# NONLINEAR OUTPUT FEEDBACK CONTROL OF A 5DOF AMB SYSTEM

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Abstract: A nonlinear dynamic output feedback control method for a 5-degree-of-freedom AMB is presented in this paper. This method is based on the system structure of AMB and a quadratic-like Lyapunov function. It is shown that backstepping and completing square techniques enable the construction of the voltage input by using measured output only. The control law guarantees the asymptotic stability of the closed loop system. 

Keywords: Magnetic bearing system, Nonlinear output feedback, Quadratic-like Lyapunov function, Power consumption

## 1. INTRODUCTION

AMB (active magnetic bearing) systems have an outstanding advantage over conventional bearings: noncontact support. For this reason, its application in high speed rotating machines is quite attractive. The price payed is that AMB is inherently an unstable system and can not be operated without feedback control.

In AMB systems, the attractive force each electromagnet exerts on the rotor has a strong nonlinearity on both the current and the air gap. In order to use linear control approaches in this system, large bias currents have to be applied to pairs of electromagnets so as to guarantee that magnetic force acting on the rotor can be approximated as a linear function of the currents (A bias current of $I_0 \approx 0.5I_{max}$ is used to linearize the system in traditional control design (Tsiotras and Velenis, 2000)). This large bias current however does not contribute to the control of magnetic bearing but is a waste of power. Furthermore, the control system designed based on linear approximation works only in a very narrow range.

Therefore, in recent years in order to design AMB systems with low power consumption, approaches called "zero-bias" or "zero-power" and "low-power", in which zero bias or low bias is used, are emerging as popular alternatives. See (Ariga *et al.*, 2000; Liu *et al.*, 2002; Charara *et al.*, 1996; Tsiotras and Velenis, 2000; Motee *et al.*, 2002; de Queiroz and Dawson, 1996*b*; de Queiroz and Dawson, 1996*a*; Knospe and Yang, 1997; Lévine *et al.*, 1996).All these aforementioned approaches are fundamentally state feedback based.

However, state feedback control approaches can not be extended to high speed rotor in which the flexibility (high order vibration modes) has to be taken into consideration and the states of these high order vibration modes are not measurable. Therefore, it is highly desired to find a nonlinear output feedback approach for AMB systems. The purpose of this paper is to show how such a nonlinear output feedback control law can be designed for a 5-degree-of freedom (5DOF) AMB system. This paper is the extension of (Liu and He, 2003) in which the idea is outlined for the 1DOF case.

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## 2. MODEL OF A 5DOF AMB SYSTEM

The rotor is assumed to be rigid for simplicity. Control of flexible rotor will be considered in a forthcoming paper.

The model in Fig.1 is composed of 10 electromagnets. The distances between the center of gravity and each acting point of magnetic forces $F_1 \sim F_8$ are set as $l$.

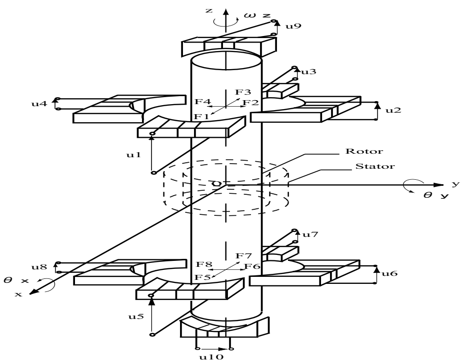


Fig. 1. Structure of a 5DOF AMB system

Let $(x_G, y_G)$ denote the displacements of the center of gravity of the rotor in $xoy$ plane, $\theta_x$, $\theta_y$ denote the angular displacements in $yoz$ plane and $xoz$ planes, respectively. Also, the air gaps in $x$ and $y$ directions at the equilibrium are denoted by $X_0$ and $Y_0$.

*Rotor Dynamics* As the motion in $z$ direction can be controlled independently as an 1DOF AMB system which is much simpler, only the control of the remaining 4DOF will be described in this paper.

