) \\ y(t) &= C(\theta)x(t) \end{aligned} \tag{1}$$

where $x(t) \in \Re^n, u(t) \in \Re^m$ and $y(t) \in \Re^l$ are the vectors of the state, the control input and the measured output, respectively and the parameter $\theta \in \Re^N$ $(\theta = (\theta_1, \cdots, \theta_N)^T)$ is a constant vector of uncertainties. Also the matrices $A(\theta), B(\theta)$ and $C(\theta)$ in eq.(1) depend affinely on the parameters θ_k for $k = 1, \cdots, N$. That is

$$\begin{aligned} \mathcal{M}(\theta) &\stackrel{\triangle}{=} \begin{pmatrix} A(\theta) & B(\theta) \\ C(\theta) & 0 \end{pmatrix} \\ &= \begin{pmatrix} A & B \\ C & 0 \end{pmatrix} + \sum_{k=1}^{N} \theta_k \begin{pmatrix} A_k & B_k \\ C_k & 0 \end{pmatrix} \end{aligned} \tag{2}$$

where the unknown parameter θ_k for $k = 1, \cdots, N$ ranges between known extremal values $\theta_k^- \leq 0$ and $\theta_k^+ \geq 0$ (i.e. $\theta_k \in [\theta_k^-, \theta_k^+]$). This assumption means that the parameter $\theta \in \Re^N$ belongs to the following parameter box (Gahinet *et al.*, 1996).

$$\Delta \stackrel{\triangle}{=} \{\theta \in \Re^N \,|\theta_k \in [\theta_k^-, \theta_k^+] \quad \text{for} \quad k = 1, \cdots, N\} \tag{3}$$

Also the matrices A, B and C in eq.(2) denote the known nominal values and the matrices A_k, B_k and C_k for $i = 1, \cdots, N$ represent the structure of the uncertainties. Additionally we assume that for $\forall\theta \in \Delta$ the pairs $(A(\theta), B(\theta))$ and $(C(\theta), A(\theta))$ are controllable and observable, respectively and for $\forall\theta \in \Delta$, the following relation holds.

$$\text{rank} \begin{pmatrix} A(\theta) - I_n & B(\theta) \\ C(\theta) & 0 \end{pmatrix} = n + l \tag{4}$$

Let $r(t) \in \Re^l$ be the reference signal or the desired output which is assumed to to be previewable. That is, we assume that the h future values of the reference signal $r(t)$ (i.e. $r(t+1), \cdots, r(t+h)$) are available at each time t as well as the present and the past values of the reference signal and the reference signal $r(t)$ satisfies following relation [1].

$$r(t+j) = r(t+h) \quad \text{for} \quad j \geq h+1 \tag{5}$$

Now in order to estimate the state $x(t)$ for the uncertain system eq.(1), we introduce the following full state observer (Hagino and H., 1991).

$$\begin{aligned} x_e(t+1) &= Ax_e(t) + Bu(t) \\ &\quad + H_r(y(t) - Cx_e(t)) \end{aligned} \tag{6}$$

where $H_r \in \Re^{n \times l}$ is the observer gain matrix. Furthermore we define the estimation error vector $z_e(t) \stackrel{\triangle}{=} x(t) - x_e(t)$, then we see from eq.(1) and (6) that the estimation error satisfies

$$\begin{aligned} z_e(t+1) &= (A(\theta) - H_r C(\theta))z_e(t) \\ &\quad + (A_e(\theta) - H_r C_e(\theta))x_e(t) \\ &\quad + B_e(\theta)u(t) \end{aligned} \tag{7}$$

where $A_e(\theta), B_e(\theta)$ and $C_e(\theta)$ are the matrices given by $A_e(\theta) = A(\theta) - A, B_e(\theta) = B(\theta) - B$ and $C_e(\theta) = C(\theta) - C$, respectively.

