SLIDING MODE CONTROL OF NON-MINIMUM PHASE NONLINEAR UNCERTAIN INPUT-DELAY CHEMICAL PROCESSES

Chyi-Tsong Chen* **and Shih-Tien Peng**

Department of Chemical Engineering, Feng Chia University
Taichung 407, Taiwan
**E-mail: ctchen@fcu.edu.tw*

Abstract: A sliding mode control scheme is developed for nonlinear, non-minimum phase, uncertain, input-delay processes. The proposed scheme, which integrates a time-advanced nonlinear predictor and a statically equivalent output map, is able to compensate the process's input-delay and to circumvent the negative effect of inverse response. The convergence properties of the proposed sliding mode control system are guaranteed theoretically by a Lyapunov-based approach. Furthermore, we applied the proposed scheme to the regulation control of a Van de Vusse reactor in the presence of diversified dynamics.

Keywords: Sliding mode control; Time-advanced nonlinear predictor; Statically equivalent output; Non-minimum phase; Input-delay; Robust stability and performance

1. INTRODUCTION

During the past few decades, the robust control system designs for uncertain processes have received considerable attention from control community. Among the established design approaches for robust process control, sliding mode control (SMC) plays an important role because it not only stabilizes certain and uncertain systems but also provides the capability of disturbance rejection and insensitivity to parameter variations (Utkin, 1992). Therefore the design of SMC control schemes has attracted considerable attention and many techniques have been proposed (Shyu and Yan, 1993; Hu et al., 1998; Roh and Oh, 2000; Li and Yurkovich, 2001; Spurgeon and Lu, 1997; Camacho et al., 1999; Herrmann et al., 2003).

However, although the SMC strategy has established many successful application in handling diversified process dynamics, the SMC control of nonlinear processes which possess simultaneously the dynamics behavior of uncertainties, input-delay and inverse response has not ever been addressed in the literature. To tackle with this difficult control problem, we propose a novel SMC control scheme which integrates a statically equivalent output map (SEOM) and a time-advanced nonlinear predictor. The main ideas are based on using the SEOM for eliminating the undesirable inverse response and a predictor for curbing the negative effect of input-delay, which therefore facilitates the design of a sliding mode controller. The convergence properties of the whole SMC control system are guaranteed by utilizing the Lyapunov stability theorem. Besides, we applied the proposed scheme to control a Van de Vusse reactor in the presence of diversified dynamics.

2. A PREDICTOR-BASED SMC SCHEME FOR NONLINEAR, UNCERTAIN, NON-MINIMUM PHASE, INPUT-DELAY PROCESSES

2.1 Control system configuration and system description

Consider a single-input/single-output non-minimum phase nonlinear input-delay process whose dynamics are modeled by the following uncertain equations:

$$\dot{\mathbf{x}}(t) = (\mathbf{f}(\mathbf{x}) + \Delta\mathbf{f}(\mathbf{x})) + (\mathbf{g}(\mathbf{x}) + \Delta\mathbf{g}(\mathbf{x}))\, u(t-\theta) \qquad (1a)$$

$$y(t) = h(\mathbf{x}) \qquad (1b)$$

where $\mathbf{x}(t) \in R^n$, $u(t) \in R$, $y(t) \in R$ and $\theta \in ([0,\infty), R)$ are state vector, control input, system output and the time delay respectively. Without loss of generality, we assume that the origin $\mathbf{x}=0$ is a uniformly asymptotically stable equilibrium point of the unforced nominal system and $h(\mathbf{x})$ vanishes at that equilibrium point. This means that y represents the tracking error. The proposed scheme shown in Fig. 1 will describe through the following individual parts.

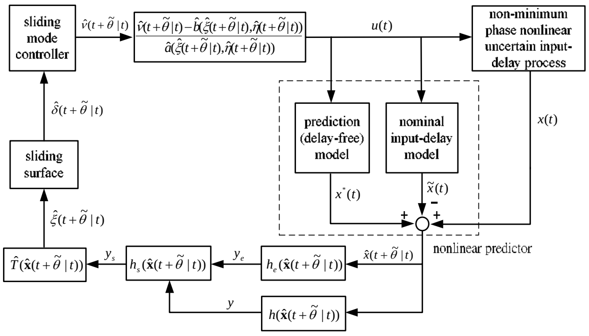

Fig. 1. Schematic diagram of the proposed predictor-based SMC scheme.

