SLIDING MODE PROXIMATE TIME-OPTIMAL SERVOMECHANISM

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Abstract: A discrete-time sliding mode proximate time-optimal servomechanism is developed using nonlinear sliding mode tangent to a reference velocity profile. Instead of a strong nonlinear function causing control chattering in discrete-time, a linear control is designed around the sliding surface. The resulting servomechanism delivers a bounded motion about the sliding surface even in the presence of plant uncertainties. Application results demonstrate accurate reference velocity tracking and uniformly excellent seek performance regardless of seek length via enforced sliding mode.

1. INTRODUCTION

Servomechanism in proximate time-optimal operation is widely used in the actuator positioning control (e.g. disk drive actuator control). Since time-optimal control which uses maximum acceleration and maximum deceleration is unfortunately known to be not robust with respect to the system uncertainties and measurement noises, it can hardly be used in practice. Removing such maximum operation from the time-optimal controller, which gives the system a finite bandwidth, is much more practical for many applications. A method for removing the infinite gain operation is introducing a linear extension to a velocity profile (e.g. the proximate time-optimal servomechanism (PTOS) (Workman, 1987; Franklin *et al.*, 1990)). By introducing a linear extension (in the phase plane) velocity profile, exponential tracking (in the time plane) of the plant velocity (position and control also) can be achieved in settling (Lee *et al.*, 2003). Although the target seek time is thus discounted, it becomes fairly robust with respect to system uncertainties and noises.

The PTOS delivers practically near minimum target seek time using maximum acceleration and maximum deceleration until the error becomes small where it switches to a linear control law for asymptotic settling down to the target point. Three basic servo modes are sequentially assigned along the reference velocity profile for better seek performance: referred to as acceleration, deceleration, and settle (Lee *et al.*, 2003). During acceleration, the actuator is controlled to approach the reference velocity profile. Once the plant trajectory reaches the reference velocity profile, the servo mode is changed to deceleration where the actuator is controlled to follow the reference velocity profile until it approaches the target. The servo mode is switched from the seek mode to the settle mode near the target. In the PTOS design, it is in general not easy to determine control gains for deceleration and settle along a reference velocity profile. Also, the resulting controller performance possibly varies on individual disk drives with different parameters. Dependent on control gains used as well as seek length, large tracking error may occur possibly causing a oscillating settle and longer target seek time. Accordingly, individually modified PTOS algorithms are widely used in the disk drive industry.

Sliding mode control (Utkin, 1992; Su *et al.*, 1996; Edwards and Spurgeon, 1998; Young *et al.*, 1999; Spurgeon, 1992), must be a good candidate in improving servo system performance and robustness, that enforces plant trajectory to slide on a certain hyperplane thereby must contribute to reduce reference velocity tracking error. In (Weerasooriya *et al.*, 1993), adaptive sliding mode disk drive control was designed using a reference velocity profile with a pivot at the zero point as a sliding surface. A servomechanism that consists of a proximate time-optimal control for seek and a sliding mode control for settle and track-follow was proposed in (Lee *et al.*, 1999). There, a state feedback controller was applied near the sliding mode, where the feedback gain can be chosen to adjust convergence rate to the sliding mode. The discrete-time sliding mode control was applied for settle and track-follow, which was designed based on the linear extension in a reference velocity profile (Lee *et al.*, 2003). In (Zhou and Wang, 2003), a discrete-time nonlinear seek controller was applied to obtain sliding mode control like results using a series of time-varying fixed pivot sliding surfaces going through the origin. There, the controller design parameters were selected for convergent sliding sequence and (proximate) time-optimal control like responses. In general, using a fixed pivot sliding mode appears to be problematic in controlling the plant trajectory to follow a reference velocity profile because these two have very different convergence directions.