Let $\overline{x}(t) \stackrel{\triangle}{=} x(t+1) - x(t)$ and $\overline{u}(t) \stackrel{\triangle}{=} u(t+1) - u(t)$ be the difference state vector and the difference control input vector respectively. Additionally, we introduce the tracking error vector $e(t) \stackrel{\triangle}{=} r(t) - y(t)$ and difference vectors $\overline{x}_e(t) \stackrel{\triangle}{=} x_e(t+1) - x_e(t), \overline{r}(t) \stackrel{\triangle}{=} r(t+1) - r(t)$ and $\overline{z}_e(t) \stackrel{\triangle}{=} z_e(t+1) - z_e(t)$. Since $\theta \in \Re^N$ is assumed to be constant, it follows from eqs.(1), (6) and (7) that

$$\begin{aligned} e(t+1) &= e(t) - C(\theta)\overline{x}_e(t) - C(\theta)\overline{z}_e(t) \\ &\quad + \overline{r}(t) \end{aligned} \tag{8}$$

Also since h future values of the reference signal $r(t+1), \cdots, r(t+h)$ are available at time t, we define the difference vector of the reference signal described by

$$\overline{r}_h(t) = \begin{pmatrix} \overline{r}^T(t) & \overline{r}^T(t+1) & \cdots & \overline{r}^T(t+h) \end{pmatrix}^T \tag{9}$$

From eq.(9), the difference vector $\overline{r}_h(t)$ satisfies

$$\overline{r}_h(t) = A_{r_h}\overline{r}_h(t) \tag{10}$$

$$A_{r_h} = \begin{pmatrix} 0 & I_l & & O \\ 0 & 0 & \ddots & 0 \\ & & \ddots & I_l \\ O & & & 0 \end{pmatrix} \tag{11}$$

Furthermore from the definition of the difference vector $\overline{r}_h(t)$, $\overline{r}(t)$ can be written as

$$\overline{r}(t) = \varGamma_r \overline{r}_h(t) \tag{12}$$

$$\varGamma_r = \begin{pmatrix} I_l & 0 & \cdots & 0 \end{pmatrix} \tag{13}$$

Now we introduce an augmented vector $\xi(t) \in \Re^{n+N_h}$ given by $\xi(t) \stackrel{\triangle}{=} \begin{pmatrix} e^T(t) & \overline{x}_e^T(t) & \overline{r}_h^T(t) & \overline{z}_e^T(t) \end{pmatrix}^T$ where $N_h \stackrel{\triangle}{=} n + l(h+2)$. Then we obtain

$$\xi(t+1) = \mathcal{F}(\theta)\xi(t) + \mathcal{G}(\theta)\overline{u}(t) \tag{14}$$

In eq.(14), $\mathcal{F}(\theta)$ and $\mathcal{G}(\theta)$ are the matrices described as

[1] The relation eq.(5) is a working assumption introduced to facilitate the mathematical development of the preview tracking controller, so that it is not required that the assumption is satisfied by the actual reference signal (Katayama *et al.*, 1985; Takaba, 1998).

$$u_r(t) = -K_{x_e}x_e(t) - K_e\sum_{j=0}^{t-1} e(j) - \sum_{j=0}^{h} K_{r_h}(j)r(t+j) \tag{19}$$

$$\Psi_{\lambda_z}(\theta) \triangleq \Phi_{\lambda_z}(\theta) + \begin{pmatrix} C(\theta) \\ H_rC(\theta) \\ 0 \end{pmatrix}^T \mathcal{W}_{\lambda_z} \begin{pmatrix} C(\theta) \\ H_rC(\theta) \\ 0 \end{pmatrix} + \mathcal{Q}_H < 0 \quad \text{for} \quad \forall\theta \in \Delta \tag{26}$$

$$\Psi_{\lambda_z}(\theta) = \Phi_{\lambda_z}(\theta) + \begin{pmatrix} C(\theta) \\ \mathcal{V}_{\lambda_z}C(\theta) \\ 0 \end{pmatrix}^T \mathcal{U}_{\lambda_z}^{-1} \begin{pmatrix} C(\theta) \\ \mathcal{V}_{\lambda_z}C(\theta) \\ 0 \end{pmatrix} + \mathcal{Q}_H < 0 \quad \text{for} \quad \forall\theta \in \Delta \tag{27}$$

$$\begin{aligned} \mathcal{F}(\theta) &= \begin{pmatrix} \mathcal{F}_{11}(\theta) & \mathcal{F}_{12}(\theta) \\ \mathcal{F}_{21}(\theta) & \mathcal{F}_{22}(\theta) \end{pmatrix} \\ \mathcal{F}_{11}(\theta) &= \begin{pmatrix} I_l & -C(\theta) & \Gamma_r \\ 0 & A + H_rC_e(\theta) & 0 \\ 0 & 0 & A_{r_h} \end{pmatrix} \\ \mathcal{F}_{12}(\theta) &= \begin{pmatrix} C^T(\theta) & C^T(\theta)H_r^T & 0 \end{pmatrix}^T \\ \mathcal{F}_{21}(\theta) &= \begin{pmatrix} 0 & A_e(\theta) - H_rC_e(\theta) & 0 \end{pmatrix} \\ \mathcal{F}_{22}(\theta) &= A(\theta) - H_rC(\theta) \end{aligned} \tag{15}$$