2.2 Design of a statically equivalent output map

An auxiliary output design method using zero assignment technique. Consider the following nonlinear non-minimum phase input-delay system

$$\dot{\mathbf{x}}(t)=\mathbf{f}(\mathbf{x})+\mathbf{g}(\mathbf{x})u(t-\theta) \tag{2a}$$

$$y(t)=h(\mathbf{x}) \tag{2b}$$

An auxiliary process which is statically equivalent to the nonlinear system of Eq. (2) can be given by

$$\dot{\mathbf{x}}(t)=\mathbf{f}(\mathbf{x})+\mathbf{g}(\mathbf{x})u(t-\theta) \tag{3a}$$

$$y_e(t)=h_e(\mathbf{x}) \tag{3b}$$

where $h_e(\cdot)$ is an auxiliary output to make the system locally minimum phase. A formulation for $h_e(\cdot)$ using the original system dynamics can be given by (Kravaris et al., 1998):

$$h_e(\mathbf{x})=h(\mathbf{x})+\sum_{j=1}^{n-1}\varepsilon_j\Psi_j(\mathbf{x}) \tag{4}$$

where

$$\Psi_j(\mathbf{x})=\det\begin{bmatrix} f_j(\mathbf{x}) & g_j(\mathbf{x}) \\ f_n(\mathbf{x}) & g_n(\mathbf{x})\end{bmatrix},\ \ j=1,2,\ldots,n-1 \tag{5}$$

are functions vanishing on the equilibrium curve and $\varepsilon_j, j=1,2,\ldots,n-1$, are constant weights being chosen such that $h_e(\mathbf{x})$ is statically equivalent to $h(\mathbf{x})$ .Let $(\mathbf{x}_s,u_s)$ be a reference equilibrium point, then we can define the zeros polynomials corresponding to $h(\mathbf{x})$ and $\Psi_j(\mathbf{x})$, $j=1,2,\ldots,n-1$, respectively, as

$$\tilde{P}(s)=\frac{\partial h(\mathbf{x}_s)}{\partial \mathbf{x}}\operatorname{Adj}\left[sI-\left(\frac{\partial \mathbf{f}(\mathbf{x}_s)}{\partial \mathbf{x}}+u_s\frac{\partial \mathbf{g}(\mathbf{x}_s)}{\partial \mathbf{x}}\right)\right]\mathbf{g}(\mathbf{x}_s) \\ =\tilde{p}_0+\tilde{p}_1 s+\cdots+\tilde{p}_{n-1}s^{n-1} \tag{6}$$

$$\tilde{Q}_j(s)=\frac{\partial \Psi_j(\mathbf{x}_s)}{\partial \mathbf{x}}\operatorname{Adj}\left[sI-\left(\frac{\partial \mathbf{f}(\mathbf{x}_s)}{\partial \mathbf{x}}+u_s\frac{\partial \mathbf{g}(\mathbf{x}_s)}{\partial \mathbf{x}}\right)\right]\mathbf{g}(\mathbf{x}_s) \\ =\tilde{q}_{j,1}s+\cdots+\tilde{q}_{j,n-1}s^{n-1},\quad j=1,2,\ldots,n-1 \tag{7}$$

Furthermore, let $z_j, j=1,2,\ldots,n-1$, be the desirable zeros for $h_e(\mathbf{x})$ at the reference equilibrium point. The given values of z_j and the requirement of static equivalence with $h(\mathbf{x})$ completely specifies the desirable zero polynomial for $h_e(\mathbf{x})$ as:

$$\tilde{P}^d(s)=\tilde{p}_0\prod_{j=1}^{n-1}\left(1-\frac{s}{z_j}\right)=\tilde{p}_0+\tilde{p}_1^d s+\cdots+\tilde{p}_{n-1}^d s^{n-1} \tag{8}$$

The necessary values of the adjustable weights ε_j can be obtained by solving the following equation

$$\tilde{P}(s)+\sum_{j=1}^{n-1}\varepsilon_j\tilde{Q}_j(s)=\tilde{P}^d(s) \tag{9}$$

Synthesis of a statically equivalent output map (SEOM) for use under process uncertainties. The purpose of this subsection is two folds. The first one is to ensure the minimum phase behaviour under the influence of process uncertainties and the other one is to guarantee the statically equivalent output property of y_s . To meet the first goal, a new algorithm for redesign of ε_j is proposed as follows:

Initialization: Choose the desired zeros, $z_j^0\in\text{LHP}$, at the reflections of the RHP zeros with respect to the imaginary axis. Also, set $\Delta z_j>0$ for pole shifting. Let $i=1$ and $z_j^i=z_j^0$.