Let the forces and torques along $x, y$ axes be denoted by $f_x, \tau_x$ and $f_y, \tau_y$ respectively. Then the motion equations for the 4DOF system are described by (Higuchi and Mizuno, 1982)

$$m\ddot{x}_G = f_x \tag{1}$$
$$m\ddot{y}_G = f_y \tag{2}$$
$$J_r\ddot{\theta}_y - J_a\omega_z\dot{\theta}_x = \tau_x \tag{3}$$
$$J_r\ddot{\theta}_x + J_a\omega_z\dot{\theta}_y = \tau_y. \tag{4}$$

Here $m$ is the mass of the rotor, $J_r$ the inertial moment around $x$ and $y$ axes and $J_a$ the inertial moment around $z$ axis. The positive rotating directions for $\theta_x$ and $\theta_y$ are as shown in Fig.1. The terms of $J_a\omega_z\dot{\theta}_x$ and $J_a\omega_z\dot{\theta}_y$ in (3) and (4) represent gyroscopic moments. In this paper, it is assumed that $\omega_z = const.$ Moreover, disturbances caused by the unbalance of the rotor are ignored.

The state variables are set as

$$x_1 = x_G,\ x_2 = \dot{x}_G,\ y_1 = y_G,\ y_2 = \dot{y}_G$$
$$z_1 = \theta_y,\ z_2 = \dot{\theta}_y,\ z_3 = \theta_x,\ z_4 = \dot{\theta}_x.$$

The measured outputs are displacements $\eta_x := x_1$, $\eta_y := y_1$, $\eta_z := [z_1 \ \ z_3]^T$ for the rotor part. Then the state equations are obtained as

$$\dot{X} = A_{xo}X + b_{xo}f_x, \quad \eta_x = c_{xo}X \tag{5}$$
$$\dot{Y} = A_{yo}Y + b_{yo}f_y, \quad \eta_y = c_{yo}Y \tag{6}$$
$$\dot{Z} = A_{zo}Z + B_{zo}\Gamma, \quad \eta_z = C_{zo}Z \tag{7}$$

where the state vectors and torque input vector are $X = [x_1\ x_2]^T$, $Y = [y_1\ y_2]^T$, $Z = [z_1\ z_2\ z_3\ z_4]^T$, $\Gamma = [\tau_y\ \tau_x]^T$ and the coefficient matrices are omitted since they can be obtained from (1)$\sim$(4) easily.

*Electromagnetic Circuit* Let the currents of magnets be denoted by $(\bar{\xi}_1, \ldots, \bar{\xi}_8)$ and the resistance of each circuit be $R$. All currents are assumed to be measured. Moreover, let $x_u$, $y_u$ and $x_l$, $y_l$ denote the gaps between the upper electromagnets and the rotor, the gaps between the lower electromagnets and the rotor respectively. Since $z_1 = \theta_1, z_3 = \phi_1$ are small enough, these gaps are approximated as

$$x_u = x_1 + lz_1, \quad x_l = x_1 - lz_1 \tag{8}$$
$$y_u = y_1 - lz_3, \quad y_l = y_1 + lz_3. \tag{9}$$

Then the state equations of electromagnetic circuits are described by $(r = 1, \ldots, 8)$

$$\dot{\bar{\xi}}_r = \frac{1}{L_r}\left(-R\bar{\xi}_r - \dot{L}_r\bar{\xi}_r + \bar{u}_r\right) \tag{10}$$

where the inductances $L_1, \ldots, L_8$ are given by

$$L_1 = \frac{2k}{X_0 - x_u},\ L_2 = \frac{2k}{Y_0 - y_u},\ L_3 = \frac{2k}{X_0 + x_u}$$
$$L_4 = \frac{2k}{Y_0 + y_u},\ L_5 = \frac{2k}{X_0 - x_l},\ L_6 = \frac{2k}{Y_0 - y_l}$$
$$L_7 = \frac{2k}{X_0 + x_l},\ L_8 = \frac{2k}{Y_0 + y_l}.$$