The aim is to develop a sliding mode proximate time-optimal servomechanism (SPTOS). A series of tangent sliding modes are designed using a reference velocity profile with linear extension (Lee *et al.*, 2003). Such sliding mode must be natural considering the reference velocity profile tracking in proximate time-optimal control. Instead of a strong nonlinear function causing problematic control chattering in discrete-time, a linear control is designed around the sliding surface. No servo mode changing is introduced as the tangent sliding mode ensures smooth convergence to the target. The resulting SPTOS must delivers a stable closed-loop as well as a bounded motion about the sliding surface in the presence of plant uncertainties. Also, the SPTOS must deliver accurate reference velocity tracking regardless of seek length that can only be produced via enforced sliding mode.

2. SLIDING MODE SERVOMECHANISM DESIGN

2.1 Actuator modeling

Consider a rigid body actuator dynamics described as

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{1}$$

with

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ a \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

where a is the acceleration constant The state vector $x^T = \begin{bmatrix} x_1 & x_2 \end{bmatrix} = \begin{bmatrix} y & v \end{bmatrix}$ for the position y and the velocity v. For a sampling period T, the zero-order-holder (ZOH) discrete-time equivalent (Franklin *et al.*, 1990) becomes

$$\begin{aligned} x(k+1) &= \Phi x(k) + \Gamma u \\ y(k) &= Cx(k) \end{aligned} \tag{2}$$

where $\Phi = e^{AT}$, $\Gamma = \int_0^T e^{A(T-\eta)} B d\eta$.

2.2 Sliding mode design with reference velocity profile

Let $k_a = 2\alpha a u_{max}$ with α being $0 < \alpha \leq 1$ denote the acceleration discount factor. Integrating the actuator model in (1) with $u = u_{\max}$ delivers a time-optimal reference velocity profile. Putting a linear extension to the profile as shown in Fig. 1 for exponential tracking near the target (Lee *et al.*, 2003), we obtain a reference velocity profile for a proximate time-optimal control

$$v_r = \begin{cases} \mathrm{sgn}(p_e)\sqrt{k_a\left(|p_e| - p_o\right)} & \text{if } |p_e| > p_l = 2p_o, \\ \dfrac{\sqrt{k_a}}{2\sqrt{p_o}} p_e & \text{otherwise} \end{cases} \tag{3}$$

where $p_e = p_r - x_1$ denotes the position error for the position reference p_r, and p_o denotes the position error shift. One can find that the profile in (3) satisfies $v_r(0) = 0$, $v_r(p_e)p_e > 0\ \forall p_e \neq 0$, $dv_r(p_e)/dp_e|_{p_e=p_l} = \frac{\sqrt{k_a}}{2\sqrt{p_o}}$.

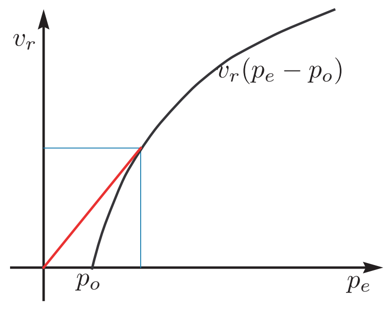


Fig. 1. Reference velocity profile with linear extension

In the proximate time-optimal control the plant trajectory is controlled to follow the reference velocity profile so as to yield an exponential tracking near the target. Whilst, in the sliding mode time-optimal control the plant trajectory is enforced to slide on the sliding surface. Accordingly, introducing a sliding mode tangent to the reference velocity profile seems to be natural for the profile tracking in proximate time-optimal control. Without loss of generality, we only consider the quadrant where $p_e \geq 0$ and $v_r \geq 0$ in the subsequent formulations. The results, however, can be generalized.

Proposition 1. The surface

$$s = S_1 p_e + v_r + S_o \tag{4}$$

with

$$S_1 = \begin{cases} -\dfrac{k_a}{2v_r} & \text{if } p_e > 2p_o, \\ -\dfrac{\sqrt{k_a}}{2\sqrt{p_o}} & \text{otherwise} \end{cases} \tag{5}$$

and

$$S_o = \begin{cases} -v_r - S_1 p_e & \text{if } p_e > 2p_o, \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

is tangent to the reference velocity profile (3) and satisfies the constraint $S_o p_e \geq 0$ such that the sliding surface can cross the origin $(0, 0)$ and stay thereafter as $p_e \to 0$.