Step 1: Set $\tilde{P}^d(s)$ based on the zeros z_j^i. Calculate ε_j^i, $j=1,2,\ldots,n-1$, from Eq. (9) and then construct $h_e^i(\mathbf{x})$ according to Eq. (4).

Step 2: Check whether $h_e^i(\mathbf{x})$ is minimum phase or not under process uncertainties by Monte Carlo simulations. If yes, stop. Otherwise, go next step.

Step 3: Shifting the desired zeros by $z_j^{i+1}=z_j^i-\Delta z_j$, then set $i=i+1$ and go back to step 1.

To achieve the statically equivalent property for the second goal, we suggest the following auxiliary output for control:

$$y_s\equiv h_s(\mathbf{x})=y+(y_e-y)e^{-\lambda_s t} \tag{10}$$

where $y_e=h_e(\mathbf{x})$, $y=h(\mathbf{x})$ and $\lambda_s>0$ is the tuning constant. The role of λ_s in this auxiliary output map is to make a smooth transition from the minimum phase one to actual process output. Actually, the selection of λ_s depends on the process dynamic characteristics. As a result, y_s appears to be a SEOM to the actual process output, which ensures no steady state offset and minimum phase behaviour despite of the influence of process uncertainties.

2.3 Design of a predictor-based sliding mode controller

Based on the synthesized auxiliary output y_s , the nonlinear uncertain input-delay model used for controller design is given by

$$\dot{\mathbf{x}}(t)=(\mathbf{f}(\mathbf{x})+\Delta\mathbf{f}(\mathbf{x}))+(\mathbf{g}(\mathbf{x})+\Delta\mathbf{g}(\mathbf{x}))u(t-\theta) \tag{11a}$$

$$y_s(t) = h_s(\mathbf{x}) \tag{11b}$$

It should be noted here that the present system is minimum phase under uncertainties and the auxiliary output y_s is statically equivalent to the actual process output y. Let the Lie derivative of a smooth function $h_s(\mathbf{x})$ along a vector field $\mathbf{g}(\mathbf{x})$ be defined as:

$$L_{\mathbf{g}} h_s(\mathbf{x}) = \frac{\partial h_s(\mathbf{x})}{\partial \mathbf{x}} \mathbf{g}(\mathbf{x}) = \sum_{i=1}^{n} \frac{\partial h_s(\mathbf{x})}{\partial x_i} g_i(\mathbf{x}) \tag{12}$$

In terms of Lie derivative, the relative degree of the system (11) is defined as $\rho = \min\{m : L_{\mathbf{g}} L_{\mathbf{f}}^{m-1} h_s(\mathbf{x}) \neq 0\}$. Similarly, let $\kappa = \min\{m : L_{\Delta\mathbf{f}} L_{\mathbf{f}}^{m-1} h_s(\mathbf{x}) \neq 0\}$ and $w = \min\{m : L_{\Delta\mathbf{g}} L_{\mathbf{f}}^{m-1} h_s(\mathbf{x}) \neq 0\}$ be the relative degrees of the uncertainties $\Delta\mathbf{f}$ and $\Delta\mathbf{g}$, respectively. Also, we assume the uncertainties satisfy the generalized matching condition, i.e., $w \geq \rho = \kappa$.