Here $k$ is a constant. The electromagnetic forces of the magnets are described by $(r = 1, \ldots, 8)$

$$F_r = \frac{1}{4k}L_r^2\bar{\xi}_r^2. \tag{11}$$

As is obvious from Fig.1, electromagnetic forces $f_x$, $f_y$ and torques $\tau_x$, $\tau_y$ are given by

$$f_x = F_1 - F_3 + F_5 - F_7 \tag{12}$$
$$f_y = F_2 - F_4 + F_6 - F_8 \tag{13}$$
$$\tau_y = (F_1 - F_3 - F_5 + F_7)\,l \tag{14}$$
$$\tau_x = (-F_2 + F_4 + F_6 - F_8)\,l. \tag{15}$$

For future use, the time derivatives of $L_i (i = 1, 3, 5, 7)$ and $L_j (j = 2, 4, 6, 8)$ are listed below

$$\dot{L}_i = \frac{\partial L_i}{\partial x_1} x_2 + \frac{\partial L_i}{\partial z_1} z_2,\ i = 1, 3, 5, 7 \qquad (16)$$

$$\dot{L}_j = \frac{\partial L_j}{\partial y_1} y_2 + \frac{\partial L_j}{\partial z_3} z_4,\ j = 2, 4, 6, 8. \qquad (17)$$

*Relative Degree of Actuator* A discussion similar to that of (Liu *et al.*, 2002) reveals that the relative degree is not defined at the equilibrium if the bias current is $\epsilon = 0$. So a bias current $\epsilon \neq 0$ is applied on the four pairs of electromagnets in order for the backstepping technique to be applicable. This renders a relative degree of 1.

To shift the equilibrium currents to zeros, new states and new inputs

$$\xi_r = \overline{\xi}_r - \epsilon, \quad u_r = \overline{u}_r - R\epsilon,\ r = 1, \ldots, 8 \qquad (18)$$

are defined. In this new coordinate the state equations of the actuator change to ($r = 1, \ldots, 8$)

$$\dot{\xi}_r = \frac{1}{L_r}\left(-R\xi_r - \dot{L}_r \overline{\xi}_r + u_r\right). \qquad (19)$$

## 3. CONTROL DESIGN

To design the control law for the system, the AMB system is decomposed into 3 parts: the first one is the rotor dynamics (5)∼(7), the second one is the static relations (11)∼(15) between the currents and electromagnetic forces, and the third one is the electromagnetic circuit (19).

Since the rotor dynamics (5)∼(7) is linear, any linear control method can be applied to design the required electromagnetic forces so as to stabilize this subsystem ($H_\infty$ linear controllers are designed in the simulation.) So the design process consists of the following 3 steps:

(1) Construct $(f_x^*, f_y^*, \tau_y^*, \tau_x^*)$ by linear dynamic output feedback of $(x, y, \theta_x, \theta_y)$.
(2) Compute the desired currents $(\xi_1^*, \ldots, \xi_8^*)$ which correspond to $(f_x^*, f_y^*, \tau_y^*, \tau_x^*)$.
(3) Use backstepping and completing square techniques to find $(\overline{u}_1, \ldots, \overline{u}_8)$ using only the measured outputs $(x, y, \theta_x, \theta_y)$ and $(\overline{\xi}_1, \ldots, \overline{\xi}_8)$.

*Structural Requirement On $K(s)$* First of all, a condition on the linear dynamic output feedback controller $K(s)$ is given, under which the input of nonlinear actuator can be realized by dynamic output feedback(Liu and He, 2003).

*Lemma 1.* The relative degree of $K(s)$ must be greater than or equal to that of the actuator in order for dynamic output feedback to be realizable.