$$\begin{aligned} \mathcal{G}(\theta) &= \begin{pmatrix} \mathcal{G}_1^T & \mathcal{G}_2^T(\theta) \end{pmatrix}^T \\ \mathcal{G}_1 &= \begin{pmatrix} 0 & B^T & 0 \end{pmatrix}^T, \ \mathcal{G}_2(\theta) = B_e(\theta) \end{aligned} \tag{16}$$

It is well known that the integral action of the controller is introduced by including the difference control in the performance index (Katayama *et al.*, 1985). Therefore we define the following performance index.

$$\mathcal{J} = \sum_{t=0}^{\infty} \{x_{e_r}^T(t)\mathcal{Q}_{e_r}x_{e_r}(t) + \overline{z}_e^T(t)\mathcal{Q}_z\overline{z}_e(t) + \overline{u}^T(t)R_r\overline{u}(t)\} \tag{17}$$

where the weighting matrices $\mathcal{Q}_{e_r} \in \Re^{N_h \times N_h}$, $\mathcal{Q}_z \in \Re^{n\times n}$ and $\mathcal{R}_r \in \Re^{m\times m}$ are positive definite which can be adjusted by designers and $x_{e_r}(t)$ is the vector given by $x_{e_r}(t) \triangleq \begin{pmatrix} e^T(t) & \overline{x}_e^T(t) & \overline{r}_h^T(t) \end{pmatrix}^T$. Now we consider the control given by

$$\begin{aligned} \overline{u}(t) &\triangleq -K_{e_r}x_{e_r}(t) \\ &= -K_ee(t) - K_{x_e}\overline{x}_e(t) - K_{r_h}\overline{r}_h(t) \end{aligned} \tag{18}$$

where K_{e_r} is the control gain matrix given by $K_{e_r} = \begin{pmatrix} K_e & K_{x_e} & K_{r_h} \end{pmatrix}$. From the definition of difference signals and the augmented vector $x_{e_r}(t)$, we obtain the control input $u(t)$ eq.(19). Here, we have used the assumption that $K_{r_h} \triangleq (K_{r_h}(0) \ K_{r_h}(1) \ \cdots \ K_{r_h}(h))$ and $x_e(j) = 0, u(j) = 0$ and $r(j) = 0$ for any $j < 0$.

Substituting eq.(18) into eq.(14) yields

$$\xi(t+1) = \mathcal{F}_{\mathcal{K}}(\theta)\xi(t) \tag{20}$$

$$\mathcal{F}_{\mathcal{K}}(\theta) = \begin{pmatrix} \mathcal{F}_{11}(\theta) - \mathcal{G}_1K_{e_r} & \mathcal{F}_{12}(\theta) \\ \mathcal{F}_{21}(\theta) - \mathcal{G}_2(\theta)K_{e_r} & \mathcal{F}_{22}(\theta) \end{pmatrix} \tag{21}$$

Therefore our control objective is to ensure robust stability and tracking without using excessive incremental variation of the control input. That is to design the observer gain matrix H_r and the control gain matrix K_{e_r} such that the performance index eq.(17) is satisfactorily small for $\forall\theta \in \Delta$.

3. DESIGN OF THE OBSERVER-BASED ROBUST PREVIEW CONTROLLER

In this section, on the basis of the design approach derived in (Oya *et al.*, 2004), we consider to design the observer-based robust controller such that the performance index eq.(17) is satisfactorily small for $\forall\theta \in \Delta$. Namely firstly, we design the observer gain matrix H_r and next, we derive the condition for the existence of the control gain matrix K_{e_r} minimizing the upper bound on the performance index eq.(17).