Design of a sliding mode controller. Based on the input-output linearization technique, there exists a local coordinate transformation as follows:

$$\begin{aligned} \left(\xi^T, \eta^T\right)^T &= \mathbf{T}(\mathbf{x}) \\ &= \left(h_s(\mathbf{x}), L_{\mathbf{f}} h_s(\mathbf{x}), \cdots, L_{\mathbf{f}}^{\rho-1} h_s(\mathbf{x}), \eta_1(\mathbf{x}), \cdots, \eta_{n-\rho}(\mathbf{x})\right)^T \end{aligned} \tag{13}$$

Then, we can transfer the nonlinear uncertain system to its normal form as

$$\dot{\xi}_i = \xi_{i+1}, \quad i = 1, 2, \cdots, \rho - 1 \tag{14a}$$

$$\dot{\xi}_\rho = [b(\xi,\eta) + \Delta b(\xi,\eta)] + [a(\xi,\eta) + \Delta a(\xi,\eta)] u(t-\theta) \tag{14b}$$

$$\dot{\eta} = \mathbf{q}(\xi,\eta) + \phi(\xi,\eta) \tag{14c}$$

$$y_s = \xi_1 \tag{14d}$$

where $a(\xi,\eta)$, $\Delta a(\xi,\eta)$, $b(\xi,\eta)$, $\Delta b(\xi,\eta)$, $\mathbf{q}(\xi,\eta)$ and $\phi_i(\xi,\eta)$ are given, respectively, by

$$a(\xi,\eta) = L_{\mathbf{g}} L_{\mathbf{f}}^{\rho-1} h_s \circ \mathbf{T}^{-1}(\xi,\eta) \tag{15}$$

$$\Delta a(\xi,\eta) = L_{\Delta\mathbf{g}} L_{\mathbf{f}}^{\rho-1} h_s \circ \mathbf{T}^{-1}(\xi,\eta) \tag{16}$$

$$b(\xi,\eta) = L_{\mathbf{f}}^{\rho} h_s \circ \mathbf{T}^{-1}(\xi,\eta) \tag{17}$$

$$\Delta b(\xi,\eta) = L_{\Delta\mathbf{f}} L_{\mathbf{f}}^{\rho-1} h_s \circ \mathbf{T}^{-1}(\xi,\eta) \tag{18}$$

$$q_i(\xi,\eta) = L_{\mathbf{f}} T_{\rho+i}(\mathbf{x}), \quad i = 1, 2, \cdots, n-\rho \tag{19}$$

$$\phi_i(\xi,\eta) = L_{\Delta\mathbf{f}} T_{\rho+i}(\mathbf{x}) + L_{\Delta\mathbf{g}} T_{\rho+i}(\mathbf{x})\, u(t-\theta), \; i = 1, 2, \ldots, n-\rho \tag{20}$$

and

$$\mathbf{x} = \mathbf{T}^{-1}(\xi,\eta) \tag{21}$$

Since the process is of internal stability, the following state feedback control law:

$$u(t-\theta) = (v(t) - b(\xi,\eta)) / a(\xi,\eta) \tag{22}$$

can be applied, where all the quantities in the right-hand side are at time t. To give the current control inputs, the control law of Eq. (22) is rewritten as

$$u(t) = \frac{v(t+\theta) - b(\xi(t+\theta), \eta(t+\theta))}{a(\xi(t+\theta), \eta(t+\theta))} \tag{23}$$

In this work, we modify the robust SMC approach of Chen and Dai (2001) to give $v(t+\theta)$ as

$$v(t+\theta) = -k\delta(t+\theta) - \mathrm{sat}(\delta(t+\theta)/\beta)[b_{\min}^{-1}(f_{\max} + |\delta(t+\theta)|)] \tag{24}$$

where the adaptive gain k is tuned by $\dot{k} = \tilde{\gamma}(\delta(t+\theta))^2$ ($\tilde{\gamma} > 0$); $f_{\max}$, $b_{\min}$, $\delta(t+\theta)$ and $\mathrm{sat}(\delta(t+\theta)/\beta)$ are given, respectively, by

$$f_{\max} = \sup_{(\xi,\eta)\in\mathbf{T}(U)} \left| \begin{aligned} &\Delta b(\xi(t+\theta), \eta(t+\theta)) \\ &- \Delta a(\xi(t+\theta), \eta(t+\theta)) \frac{b(\xi(t+\theta), \eta(t+\theta))}{a(\xi(t+\theta), \eta(t+\theta))} \end{aligned} \right| \tag{25}$$

$$b_{\min} = 1 - \sup_{(\xi,\eta)\in\mathbf{T}(U)} \left| \frac{\Delta a(\xi(t+\theta), \eta(t+\theta))}{a(\xi(t+\theta), \eta(t+\theta))} \right| \tag{26}$$