Proof: Skipped due to space limitation. ■

2.3 Control design using the sliding mode

In order to design a discrete-time controller, let us build a discrete-time sliding mode from (4) as

$$s(k) = \begin{bmatrix} S_1 & 1 \end{bmatrix} x(k) + s_o(k). \tag{7}$$

Introducing an approximation of stable sliding sequence

$$s(k) = Sx(k) - \rho Sx(k-1) \tag{8}$$

where $S = \begin{bmatrix} \xi S_1 & 1 \end{bmatrix}$ for some $\xi \geq 1$ and $0 \leq \rho < 1$. It should be noted that $\xi = 1$ and $\rho = 0$ for fixed sliding mode. Also notice that introducing time varying parameters $\xi(k)$ and $\rho(k)$ may yield exact equivalence, although not used here. Considering that the fixed sliding mode (to deliver exponential decay while mostly affecting seek performance) must be used in the end of seek, the above approximation using constant parameters is very practical. The parameters ξ and ρ for tangent sliding mode can be chosen to attain smaller reference velocity tracking error during deceleration

$$\min_{\xi,\rho} \|v_r(k) - s(k)\| \text{ for all } k > 0, \; s(k)u(k) \leq 0. \tag{9}$$

Then, the ideal sliding mode condition $s(k+1) = s(k) = 0$ gives the optimal equivalent control

$$u_{eq}(k) = -K_{eq} x(k) = -\left(S\Gamma\right)^{-1} S\left(\Phi - \rho I\right) x(k) \tag{10}$$

minimizing reference velocity tracking error during deceleration. The ideal sliding mode is then described by

$$x(k+1) = \Phi_{eq} x(k) = \left(\Phi - \Gamma\left(S\Gamma\right)^{-1} S\left(\Phi - \rho I\right)\right) x(k) \tag{11}$$

with its eigenvalue matrix $\Lambda_{eq} = \operatorname{diag}(\lambda_s, \rho)$ and the corresponding eigenvector matrix $V_{eq} = \begin{bmatrix} v_s & v_n \end{bmatrix}$. The nonzero eigenvalue λ_s satisfies $|\lambda_s| < 1$ as it is designed for and the corresponding eigenvector matrix v_s belongs to the null space $\mathcal{N}(S)$.

Corollary 1. A sufficient condition for the existence of the discrete-time sliding mode is that there exists a positive integer k_o such that

$$|s(k+1)| \leq w(k)\,|s(k)|\,, \; 0 \leq w(k) < 1, \; k \geq k_o \tag{12}$$

in the region $\mathcal{N}_\epsilon = \{|s(k)| = |Sx(k)| < \epsilon\}$.

The sufficient condition (12) is equivalent to

$$-w(k)\,|s(k)| \leq s(k+1) \leq w(k)\,|s(k)|\,, \; 0 \leq w(k) < 1.$$

Here, we let $s(k+1) = w(k)|s(k)|$. Then clearly, a sufficient condition for the existence of the sliding mode is $|w(k)| < 1$. Using $s(k+1) = Sx(k+1) = -S\Gamma u_{eq}(k) + S\Gamma u(k)$ we have

$$u(k) = u_{eq}(k) + \left(S\Gamma\right)^{-1} w(k)|s(k)|. \tag{13}$$

Then, we have

$$u(k) = u_{eq}(k) + \left(S\Gamma\right)^{-1} w(k)\operatorname{sgn}[s(k)]s(k). \tag{14}$$