3.1 Design of the Observer Gain Matrix

From eq.(7), the estimation error satisfies

$$\begin{aligned} \overline{z}_e(t+1) = (A(\theta) - H_rC(\theta))\overline{z}_e(t) \\ + (A_e(\theta) - H_rC_e(\theta))\overline{x}_e(t) \\ + B_e(\theta)\overline{u}(t) \end{aligned} \tag{22}$$

In this paper, we consider to design the observer gain matrix H_r which stabilizes the following system obtained by ignoring the $\overline{u}(t)$ and $\overline{x}_e(t)$ in eq.(21)

$$\lambda_z(t+1) = (A(\theta) - H_rC(\theta))\lambda_z(t) \tag{23}$$

Now we introduce the quadratic function $\mathcal{V}_H(\lambda_z, t) \triangleq \lambda_z^T(t)\mathcal{Y}_{\lambda_z}\lambda_z(t)$ as a Lyapunov function candidate where the matrix $\mathcal{Y}_{\lambda_z} \in \Re^{n\times n}$ is the symmetric positive definite. Then the first order difference $\Delta\mathcal{V}_H(\lambda_z, t) \triangleq \mathcal{V}_H(\lambda_z, t+1) - \mathcal{V}_H(\lambda_z, t)$ along the trajectory of the system eq.(23) satisfies

$$\Delta V_H(\lambda_z, t) = \lambda_z^T(t)\Phi_{\lambda_z}(\theta)\lambda_z(t) \tag{24}$$

$$\begin{aligned} \Phi_{\lambda_z}(\theta) &= \mathcal{A}_H(\theta)^T\mathcal{Y}_{\lambda_z}\mathcal{A}_H(\theta) - \mathcal{Y}_{\lambda_z} \\ \mathcal{A}_H(\theta) &= (A(\theta) - H_rC(\theta)) \end{aligned} \tag{25}$$

Therefore if there exist the matrices H_r and $\mathcal{Y}_{\lambda_z}$ satisfying the condition $\Phi_{\lambda_z}(\theta) < 0$ for $\forall\theta \in \Delta$, then the quadratic stability of the system eq.(23) is ensured. Namely, the quadratic function $\mathcal{V}_H(\lambda_z, t)$ becomes a Lyapunov function for the system eq.(23). However by introducing a symmetric positive definite matrix $\mathcal{W}_{\lambda_z} \in \Re^{N_h\times N_h}$ and a design parameter $\mathcal{Q}_H \in \Re^{n\times n}$ which is a symmetric positive definite matrix, we consider the condition eq.(26) (see **Remark 1**).

The condition eq.(26) is also written as eq.(27). In eq.(27), $\mathcal{V}_{\lambda_z}$ is a matrix satisfying $\mathcal{V}_{\lambda_z} \triangleq \mathcal{Y}_{\lambda_z}H_r$

$$
\left(\begin{array}{c|c|ccc}
-\mathcal{Y}_{\lambda_z}+\mathcal{Q}_H & A^T(\theta)\mathcal{Y}_{\lambda_z}-C^T(\theta)\mathcal{V}^T_{\lambda_z} & C^T(\theta) & C^T(\theta)\mathcal{V}^T_{\lambda_z} & 0 \\
\hline
\mathcal{Y}_{\lambda_z}A(\theta)-\mathcal{V}_{\lambda_z}C(\theta) & -\mathcal{Y}_{\lambda_z} & 0 & 0 & 0 \\
\hline
C(\theta) & 0 & & & \\
\mathcal{V}_{\lambda_z}C(\theta) & 0 & & -\mathcal{U}_{\lambda_z} & \\
0 & 0 & & &
\end{array}\right) < 0 \quad \text{for} \quad \forall\theta\in\Delta_{\text{vex}} \qquad (29)
$$

and $\mathcal{T}_{\lambda_z}$ and $\mathcal{U}^{-1}_{\lambda_z}$ are the positive definite matrices expressed as

$$
\begin{aligned}
\mathcal{U}^{-1}_{\lambda_z} &\triangleq \mathcal{T}^{-1}_{\lambda_z}\mathcal{W}_{\lambda_z}\mathcal{T}^{-1}_{\lambda_z} \\
\mathcal{T}_{\lambda_z} &\triangleq diag\left(\ I_l,\ \mathcal{Y}_{\lambda_z},\ I_n\ \right)
\end{aligned} \qquad (28)
$$