*Output Feedback Design of Magnetic Force* Since the relative degree of the electromagnetic actuator is 1, the linear controller must be strictly proper. Here it is assumed that the linear controller is designed as follows:

$$\dot{X}_k = A_{xk} X_k + b_{xk}\eta_x, \quad f_x^* = c_{xk} X_k \qquad (20)$$

$$\dot{Y}_k = A_{yk} Y_k + b_{yk}\eta_y, \quad f_y^* = c_{yk} Y_k \qquad (21)$$

$$\dot{Z}_k = A_{zk} Z_k + B_{zk}\eta_z, \quad \Gamma^* = C_{zk} Z_k. \qquad (22)$$

This controller yields 3 decoupled closed-loop subsystems corresponding to each motion. As an example, the closed-loop subsystem for the motion in $x$ direction is shown below

$$\begin{bmatrix} \dot{X} \\ \dot{X}_k \end{bmatrix} = A_x \begin{bmatrix} X \\ X_k \end{bmatrix} + b_x f_{xe}, f_{xe} := f_x - f_x^* \qquad (23)$$

in which

$$A_x = \begin{bmatrix} A_{xo} & b_{xo} c_{xk} \\ b_{xk} c_{xo} & A_{xk} \end{bmatrix}, \quad b_x = \begin{bmatrix} b_{xo} \\ 0 \end{bmatrix}.$$

The coefficient matrices $A_y$, $b_y$, $A_z$ and $B_z$ for the closed loop of other 2 motions are obtained similarly.

To find a Lyapunov function for the whole linear closed-loop subsystem, let us define the following notations:

$$\zeta = \begin{bmatrix} X^T & X_k^T & Y^T & Y_k^T & Z^T & Z_k^T \end{bmatrix}^T$$
$$T_e = \begin{bmatrix} f_{xe} & f_{ye} & \tau_{ye} & \tau_{xe} \end{bmatrix}^T$$

in which

$$f_{ye} = f_y - f_y^*,\ \tau_{ye} = \tau_y - \tau_y^*,\ \tau_{xe} = \tau_x - \tau_x^*.$$

Then it is easy to verify that the state equation of the linear closed-loop subsystem is described by

$$\dot{\zeta} = A_c \zeta + B_c T_e \qquad (24)$$

where

$$A_c = \mathrm{diag}(A_x \quad A_y \quad A_z), \quad B_c = \mathrm{diag}(b_x \quad b_y \quad B_z).$$

Further, since $A_c$ is stable, there exists a matrix $P > 0$ satisfying the following Lyapunov equation

$$A_c^T P + P A_c + I = 0 \qquad (25)$$

and $P$ has a structure of $P = \mathrm{diag}(P_x \quad P_y \quad P_z)$.

Then it is well-known that the quadratic function

$$V_1(\zeta) = \zeta^T P \zeta \qquad (26)$$

provides a Lyapunov function for the linear subsystem (24). The derivative of $V_1(\zeta)$ is found to be

$$\dot{V}_1 = -||\zeta||^2 + \zeta^T (P B_c T_e) + (P B_c T_e)^T \zeta. \qquad (27)$$

*Computation of Desired Currents* Next, to find the desired currents $(\xi_1^*, \ldots, \xi_8^*)$ corresponding to

$(f_x^*, f_y^*, \tau_y^*, \tau_x^*)$, let us begin with the analysis of equations (11)~(15). Obviously, the solution is not unique. The solution that minimizes the total electric power is to be determined. (12) and (14) can be transformed into

$$\left(\alpha f_x^* + \beta\tau_y^*\right)/2 = L_1^2(\overline{\xi_1}^*)^2 - L_3^2(\overline{\xi_3}^*)^2 \quad (28)$$
$$\left(\alpha f_x^* - \beta\tau_y^*\right)/2 = L_5^2(\overline{\xi_5}^*)^2 - L_7^2(\overline{\xi_7}^*)^2 \quad (29)$$

in which

$$\alpha = 4k,\ \beta = 4k/l,\ \overline{\xi}_r^* = \xi_r^* + \epsilon\ (r = 1, \ldots, 8).$$

Define

$$S_1 = \alpha f_x^* + \beta\tau_y^*,\ S_2 = \alpha f_x^* - \beta\tau_y^* \quad (30)$$

as 2 switching functions. In order to minimize the electric power, there must hold

$$\begin{cases} S_1 > 0,\ \xi_3^* = 0 \\ S_1 = 0,\ \xi_1^* = \xi_3^* = 0 \\ S_1 < 0,\ \xi_1^* = 0 \end{cases},\ \begin{cases} S_2 > 0,\ \xi_7^* = 0 \\ S_2 = 0,\ \xi_5^* = \xi_7^* = 0 \\ S_2 < 0,\ \xi_5^* = 0 \end{cases}$$