$$\delta(t+\theta) = \mathbf{c}^T \xi(t+\theta) = \sum_{i=1}^{\rho} c_i \xi_i(t+\theta), \; c_\rho = 1 \tag{27}$$

$$\mathrm{sat}(\delta(t+\theta)/\beta) = \begin{cases} \delta(t+\theta)/\beta, & \text{if } |\delta(t+\theta)/\beta| < 1 \\ \mathrm{sign}(\delta(t+\theta)/\beta), & \text{if } |\delta(t+\theta)/\beta| \geq 1 \end{cases} \tag{28}$$

In the control law, β is the user-specified boundary layer thickness used to eliminate the input chattering, and coefficients c_i in $\delta(t+\theta)$ are chosen such that the polynomial $\Gamma(\lambda) = \lambda^{\rho-1} + c_{\rho-1}\lambda^{\rho-2} + \cdots + c_2\lambda + c_1$ has all roots in the LHP. At this stage, the SMC control law (23) for input-delay processes has been constructed. However, this controller can not be directly implemented without having the predictive states. Therefore, in the next subsection we shall introduce a nonlinear predictor to obtain time-advanced predictive states.

A nonlinear predictor and the robustness of the predictor-based sliding mode control scheme. To compensate the time delay of the process and therefore estimate the process's time-advanced states, we suggest the use of the following nonlinear predictor:

$$\dot{\mathbf{x}}^*(t) = \mathbf{f}(\mathbf{x}^*(t)) + \mathbf{g}(\mathbf{x}^*(t)) u(t) \tag{29a}$$

$$\dot{\tilde{\mathbf{x}}}(t) = \mathbf{f}(\tilde{\mathbf{x}}(t)) + \mathbf{g}(\tilde{\mathbf{x}}(t)) u(t-\tilde{\theta}) \tag{29b}$$

$$\hat{\mathbf{x}}(t+\tilde{\theta} \mid t) = \mathbf{x}(t) + \mathbf{x}^*(t) - \tilde{\mathbf{x}}(t) \tag{29c}$$

where $\mathbf{x}^*(t)$, $\tilde{\mathbf{x}}(t)$ and $\mathbf{x}(t) \in R^n$ denote, respectively, the model state vector, the nominal state vector, and the actual plant's state vector; $\tilde{\theta} \geq 0$ is the estimated time delay in the manipulated input and $\hat{\mathbf{x}}(t+\tilde{\theta} \mid t) \in R^n$ represents the corrected time-advanced predictive state vector. By comparing Eqs. (29a) with (29b), it follows that $\mathbf{x}^*(t) = \tilde{\mathbf{x}}(t+\tilde{\theta})$ if the predictor is initialized as $\mathbf{x}^*(0) = \tilde{\mathbf{x}}(\tilde{\theta})$. This initialization can be achieved at steady state because in this case $\tilde{\mathbf{x}}(\tilde{\theta}) = \tilde{\mathbf{x}}(0)$. As a result, in the absence of plant/model mismatch the prediction model yields the plant state vector one time delay ahead, i.e. $\hat{\mathbf{x}}(t+\tilde{\theta} \mid t) = \mathbf{x}(t+\theta)$ if $\tilde{\theta} = \theta$. With the introduction of the nonlinear predictor, the transformed system (14) can be represented as

$$\dot{\hat{\xi}}_i = \hat{\xi}_{i+1}, \quad i = 1, 2, \cdots, \rho - 1 \tag{30a}$$

$$\dot{\hat{\xi}}_\rho = [\hat{b}(\hat{\xi},\hat{\eta}) + \Delta\hat{b}(\hat{\xi},\hat{\eta})] + [\hat{a}(\hat{\xi},\hat{\eta}) + \Delta\hat{a}(\hat{\xi},\hat{\eta})] u(t-\tilde{\theta}) \tag{30b}$$

$$\dot{\hat{\eta}} = \hat{\mathbf{q}}(\hat{\xi},\hat{\eta}) + \hat{\phi}(\hat{\xi},\hat{\eta}) \tag{30c}$$

$$\hat{y}_s = \hat{\xi}_1 \tag{30d}$$

where $\hat{\xi}$, $\hat{\eta}$, $\hat{a}$, $\Delta\hat{a}$, $\hat{b}$, $\Delta\hat{b}$, $\hat{\mathbf{q}}$ and $\hat{\phi}$ are defined similarly as those in ξ, η, a, Δa, b, Δb, $\mathbf{q}$ and