Considering the problematic control chattering in digital implementation of sliding mode control, discontinuous control is not used. Introducing $W = w(k)\operatorname{sgn}[s(k)]$, referred to as convergence rate factor, we build

$$u(k) = u_{eq}(k) + \left(S\Gamma\right)^{-1} W s(k). \tag{15}$$

Furthermore, introducing a control bound, we have

$$u(k) = \operatorname{sat}\left[u_{eq}(k) + \left(S\Gamma\right)^{-1} W s(k)\right] \tag{16}$$

where

$$\operatorname{sat}[u] = \begin{cases} \operatorname{sgn}[u]u_{\max} & \text{if } |u| > u_{\max}, \\ u & \text{otherwise} \end{cases}$$

is the saturation function and $u_{\max}$ denotes the maximum control input. Accordingly, the control regions are defined as follows:

$\mathcal{S}_+ = \{x : u = u_{\max}\}$;
$\mathcal{S}_- = \{x : u = -u_{\max}\}$;
$\mathcal{U} = \{x : |u| < u_{\max}\}$: boundary layer.

Observation 1. The resulting SPTOS system becomes linear in the boundary layer $\mathcal{U}$ with the state feedback equivalent sliding mode control $u(k) = -Kx(k)$ where

$$K = K_{eq} - \left(S\Gamma\right)^{-1} WS. \tag{17}$$

As a matter of fact, a linear control is applied around the sliding surface where the error is small. A strong nonlinear function may be preferred to a linear control because of its strong acting control around the sliding surface. The strong control achieves disturbance rejection. In discrete-time however, the discontinuous control action can become impractically fast, this limits some benefits of the sliding mode control, causing control chattering. Thereby in this paper, instead of a strong nonlinear function like switching, a linear control is designed around the sliding surface to eliminate the chattering problem. A question that may naturally arise at this point is if the resulting SPTOS then guarantees a bounded motion about the sliding surface, a very useful characteristic of sliding mode control. The linear control in general does not have such a desirable property. The next section is dedicated to discussing a bounded motion about the sliding surface.

2.4 Analysis of the SPTOS

Let us examine if the resulting SPTOS delivers a stable closed-loop and a bounded motion about the sliding surface. For this purpose, it is necessary to examine the eigenvalues of the closed-loop in the boundary layer $\mathcal{U}$ where the error is small and the control becomes linear.

Let the control closed-loop eigenvalue matrix $\tilde{\Lambda} = \operatorname{diag}\left(\tilde{\lambda}_s, \tilde{\lambda}_n\right)$ corresponding to the eigenvector matrix $\tilde{V} = \left[\, \tilde{v}_s \;\; \tilde{v}_n \,\right]$. The we find the followings.

Proposition 2. Introducing the convergence rate factor W satisfies

$$\tilde{\lambda}_s = \lambda_s, \; \tilde{\lambda}_n = W + \rho. \tag{18}$$

Proof: Skipped due to space limitation. ■

Theorem 1. Suppose that an actuator in the form of (1) with an initial condition $(x_i, 0)$ is given. A reference velocity profile

$$v_r = \begin{cases} \operatorname{sgn}(p_e)\sqrt{k_a\left(|p_e| - p_o\right)} & \text{if } |p_e| > p_l = 2p_o, \\ \dfrac{\sqrt{k_a}}{2\sqrt{p_o}} p_e & \text{otherwise} \end{cases}$$

is designed such that a sliding mode $s(k) = Sx(k) + \rho S_o$ delivers the equivalent control

$$u_{eq}(k) = -\left(S\Gamma\right)^{-1} S\left(\Phi - \rho I\right) x(k).$$

Then, for any factor W with $|W + \rho| < 1$, producing convergent discrete sliding modes, the resulting sliding mode control law

$$u(k) = \operatorname{sat}\left[u_{eq}(k) + \left(S\Gamma\right)^{-1} W s(k)\right]$$

delivers a globally asymptotically stable discrete-time closed-loop system with $|\Lambda| < I$.