Consequently, the optimal odd numbered desired currents are obtained as

$$\xi_1^* = \frac{1}{L_1}\sqrt{\frac{S_1}{2} + L_3^2\epsilon^2} - \epsilon\ (S_1 > 0),\ 0\ (S_1 \le 0)$$
$$\xi_3^* = 0\ (S_1 \ge 0),\ \frac{1}{L_3}\sqrt{\frac{-S_1}{2} + L_1^2\epsilon^2} - \epsilon\ (S_1 < 0)$$
$$\xi_5^* = \frac{1}{L_5}\sqrt{\frac{S_2}{2} + L_7^2\epsilon^2} - \epsilon\ (S_2 > 0),\ 0\ (S_2 \le 0)$$
$$\xi_7^* = 0\ (S_2 \ge 0),\ \frac{1}{L_7}\sqrt{\frac{-S_2}{2} + L_5^2\epsilon^2} - \epsilon\ (S_2 < 0).$$

Similarly, the optimal even numbered desired currents are

$$\xi_2^* = \frac{1}{L_2}\sqrt{\frac{S_3}{2} + L_4^2\epsilon^2} - \epsilon\ (S_3 > 0),\ 0\ (S_3 \le 0)$$
$$\xi_4^* = 0\ (S_3 \ge 0),\ \frac{1}{L_4}\sqrt{\frac{-S_3}{2} + L_2^2\epsilon^2} - \epsilon\ (S_3 < 0)$$
$$\xi_6^* = \frac{1}{L_6}\sqrt{\frac{S_4}{2} + L_8^2\epsilon^2} - \epsilon\ (S_4 > 0),\ 0\ (S_4 \le 0)$$
$$\xi_8^* = 0\ (S_4 \ge 0),\ \frac{1}{L_8}\sqrt{\frac{-S_4}{2} + L_6^2\epsilon^2} - \epsilon\ (S_4 < 0)$$

where

$$S_3 = \alpha f_y^* - \beta\tau_x^*,\ S_4 = \alpha f_y^* + \beta\tau_x^*. \quad (31)$$

*Design of Voltage Inputs* In this part, the voltage inputs will be designed by using backstepping and completing square techniques.

*Error Equations* As in the standard backstepping, the errors between real currents $(\xi_1, \ldots, \xi_8)$ and the desired currents $(\xi_1^*, \ldots, \xi_8^*)$ are defined as

$$e_r = \xi_r - \xi_r^*, \quad i = 1, \ldots, 8.$$

Then their dynamics are described by

$$\dot{e}_r = \frac{1}{L_r}\left[-R\xi_r - \dot{L}_r\overline{\xi}_r + u_r - L_r\dot{\xi}_r^*\right]. \quad (32)$$

Further, it can be verified that equations

$$\sum_{i\in I}\frac{\partial L_i}{\partial x_1}\left(\frac{1}{2}e_i - \overline{\xi}_i\right)e_i = -f_{xe} \quad (33)$$
$$\sum_{j\in J}\frac{\partial L_j}{\partial y_1}\left(\frac{1}{2}e_j - \overline{\xi}_j\right)e_j = -f_{ye} \quad (34)$$
$$\sum_{i\in I}\frac{\partial L_i}{\partial z_1}\left(\frac{1}{2}e_i - \overline{\xi}_i\right)e_i = -\tau_{ye} \quad (35)$$
$$\sum_{j\in J}\frac{\partial L_j}{\partial z_3}\left(\frac{1}{2}e_j - \overline{\xi}_j\right)e_j = -\tau_{xe} \quad (36)$$

hold where $I$ and $J$ are 2 integer sets

$$I = \{1, 3, 5, 7\}, \quad J = \{2, 4, 6, 8\}.$$

*Lyapunov Function* The key point in obtaining an output control law for this AMB system is to use the following quadratic-like function