If there exist the matrices $\mathcal{Y}_{\lambda_z}$,$\mathcal{U}_{\lambda_z}$ and $\mathcal{V}_{\lambda_z}$ satisfying the condition eq.(27), then quadratic stability of the system eq.(23) is ensured. We now define the set of the 2^N vertices of the parameter box Δ eq.(3) such as $\Delta_{\text{vex}} \triangleq \{\omega \in \Re^N \mid \omega_k \in \{\theta^-_k, \theta^+_k\}$ for $k = 1,\cdots,N\}$. Furthermore using Schur complement formula, the design problem of the observer gain matrix H_r is reduced to the problem of finding the matrices $\mathcal{Y}_{\lambda_z}$,$\mathcal{U}_{\lambda_z}$ and $\mathcal{V}_{\lambda_z}$ which satisfy the linear matrix inequalities (LMIs) eq.(29). Thus, if the solution of the linear matrix inequalities (LMIs) eq.(29) exists, then using the solution, the observer gain H_r can be obtained as

$$
H_r = \mathcal{Y}^{-1}_{\lambda_z}\mathcal{V}_{\lambda_z} \qquad (30)
$$

3.2 Design of the Control Gain Matrix

In the previous section, the observer gain matrix H_r has been derived. Hence, we consider to design the control gain matrix K_{e_r} minimizing the upper bound on the performance index eq.(17)

Using the vector $\overline{u}(t)$ eq.(18), the performance index eq.(17) is rewritten as

$$
\mathcal{J} = \sum_{t=0}^{\infty} \xi^T(t)\mathcal{Q}_\xi\xi(t) \qquad (31)
$$

where $\mathcal{Q}_\xi$ is the positive definite matrix expressed as $\mathcal{Q}_\xi = diag\left(\ \mathcal{Q}_{e_z} + K^T_{e_r}\mathcal{R}_{e_r}K_{e_r},\ \mathcal{Q}_z\ \right)$.

Here we shall give a theorem for quadratic stability of the augmented system eq.(20) and the upper bound on the performance index eq.(31).

Theorem 1. If there exist the control gain matrix K_{e_r} and symmetric positive definite matrix $\mathcal{X}_\xi \in \Re^{(n+N_h)\times(n+N_h)}$ satisfying the following inequality, then quadratic stability of the augmented system eq.(20) is ensured.

$$
\begin{aligned}
\Phi_\xi(\theta) &\triangleq \mathcal{F}^T_K(\theta)\mathcal{X}_\xi\mathcal{F}_K(\theta) - \mathcal{X}_\xi + \mathcal{Q}_\xi \\
&\leq 0 \ \ \text{for} \ \ \forall\theta\in\Delta
\end{aligned} \qquad (32)
$$

Furthermore the upper bound on the performance index eq.(31), denoted by $\mathcal{J}^*$, is given as

$$
\begin{aligned}
\mathcal{J} &\leq \xi^T(0)\mathcal{X}_\xi\xi(0) \\
&\triangleq \mathcal{J}^* \ \ \text{for} \ \ \forall\theta\in\Delta
\end{aligned} \qquad (33)
$$

Proof: Now we condider the quadratic function $\mathcal{V}_K(\xi,t) \triangleq \xi^T(t)\mathcal{X}_\xi\xi(t)$ as a Lyapunov function candidate.Then the first order difference $\Delta\mathcal{V}_K(\xi,t) \triangleq \mathcal{V}_K(\xi,t+1) - \mathcal{V}_K(\xi,t)$ along the trajectory of the system eq.(20) satisfies

$$
\Delta\mathcal{V}_K(\xi,t) = \xi^T(t)\Phi_\xi(\theta)\xi(t) \qquad (34)
$$

Thus, if the matrices K_{e_r} and $\mathcal{X}_\xi$ satisfying $\Phi_\xi(\theta) < 0$ for $\forall\theta\in\Delta$, then the quadratic stability of the augmented system eq.(20) is ensured. If there exist the control gain matrix K_{e_r} and the symmetric positive definite matrix $\mathcal{X}_\xi$ which satisfy the condition eq.(32), then the quadratic function $\mathcal{V}_K(\xi,t)$ satisfies the following relation and the quadratic function $\mathcal{V}_K(\xi,t)$ becomes a Lyapunov function for the augmented system eq.(20).

$$
\Delta\mathcal{V}_K(\xi,t) \leq -\xi^T(t)\mathcal{Q}_\xi\xi(t) \qquad (35)
$$

Furthermore since the augmented system eq.(20) is quadratically stable, summing up inequality eq.(35) over the period $t = 0 \rightarrow \infty$, we get

$$
\sum_{t=0}^{\infty} \xi^T(t)\mathcal{Q}_\xi\xi(t) \leq \xi^T(0)\mathcal{X}_\xi\xi(0) \qquad (36)
$$

It follows that the result of the theorem is true. The proof of **Theorem 1** is completed. ∎