ϕ, respectively. Besides, the current input $u(t)$ is modified from Eq. (23) to be computed by

$$u(t)=\frac{\hat{v}(t+\tilde{\theta}\mid t)-\hat{b}(\hat{\xi}(t+\tilde{\theta}\mid t),\hat{\eta}(t+\tilde{\theta}\mid t))}{\hat{a}(\hat{\xi}(t+\tilde{\theta}\mid t),\hat{\eta}(t+\tilde{\theta}\mid t))} \tag{31}$$

of which the future input $\hat{v}(t+\tilde{\theta}\mid t)$ is calculated as

$$\begin{aligned}\hat{v}(t+\tilde{\theta}\mid t)&=-\hat{k}\hat{\delta}(t+\tilde{\theta}\mid t)\\ &-\mathrm{sat}(\hat{\delta}(t+\tilde{\theta}\mid t)/\beta)[\hat{b}_{\min}^{-1}(\hat{f}_{\max}+\left|\hat{\delta}(t+\tilde{\theta}\mid t)\right|)]\end{aligned} \tag{32}$$

where the adaptive gain $\hat{k}$ is tuned by $\dot{\hat{k}}=\tilde{\gamma}(\hat{\delta}(t+\tilde{\theta}\mid t))^2$ ($\tilde{\gamma}>0$); $\hat{\delta}(t+\tilde{\theta}\mid t)$, $\hat{f}_{\max}$, and $\hat{b}_{\min}$ are given, similarly as $\delta(t+\theta)$, $f_{\max}$ and $b_{\min}$. With the incorporation of the nonlinear predictor and the insertion of the control law (31), the resultant closed-loop system can be formulated as follows:

$$\dot{\hat{\xi}}=\mathbf{A}_c\hat{\xi}+\mathbf{B}_c[\hat{v}+\hat{b}_s^{-1}(\hat{f}_s+\mathbf{c}^T\hat{\xi})] \tag{33a}$$

$$\dot{\hat{\eta}}=\hat{\mathbf{q}}(\hat{\xi},\hat{\eta})+\hat{\phi}(\hat{\xi},\hat{\eta}) \tag{33b}$$

$$\dot{\mathbf{x}}^*=\mathbf{f}(\mathbf{x}^*)+\mathbf{g}(\mathbf{x}^*)u(t) \tag{33c}$$

$$\dot{\tilde{\mathbf{x}}}=\mathbf{f}(\tilde{\mathbf{x}})+\mathbf{g}(\tilde{\mathbf{x}})u(t-\tilde{\theta}) \tag{33d}$$

where

$$\mathbf{A}_c=\begin{bmatrix}0&1&0&0&\dots&0&0\\0&0&1&0&\dots&0&0\\\vdots&\vdots&\vdots&\vdots&\ddots&\vdots&\vdots\\0&0&0&0&\dots&1&0\\0&0&0&0&\dots&0&1\\-c_1&-c_2&-c_3&-c_4&\dots&-c_{\rho-1}&-1\end{bmatrix} \tag{34}$$

$$\mathbf{B}_c=\begin{bmatrix}0&\cdots&0&\hat{b}_s\end{bmatrix}^T \tag{35}$$

$$\hat{f}_s:=\hat{f}_s(\hat{\xi},\hat{\eta})=\Delta\hat{b}(\hat{\xi},\hat{\eta})-\Delta\hat{a}(\hat{\xi},\hat{\eta})\frac{\hat{b}(\hat{\xi},\hat{\eta})}{\hat{a}(\hat{\xi},\hat{\eta})} \tag{36}$$

and $\hat{b}_s:=\hat{b}_s(\hat{\xi},\hat{\eta})=1+\dfrac{\Delta\hat{a}(\hat{\xi},\hat{\eta})}{\hat{a}(\hat{\xi},\hat{\eta})}$ (37)

The robust stability and desired behaviour of the closed-loop system are described in the following theorem.