$$V(\zeta, e_1, \ldots, e_8) = V_1 + \frac{1}{2}\sum_{r=1}^{8} L_r e_r^2 \quad (37)$$

as a candidate of Lyapunov function for the whole system. Note that $L_r$ is a (positive) function of the air gap, so this Lyapunov function is different from the one provided by standard backstepping method. Derivation of $V$ along the trajectory yields

$$\dot{V} = \dot{V}_1 + \sum_{r=1}^{8}\dot{L}_r\left(\frac{1}{2}e_i - \overline{\xi}_i\right)e_r + \sum_{r=1}^{8} e_r(u_r - R\xi_r - L_r\dot{\xi}_r^*)$$

based on (32). Further, substitution of (16), (17), (27) as well as (33)~(36) leads to

$$\begin{aligned}\dot{V} = & -\|\zeta\|^2 + \zeta^T(PB_cT_e) + (PB_cT_e)^T\zeta \\ & - f_{xe}x_2 - f_{ye}y_2 - \tau_{ye}z_2 - \tau_{xe}z_4 \\ & + \sum_{r=1}^{8} e_r(u_r - R\xi_r - L_r\dot{\xi}_r^*)\end{aligned} \quad (38)$$

*First Completion of Square* As $(x_2, y_2, z_2, z_4)$ are not measured, the terms containing them can not be cancelled by the voltage inputs $u_r (r = 1, \ldots, 8)$. What is possible is to cover these terms by the negative term $-\|\zeta\|^2$. Define

$$\overline{P} = PB_c - \frac{1}{2}B_cM,\ M = \mathrm{diag}(m\ \ m\ \ J_r\ \ J_r).$$

Since $[x_2 \;\; y_2 \;\; z_2 \;\; z_4]^T = MB_c^T\zeta$, completion of square yields

$$\dot{V} = -||\zeta - \overline{P}T_e||^2 + ||\overline{P}T_e||^2 + \sum_{r=1}^{8} e_r(u_r - R\xi_r) - \sum_{r=1}^{8} e_r L_r \dot{\xi}_r^*. \quad (39)$$

*Second Completion of Square* Further, let us deal with the last term in (39). Tedious but straightforward computation yields

$$L_i\dot{\xi}_i^* = w_i x_2 + \tilde{w}_i l z_2 + v_i, \;\; i = 1,3,5,7 \quad (40)$$
$$L_j\dot{\xi}_j^* = w_j y_2 + \tilde{w}_j l z_4 + v_j, \;\; j = 2,4,6,8. \quad (41)$$

The signals $w_r, \; \tilde{w}_r, \; v_r \; (r = 1, \ldots, 8)$ are functions of measured outputs. Then it is obtained that

$$\sum_{r=1}^{8} e_r L_r \dot{\xi}_r^* = \sum_{r=1}^{8} e_r v_r + w_{od}x_2 + w_{ev}y_2 + \tilde{w}_{od}z_2 + \tilde{w}_{ev}z_4$$

where $w_{od} = \sum_{i\in I} e_i w_i$, $\tilde{w}_{od} = l\sum_{i\in I} e_i\tilde{w}_i$, $w_{ev} = \sum_{j\in J} e_j w_j$ and $\tilde{w}_{ev} = l\sum_{j\in J} e_j\tilde{w}_j$. In order to get rid of the unmeasured states in this term, completion of square will be executed one more time. For simplicity of presentation, define

$$W = [w_{od} \;\; w_{ev} \;\; \tilde{w}_{od} \;\; \tilde{w}_{ev}].$$

Then, this equation can be further written as

$$\begin{aligned}&\sum_{r=1}^{8} e_r L_r \dot{\xi}_r^* \\ &= -||\zeta - \overline{P}T_e||^2 + ||\zeta - \overline{P}T_e + \frac{1}{2}B_c(WM)^T||^2 \\ &\quad -||\frac{1}{2}B_c(WM)^T||^2 + WMB_c^T\overline{P}T_e + \sum_{r=1}^{8} e_r v_r.\end{aligned}$$