Proof: Skipped due to space limitation. ■

What is then the best value of the closed-loop eigenvalue $\tilde{\lambda}_n$? In the case of using nonlinear tangent sliding mode, this is equivalent to finding the best value of W that may deliver the best value of the closed-loop eigenvalue $\tilde{\lambda}_n = W + \rho$ together with ρ from (9). Observing the control scheme (that must be designed for accurate reference tracking) eventually expressed in terms of the closed-loop eigenvalue $\tilde{\lambda}_n = W + \rho$, one can rewrite the minimization problem (9) in terms of the parameters ξ and $\tilde{\lambda}_n$ as

$$\min_{\xi, \tilde{\lambda}_n} \|v_r(k) - s(k)\| \text{ for all } k > 0, \; s(k)u(k) \le 0. \tag{19}$$

This expression is preferred to (9) because one can also determine the best value of the closed-loop eigenvalue $\tilde{\lambda}_n = W + \rho$. The values of each of the parameters W and ρ are in fact not required in the controller design. The state feedback equivalent control gain in (17) must be optimal with the parameters from (19). In the case of using fixed sliding mode, on the other hand, the convergence rate factor W (that equals $\tilde{\lambda}_n$ since $\rho = 0$) is determined for fast yet smooth access to it. Applying the minimization problem (19) to the fixed sliding mode must deliver $W = \tilde{\lambda}_n = 0$ producing the equivalent control u_{eq} and instantaneous access to the fixed sliding mode, that is undesirable in practice considering the actuators' flexible dynamics.

Let us consider a case of using the state estimator (Franklin *et al.*, 1990)

$$\begin{aligned} \bar{x}(k+1) &= \Phi\hat{x}(k) + \Gamma u(k) \\ \hat{x}(k) &= \bar{x}(k) + L_c\left(y(k) - C\bar{x}(k)\right) \end{aligned} \tag{20}$$

where $\bar{x}(\cdot)$ and $\hat{x}(\cdot)$ are predicted and corrected state estimates, respectively, L_c is the state estimator gain to be determined for desired state estimation with a diagonal eigenvalue matrix Λ_{est}.

The sliding mode in terms of the state estimate is

$$s(k) = S\hat{x}(k) + \rho S_o. \tag{21}$$

Also, the equivalent control in terms of the state estimate is

$$u_{eq}(k) = -\left(S\Gamma\right)^{-1} S\left(\Phi - \rho I\right)\hat{x}(k). \tag{22}$$

Let us consider the closed-loop system subject to a bounded matched plant uncertainty $g(k)$ described by

$$\mathbf{x}(k+1) = \Phi_{cl}\mathbf{x}(k) + \tilde{g}(k) \tag{23}$$

where $\mathbf{x} = \begin{bmatrix} x(k) \\ x(k) - \bar{x}(k) \end{bmatrix}$ and

$$\Phi_{cl} = \begin{bmatrix} \Phi - \Gamma K & \Gamma K\left(I - L_c C\right) \\ 0 & \Phi - \Phi L_c C \end{bmatrix}, \; \tilde{g} = \begin{bmatrix} g(x) \\ g(x) \end{bmatrix}.$$

The uncertainty is bounded such that $\|g(k)\| \le \epsilon\|\mathbf{x}(k)\|$ for a positive constant ϵ. This uncertainty bound is realistic since state estimation error can become sufficiently small even in the presence of unknown plant uncertainty (Lee and Chung, 2003). Let $\Lambda_{cl} = \text{diag}(\Lambda, \Lambda_{est})$, the closed-loop eigenvalue matrices, then the robust closed-loop stability can be stated as follows.

Corollary 2. If $\|\Lambda_{cl}\| + \sqrt{2}\epsilon < 1$, for $0 \le \|\mathbf{x}(k_o)\| < r$, the uncertainty closed-loop system (23) is globally uniformly asymptotically stable about the ball $\mathcal{B}(d) = \{\mathbf{x} \in \mathbb{R}^{2n} : \|\mathbf{x}\| \le d\}$ such that $\|\mathbf{x}(k)\| \le d(r, \Lambda_{cl}, \epsilon, k) = \left(\|\Lambda_{cl}\| + \sqrt{2}\epsilon\right)^{k-k_o} r$ for all $k \ge k_o$.