Theorem 1. Suppose that the system (11) is subject to the control law (31) and the stable nonlinear predictor (29). If $\hat{b}_{\min}>0$, then the closed-loop system (33) possesses the following properties.

(P1) Uniform stability: For each $\bar{d}\geq\underline{d}$, given any $\bar{\eta}(\cdot):[t_0,\infty)\rightarrow R^n$, and $\bar{\eta}(t_0)=\bar{\eta}_0$ of the closed-loop system (33), there exists a constant $\vartheta(\bar{d})>0$ such that $\|\bar{\eta}_0\|\leq\vartheta(\bar{d})$ implies that $\|\bar{\eta}(t)\|\leq\bar{d}$ for all $t\geq t_0$.

(P2) Uniform boundedness: Let $\bar{\eta}=[\hat{\xi}^T,\hat{\eta}^T]^T$ and $\bar{\eta}_0=[\hat{\xi}_0^T,\hat{\eta}_0^T]^T$. Given any $r>0$ and any $\bar{\eta}(\cdot)$: $[t_0,t_1)\rightarrow R^n$ and $\bar{\eta}(t_0)=\bar{\eta}_0$ of the closed-loop system (33) with $\|\bar{\eta}_0\|\leq r$, there exists a constant $d(r)>0$ such that $\|\bar{\eta}(t)\|\leq d(r)$ for all $t\in[t_0,t_1)$.

(P3) Uniform ultimate boundedness: For each $\bar{d}\geq\underline{d}$ and $r>0$, given any $\bar{\eta}(\cdot)$: $[t_0,\infty)\rightarrow R^n$ and $\bar{\eta}(t_0)=\bar{\eta}_0$ of the closed-loop system (33) with $\|\bar{\eta}_0\|\leq r$, there exists a finite time $\bar{t}(\bar{d},r)\geq 0$ such that $\|\bar{\eta}(t)\|\leq\bar{d}$ for all $t\geq t_0+\bar{t}(\bar{d},r)$.

Proof: Since the system (11) is of internal stability, the nominal system of system (11) possesses the property of hyperbolically minimum phase. Besides, for internal stability, the function $\hat{\mathbf{q}}(\hat{\xi},\hat{\eta})$ is assumed to be Lipschitz in $\hat{\xi}$ uniformly in $\hat{\eta}$, i.e.

$$\left\|\hat{\mathbf{q}}(\hat{\xi},\hat{\eta})-\hat{\mathbf{q}}(0,\hat{\eta})\right\|\leq L\left\|\hat{\xi}\right\| \tag{38}$$

where L is called a Lipschitz constant of $\hat{\mathbf{q}}(\hat{\xi},\hat{\eta})$. Under the condition of Eq. (38) and using a converse theorem of Lyapunov, there exists a Lyapunov function $V_0(\hat{\eta})$ which satisfies the following inequalities:

$$\sigma_1\|\hat{\eta}\|^2\leq V_0(\hat{\eta})\leq\sigma_2\|\hat{\eta}\|^2 \tag{39}$$

$$\frac{\partial V_0}{\partial\hat{\eta}^T}\hat{\mathbf{q}}(0,\hat{\eta})\leq-\lambda_1\|\hat{\eta}\|^2 \tag{40}$$

$$\left\|\partial V_0/\partial\hat{\eta}\right\|\leq\lambda_2\|\hat{\eta}\| \tag{41}$$

where σ_1, σ_2, λ_1 and λ_2 are positive constants. To include the effects of uncertainties on the dynamics of $\hat{\eta}$, we also make the supposition on $\hat{\phi}(\hat{\xi},\hat{\eta})$ as

$$\left\|\hat{\phi}(\hat{\xi},\hat{\eta})\right\|\leq l_1(\|\hat{\xi}\|+\|\hat{\eta}\|)+l_2 \tag{42}$$

for all $(\hat{\xi},\hat{\eta})\in\hat{\mathbf{T}}(U)$, where l_1 and l_2 are positive constants. Now, let's consider the Lyapunov candidate as