Substitution of this equation into (39) gives

$$\begin{aligned}\dot{V} = &-||\zeta - \overline{P}T_e + \frac{1}{2}B_c(WM)^T||^2 \\ &+||\overline{P}T_e||^2 + ||\frac{1}{2}B_c(WM)^T||^2 - WMB_c^T\overline{P}T_e \\ &+\sum_{r=1}^{8} e_r(u_r - R\xi_r - v_r). \end{aligned} \quad (42)$$

The 2nd to the 4th terms can be expressed in terms of $e_r$ explicitly. After the cancellation of these terms by the voltage input $u_r$ (see Theorem 2 for the formula), a nonpositive

$$\dot{V} = -||\zeta - \overline{P}T_e + \frac{1}{2}B_c(WM)^T||^2 - \sum_{r=1}^{8}(c_r + R)e_r^2$$

is obtained. Here $c_r > 0$ $(r = 1, \cdots, 8)$ is a control gain. Moreover, $\dot{V} \equiv 0$ iff

$$\zeta - \overline{P}T_e + \frac{1}{2}B_c(WM)^T = 0, \;\; e_1 = \cdots = e_8 = 0.$$

As $e_1 = \cdots = e_8 = 0$ implies $T_e = 0$ and $W = 0$, it follows from the above equation that $\zeta = 0$. Therefore, asymptotic stability is guaranteed by LaSalle's invariance principle.

*Theorem 2.* The asymptotic stability of the AMB system is guaranteed by the following dynamic output feedback voltage inputs:

$$\begin{aligned}u_i = &R\xi_i^* + v_i - c_i e_i - \frac{1-l^2}{4}(w_i w_5 e_5 + w_i w_7 e_7) \\ &\pm\left(\frac{\kappa_1}{\alpha} + \frac{\kappa_3}{\beta} - \lambda_1 f_{xe} - \lambda_3\tau_{ye} - \lambda_5\tau_{xe}\right) L_i^2(\overline{\xi}_i + \overline{\xi}_i^*) \\ &-\frac{1+l^2}{4}w_i^2 e_i \;\; (i = 1:+, \;\; i = 3:-) \\ u_j = &R\xi_j^* + v_j - c_j e_j - \frac{1-l^2}{4}(w_j w_6 e_6 + w_j w_8 e_8) \\ &\pm\left(\frac{\kappa_2}{\alpha} - \frac{\kappa_4}{\beta} - \lambda_2 f_{ye} + \lambda_4\tau_{xe} + \lambda_5\tau_{ye}\right) L_j^2(\overline{\xi}_j + \overline{\xi}_j^*) \\ &-\frac{1+l^2}{4}w_j^2 e_j \;\; (j = 2:+, j = 4:-) \\ u_p = &R\xi_p^* + v_p - c_p e_p - \frac{1-l^2}{4}(w_p w_1 e_1 + w_p w_3 e_3) \\ &+\left(\frac{\kappa_1}{\alpha} - \frac{\kappa_3}{\beta} - \lambda_1 f_{xe} + \lambda_3\tau_{ye} + \lambda_5\tau_{xe}\right) L_p^2(\overline{\xi}_p + \overline{\xi}_p^*) \\ &-\frac{1+l^2}{4}w_p^2 e_p \;\; (p = 5:+, p = 7:-) \\ u_q = &R\xi_q^* + v_q - c_q e_q - \frac{1-l^2}{4}(w_q w_2 e_2 + w_q w_4 e_4) \\ &+\left(\frac{\kappa_2}{\alpha} + \frac{\kappa_4}{\beta} - \lambda_2 f_{ye} - \lambda_4\tau_{xe} - \lambda_5\tau_{ye}\right) L_q^2(\overline{\xi}_q + \overline{\xi}_q^*) \\ &-\frac{1+l^2}{4}w_q^2 e_q \;\; (q = 6:+, q = 8:-).\end{aligned}$$

Note that $w_r$, $v_r$ and $\tilde{w}_r$ are bounded since $\overline{\xi}_r^* > 0$ $(r = 1, \ldots 8)$. Therefore, the given control inputs are bounded.