In eq.(33), the upper bound $\mathcal{J}^*$ depends on the initial value $\xi(0)$. Thus we assume that the initial value $\xi(0)$ is zero mean random vector satisfying $E\{\xi(0)\xi^T(0)\} = I_{n+N_h}$ and $E\{\xi(0)\} = 0$, because the control gain matrix cannot be designed by using the vector $\xi(0)$. Then the upper bound on the performance index eq.(33) is given as $E\{\mathcal{J}^*\} = Tr\{\mathcal{X}_\xi\}$. Therefore we seek to minimize $Tr\{\mathcal{X}_\xi\}$ subject to the constraint eq.(32). Namely the problem of designing the control gain matrix to minimize the upper bound on the performance index eq.(33) is reduced to the following constrained optimization problem.

$$
\begin{aligned}
&\underset{\mathcal{X}_\xi, K_{e_r}}{\text{Minimize}}[Tr\{\mathcal{X}_\xi\}] \ \ \text{subject to} \\
&\qquad \text{eq.(32) and } \ \mathcal{X}_\xi > 0
\end{aligned} \qquad (37)
$$

Now we introduce a symmetric positive definite matrix $\mathcal{S}_\xi \triangleq diag(\mathcal{S}_{e_r}, \mathcal{S}_z) = \mathcal{X}^{-1}_\xi$ and consider the change of variable $\mathcal{W}_{e_r} \triangleq K_{e_r}\mathcal{S}_{e_r}$. Then pre- and post-multiplying eq.(32) by $\mathcal{S}_\xi$ and simple algebraic manipulation, the condition eq.(32) can be written as eq.(38). Furthermore by using the Schur complement formula, the inequality eq.(38)

$$
-\mathcal{S}_\xi+\mathcal{S}_\xi\mathcal{F}_\mathcal{K}^T(\theta)\mathcal{S}_\xi^{-1}\mathcal{F}_\mathcal{K}(\theta)\mathcal{S}_\xi+\begin{pmatrix}\mathcal{S}_{e_r} & 0\\ 0 & \mathcal{S}_z\\ \mathcal{W}_{e_r} & 0\end{pmatrix}^T\begin{pmatrix}\mathcal{Q}_{e_r} & 0 & 0\\ 0 & \mathcal{Q}_z & 0\\ 0 & 0 & \mathcal{R}_{e_r}\end{pmatrix}\begin{pmatrix}\mathcal{S}_{e_r} & 0\\ 0 & \mathcal{S}_z\\ \mathcal{W}_{e_r} & 0\end{pmatrix}<0 \text{ for } \forall\theta\in\Delta \tag{38}
$$

$$
(A(\theta)-H_rC(\theta))^T\mathcal{X}_z(A(\theta)-H_rC(\theta))-\mathcal{X}_z+\begin{pmatrix}C(\theta)\\ H_rC(\theta)\\ 0\end{pmatrix}^T\mathcal{X}_{e_r}\begin{pmatrix}C(\theta)\\ H_rC(\theta)\\ 0\end{pmatrix}+\mathcal{Q}_z
$$

$$
<0 \text{ for } \forall\theta\in\Delta \tag{45}
$$

$$
H_r=\begin{pmatrix}1.2894 & 0.8150\end{pmatrix}^T \tag{46}
$$

$$
K_{e_r}=\begin{pmatrix}-0.1315 & 0.9077 & 1.0249 & -0.1315 & -0.2161 & -0.2031 & -0.1390 & -0.0786 & -0.0381\end{pmatrix} \tag{47}
$$

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is reduced to the following condition where $\mathcal{Q}^*$ is the matrix given by $\mathcal{Q}^*=diag\left(\ \mathcal{Q}_{e_r},\ \mathcal{Q}_z,\ \mathcal{R}_{e_r}\ \right)$.

$$
\Upsilon(\theta)\triangleq\left(\begin{array}{c|c|c}-\mathcal{S}_\xi & \mathcal{S}_\xi\mathcal{F}_\mathcal{K}^T(\theta) & \begin{matrix}\mathcal{S}_{e_r} & 0 & \mathcal{W}_{e_r}^T\\ 0 & \mathcal{S}_z & 0\end{matrix}\\ \hline \mathcal{F}_\mathcal{K}(\theta)\mathcal{S}_\xi & -\mathcal{S}_\xi & \begin{matrix}0 & 0 & 0\end{matrix}\\ \hline \begin{matrix}\mathcal{S}_{e_r} & 0\\ 0 & \mathcal{S}_z\\ \mathcal{W}_{e_r} & 0\end{matrix} & \begin{matrix}0\\0\\0\end{matrix} & -\left(\mathcal{Q}^*\right)^{-1}\end{array}\right)<0 \text{ for } \forall\theta\in\Delta \tag{39}
$$