Proof: Skipped due to space limitation. ■

This result can be applied to state bounded motion about the sliding surface.

Corollary 3. The distance from the sliding mode is bounded by

$$\|s\| \le \|S\| d(r, \Lambda, \epsilon, k) + \|s_o\| < 2\|S\| d(r, \Lambda, \epsilon, k-1).$$

Furthermore, $\|s\| \le \|S\| d(r, \Lambda, \epsilon, k)$ for $|p_e| \le 2p_o$.

Proof: Skipped due to space limitation. ■

The resulting SPTOS is shown to deliver a bounded motion about the sliding surface even in the presence of plant uncertainties. Although not producing a strong acting control around the sliding surface, the SPTOS that becomes linear in the boundary layer $\mathcal{U}$ is shown to enforce the plant trajectory points robustly converge to the sliding surface against uncertainties in Corollaries 2 and 3, that can not happen in the linear control design. The resulting SPTOS thereby must deliver a desired plant trajectory that can only be produced via enforced sliding mode. Such bounded motion about the sliding surface then eventually guarantees bounded motion about the corresponding reference velocity profile. As mentioned, the design parameters ξ and $\tilde{\lambda}_n$ for tangent sliding mode can be chosen to attain a minimized reference velocity tracking error during deceleration. Solving the minimization problem (19) delivers optimal parameter values for tangent sliding mode to reduce the maximum reference tracking error. Larger residual tracking error in general causes oscillating settle and results in a slower seek time. Therefore, accurate reference velocity tracking by the SPTOS must improve seek performance. This is the key benefit of the proposed SPTOS.

3. AN APPLICATION EXAMPLE

The proposed SPTOS is applied to a disk drive actuator expressed by a triplet (A, B, C):

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ a \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

with $a = 2.4194 \times 10^4$ Amp^{-1}sec^{-2}. For a sampling rate of 150 μsec, a control limit of $u_{\max} = 0.7$ Amp, a track density of 47,000 tracks per inch, a SPTOS is designed based on a discrete-time equivalent triplet (Φ, Γ, C) following the given design procedure.

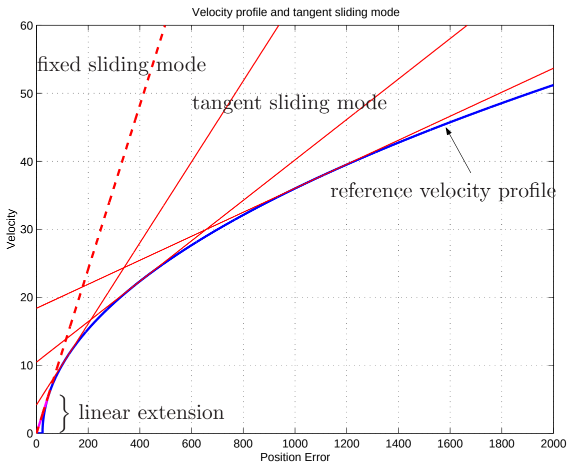
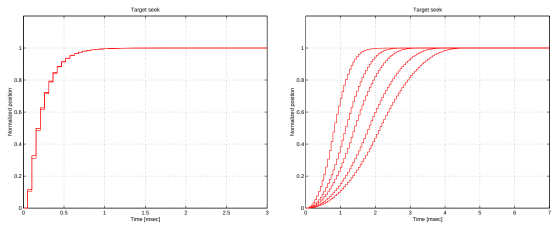


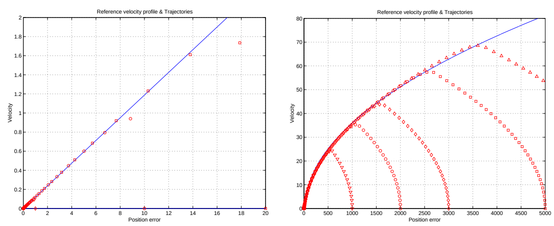
Fig. 2. Exemplary sliding mode tangent to reference velocity profile

Exemplary sliding mode tangent to the reference velocity profile (designed for $p_o = 24$ tracks and $\alpha = 0.975$) is shown in Fig. 2.