## 4. NUMERICAL SIMULATION

Parameters used in simulations are listed below: $m = 14.46$, $l = 0.13$, $J_a = 0.0136$, $J_r = 0.333$, $k = 0.00769$, $X_0 = Y_0 = 0.55$, $R = 14.7$.

In simulation, initial displacements are set as $x(0) = -5 \times 10^{-2}$[mm], $y(0) = 5 \times 10^{-2}$[mm], $\theta_x(0) = -5\times 10^{-4}$[rad] and $\theta_y(0) = 5\times 10^{-4}$[rad]. Initial speeds are all set as zeros. The bias current applied is $\epsilon = 0.2$[A] and the axial rotation speed $\omega_z$ is set as 1000[rad/s]. Some of the responses are shown in Fig.2∼Fig.5. Here, control gains used are $c_1 = \ldots = c_8 = 20$. The linear controller is designed using $H_\infty$ control to suppress the effect of torque disturbance on the displacement output and electromagnetic force input. The detail is omitted.

The displacements and velocities of rotor settle in about 4[ms], the currents of electromagnets also settle to the bias currents in about 4[ms]. Further, as can be seen from Fig.4∼Fig.5, the control voltages switch and only one of each pair of magnets is active. The switching works well because it is determined based on the change of directions of the synthesized magnetic forces necessary for the control of the rotor. Simulations under other bias currents level and initial conditions show similar trend.

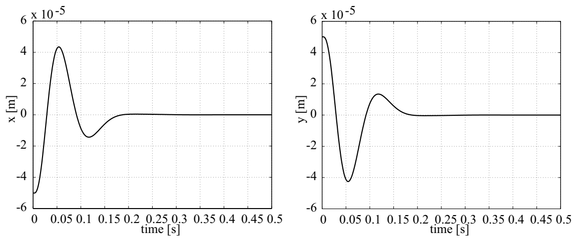


Fig. 2. $x_G$ and $y_G$

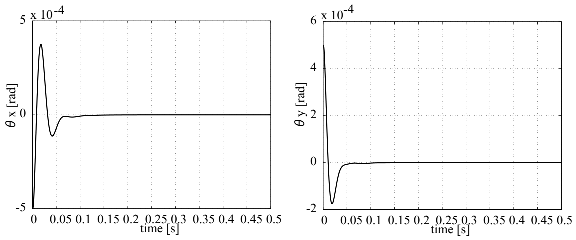


Fig. 3. $\theta_x$ and $\theta_y$

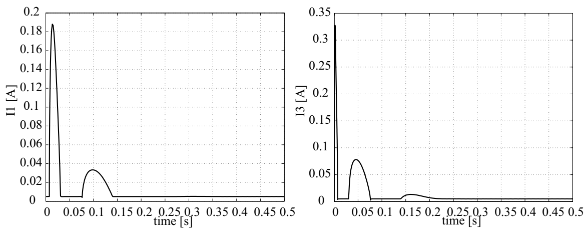


Fig. 4. Currents 1 and 3

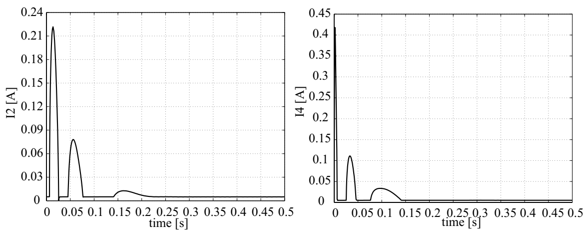


Fig. 5. Currents 2 and 4

## 5. CONCLUDING REMARKS

In this paper, a nonlinear dynamic output feedback control method has been presented for a 5DOF AMB system. This method guarantees the asymptotic stability.

The extention of this approach to more general cases is under study, where the axial rotation speed $\omega_z$ is time-varying and the rotor is flexible. The result will be reported in a forthcoming paper.

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