The condition eq.(39) is linear matrix inequality (LMI) in $\mathcal{S}_{e_r},\mathcal{S}_z$ and $\mathcal{W}_{e_z}$ because the matrix $\mathcal{F}_\mathcal{K}(\theta)\mathcal{S}_\xi$ is expressed as

$$
\mathcal{F}_\mathcal{K}(\theta)\mathcal{S}_\xi=\begin{pmatrix}\mathcal{F}_{11}(\theta)\mathcal{S}_{e_r}-\mathcal{G}_1\mathcal{W}_{e_r} & \mathcal{F}_{12}(\theta)\mathcal{S}_z\\ \mathcal{F}_{21}(\theta)\mathcal{S}_{e_r}-\mathcal{G}_2(\theta)\mathcal{W}_{e_r} & \mathcal{F}_{22}(\theta)\mathcal{S}_z\end{pmatrix} \tag{40}
$$

Thus the condition eq.(39) is equivalent to

$$
\Upsilon(\theta)\le 0 \text{ for } \forall\theta\in\Delta_{\text{vex}} \tag{41}
$$

Here by introducing a complementary variable $\mathcal{Z}_\xi\in\Re^{(n+N_h)\times(n+N_h)}$

$$
\begin{pmatrix}\mathcal{Z}_\xi & I_{n+N_h}\\ I_{n+N_h} & \mathcal{S}_\xi\end{pmatrix}\ge 0 \tag{42}
$$

the minimization problem of $Tr\{\mathcal{X}_\xi\}$ can be transformed into that of $Tr\{\mathcal{Z}_\xi\}$. Consequently, the optimization problem eq.(37) is reduced to the following constrained convex optimization problem, because the condition eq.(42) is also the linear matrix inequality (LMI) in $\mathcal{Z}_\xi$ and $\mathcal{S}_\xi$.

$$
\underset{\mathcal{Z}_\xi,\mathcal{S}_{e_r},\mathcal{S}_z,\mathcal{W}_{e_r}}{\text{Minimize}}\ [Tr\{\mathcal{Z}_\xi\}] \text{ subject to}
$$

$$
\text{eqs.(41) and (42)},\quad \mathcal{S}_{e_r}>0 \text{ and } \mathcal{S}_z>0 \tag{43}
$$

If the solution $\mathcal{Z}_\xi>0,\mathcal{S}_{e_r}>0,\mathcal{S}_z>0$ and $\mathcal{W}_{e_r}$ of the optimization problem eq.(43) is obtained, then the control gain matrix K_{e_r} is obtained as

$$
K_{e_r}=\mathcal{W}_{e_r}\mathcal{S}_{e_r}^{-1} \tag{44}
$$

Note that, the constrained convex optimization problem eq.(42) can be solved by using software such as MATLAB's LMI Control Toolbox, Scilab's LMITOOL and so on.

As a result, the following theorem is obtained.

Theorem 2. There exists the control gain matrix K_{e_r} minimizing the upper bound on the performance index eq.(33), if there exist the optimal solution $\mathcal{Z}_\xi>0,W_{e_r},\mathcal{S}_{e_r}>0$ and $\mathcal{S}_z>0$ of the optimization problem eq.(43). Also, using the solution of the linear matrix inequality (LMI) condition eq.(29), the observer gain matrix H_e is designed in advance such as $H_r=\mathcal{Y}_{\lambda_z}^{-1}\mathcal{V}_{\lambda_z}$.

If the solution $\mathcal{Z}_\xi,W_{e_r},\mathcal{S}_{e_r}$ and $\mathcal{S}_z$ is obtained, then the control gain matrix K_{e_r} is given by

$$
K_{e_r}=\mathcal{W}_{e_r}\mathcal{S}_{e_r}^{-1}
$$

Remark 1. In this paper, the observer eq.(6) is designed such that quadratic stability of the system eq.(23) is ensured and the condition eq.(29) is satisfied, because in order to get the control gain matrix K_{e_r} and the symmetric positive definite matrix $\mathcal{X}_\xi$ satisfying the condition eq.(32), $(2,2)$-block of the condition $\Phi_\xi(\theta)\le 0$ must be negative semidefinite. Namely, the observer gain matrix H_r has to be determined, making allowance for the inequality eq.(45). Thus we introduce a symmetric positive definite matrix $\mathcal{W}_{\lambda_z}$ and a design parameter $\mathcal{Q}_H$ and consider the condition eq.(26).