(a) 1, 10, 20 track seeks (b) 1000, 2000, 3000, 5000, 7000 track seeks

Fig. 3. Normalized target seek profiles



(a) 1, 10, 20 track seeks (b) 1000, 2000, 3000, 5000, 7000 track seeks

Fig. 4. Target seek profiles in phase plane

The SPTOS yields typical track seek profile as shown in Figs. 3 and 4. The plant trajectory points near the target in the phase plane (Fig. 4(a)) shows perfect reference (i.e., the fixed sliding mode or linearly extended reference velocity) tracking. Similar perfect tracking was observed regardless of seek length. This is fairly different from the PTOS results in which quite large error even causing oscillating settle may exist dependent on

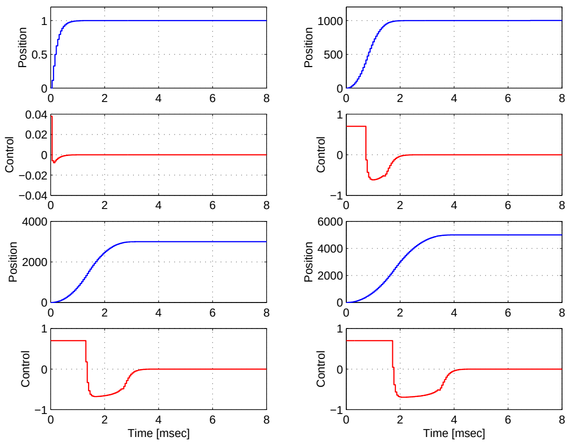


Fig. 5. Position and control profiles in target seek: 1, 1000, 3000, and 5000 tracks.

the control gains used as well as seek length. Considering that one of the design objectives of the settle mode is to provide a small velocity/position error initial condition for the regulation control at the target, the SPTOS must improve control performance (Fig. 3(a)). In addition, the SPTOS also shows accurate reference tracking in the tangent sliding mode region (Fig. 4(b)) and delivers excellent track seek performance (Fig. 3(b)). Considering the trajectories of acceleration (with full control) and deceleration, one can find the SPTOS delivers an (almost symmetric) output shape very close to the time-optimal one. Fig. 5 shows the position and control profiles in target seek tests. Uniformly excellent seek performance is observed regardless of seek length, that can only be produced via enforced sliding mode.

In the SPTOS, the plant trajectory follows the reference velocity profile quite accurately regardless of seek length. The reference tracking error disappears rapidly dependent on the convergence rate factor W as the fixed sliding mode region is reached. The SPTOS using the state feedback equivalent control gain K uniquely determined and updated along the sliding mode (tangent to the reference velocity profile) determines the plant trajectory points converging to the target.

4. CONCLUSIONS

This paper has presented a discrete-time sliding mode proximate time-optimal servomechanism developed using nonlinear sliding mode tangent to a reference velocity profile. Instead of a strong nonlinear function causing control chattering in discrete-time, a linear control was designed around the sliding surface. The sliding mode servomechanism that becomes linear in the boundary layer was shown to deliver a bounded motion about the sliding surface even in the presence of plant uncertainties, that can not happen in the linear control design. The utility of the proposed servomechanism was demonstrated through an application example. Accurate reference velocity tracking and uniformly excellent seek performance regardless of seek length, that can only be produced via enforced sliding mode, promises improved seek performance yet simple design.

Note: Complete proofs and application example appear in a full paper version.

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