4. NUMERICAL EXAMPLE

Consider the following uncertain system.

$$
\begin{aligned}
x(t+1)&=\begin{pmatrix}1.0 & 1.0\\ 0.5 & 0.85+\delta_1\end{pmatrix}x(t)\\
&\quad+\begin{pmatrix}1.0\\ 1.0\end{pmatrix}u(t)\\
y(t)&=\begin{pmatrix}1.0+\delta_2 & 0\end{pmatrix}x(t)
\end{aligned}
$$

where the parameters δ_1 and δ_2 are the uncertainties and are assumed to vary within the interval $[-0.15,0.15]$ and $[-0.10,0.10]$, respectively.

By selecting the design parameter $\mathcal{Q}_H=I_2$ and solving the linear matrix inequality (LMI) condition eq.(28), we get the observer gain matrix H_r eq.(45). Next, let $\mathcal{Q}_{e_r}=I_3,\mathcal{Q}_z=4.0I_2,\mathcal{R}_r=1.0$ and $h=5$, then by solving the constrained convex optimization problem eq.(42), we obtain the control gain matrix K_{e_r} eq.(46).

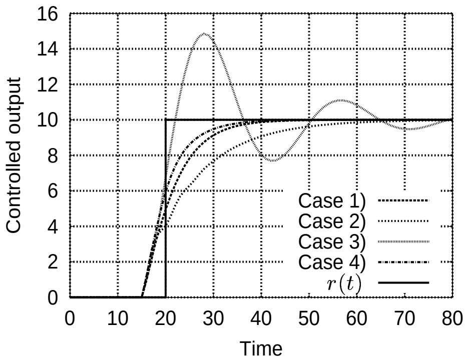

Fig. 1. Time histories of the controlled output $y(t)$

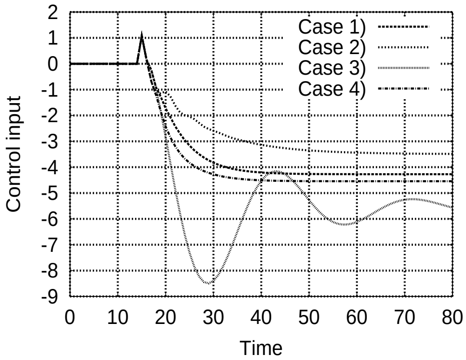

Fig. 2. Time histories of the control input $u(t)$

In this example, we assume that the reference signal $r(t)$ varies as

$$r(t) = \begin{cases} 0.0 & \text{for} \quad t < 20 \\ 10.0 & \text{for} \quad t \geq 20 \end{cases} \tag{45}$$

Also, to examine the robustness of the proposed controller, we consider the following cases.

- Case 1) : $\delta_1 = -0.15$ and $\delta_2 = -0.1$
- Case 2) : $\delta_1 = -0.15$ and $\delta_2 = 0.1$
- Case 3) : $\delta_1 = 0.15$ and $\delta_2 = -0.1$
- Case 4) : $\delta_1 = 0.15$ and $\delta_2 = 0.1$

The results of the simulation of this example are shown in figure 2-4. We see from these figures that the proposed robust preview controller achieves performance robustness.

5. CONCLUSIONS

In this paper for uncertain discrete-time systems, we present a design method of an observer-based robust preview controller. The observer-based robust controller is easily obtained , because adopting 2-stage design approach, the design problem of the observer-based controller is reduced to linear matrix inequalities (LMIs).

In future work, we will examine the controller design algorithm for the minimization of the upper bound on the performance index, because the observer-based controller derived by our design method is not optimal.

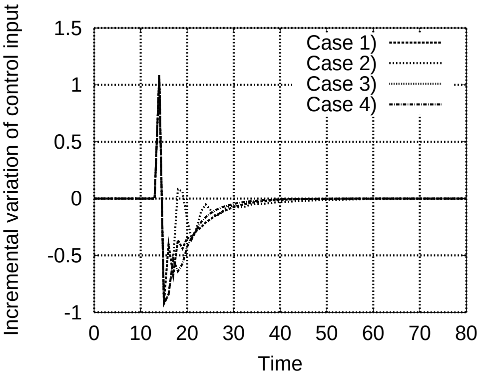

Fig. 3. Time histories of incremental variation of the control input $\overline{u}(t)$

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