$$V(\hat{\xi},\hat{\eta})=\mu_1V_1(\hat{\xi})+\mu_0V_0(\hat{\eta}) \tag{43}$$

where $V_1(\hat{\xi})=1/2\,\hat{\delta}^2+\gamma/2\tilde{k}^2$ and $\tilde{k}\equiv\hat{k}-k^*$ with k^* being the desired steady state feedback gain; μ_1 and μ_0 are positive constants to be specified later. The time derivative of $\dot{V}$ is given by

$$\begin{aligned}&\dot{V}(\hat{\xi},\hat{\eta})=\mu_1\dot{V}_1(\hat{\xi})+\mu_0\dot{V}_0(\hat{\eta})\\
&=\mu_1(\hat{\xi}^T\mathbf{c}\mathbf{c}^T\mathbf{A}_c\hat{\xi}+\hat{b}_s\hat{\delta}\{\hat{b}_s^{-1}(\hat{f}_s+\mathbf{c}^T\hat{\xi})-\mathrm{sat}(\hat{\delta}/\beta)[\hat{b}_{\min}^{-1}(\hat{f}_{\max}+|\hat{\delta}|)]\}\\
&+\hat{b}_s\tilde{k}(-\hat{\delta}^2+\dot{\hat{k}}\hat{b}_s^{-1}\gamma)-\hat{b}_sk^*\hat{\delta}^2)+\mu_0\frac{\partial V_0}{\partial\hat{\eta}^T}(\hat{\mathbf{q}}(0,\hat{\eta})+\hat{\mathbf{q}}(\hat{\xi},\hat{\eta})\\
&-\hat{\mathbf{q}}(0,\hat{\eta})+\hat{\phi}(\hat{\xi},\hat{\eta}))\leq\mu_1(-\hat{\xi}^T\mathbf{Q}\hat{\xi}-\hat{b}_sk^*\hat{\delta}^2)+\mu_0(\frac{\partial V_0}{\partial\hat{\eta}^T}\hat{\mathbf{q}}(0,\hat{\eta})\\
&+\left\|\frac{\partial V_0}{\partial\hat{\eta}^T}\right\|\times\left\|\hat{\mathbf{q}}(\hat{\xi},\hat{\eta})-\hat{\mathbf{q}}(0,\hat{\eta})\right\|+\left\|\frac{\partial V_0}{\partial\hat{\eta}^T}\right\|\times\left\|\hat{\phi}(\hat{\xi},\hat{\eta})\right\|)\\
&\leq\mu_1(-\hat{\xi}^T\mathbf{Q}\hat{\xi}-\hat{b}_sk^*\hat{\delta}^2)-(\mu_0\lambda_1-\mu_0\lambda_2l_1)\times\|\hat{\eta}\|^2\\
&+(\mu_0\lambda_2L+\mu_0\lambda_2l_1)\|\hat{\eta}\|\|\hat{\xi}\|+\mu_0\lambda_2l_2\|\hat{\eta}\|\\
&\leq-[\mu_1\lambda_{\min}(\mathbf{Q})-\frac{1}{4}]\|\hat{\xi}\|^2-(e_1-e_2^2)\times\|\hat{\eta}\|^2+\mu_0\lambda_2l_2\|\hat{\eta}\|\end{aligned} \tag{44}$$

where $\dot{\hat{k}}=(\hat{b}_s/\gamma)\hat{\delta}^2=\tilde{\gamma}\hat{\delta}^2$ with $\tilde{\gamma}>0$ and a non-negative value of $\hat{k}(0)$ has been applied in the above derivation. In Eq. (44), $\mathbf{Q} \equiv -\mathbf{c}\mathbf{c}^T \mathbf{A}_c$ is a positive definite matrix, $\lambda_{\min}(\mathbf{Q})$ denotes the minimum eigenvalue of $\mathbf{Q}$, and $e_1 = \mu_0(\lambda_1 - \lambda_2 l_1)$ and $e_2 = \mu_0 \lambda_2 (L + l_1)$ are set with the inequality of $e_2 \|\hat{\eta}\| \|\hat{\xi}\| \le 0.25 \|\hat{\xi}\|^2 + e_2^2 \|\hat{\eta}\|^2$. Since $l_1 < \lambda_1 / \lambda_2$, we can appropriately chose μ_0 and μ_1 such that all the square terms in Eq. (44) are negative. Finally, by letting $\overline{\eta} = [\hat{\xi}^T, \hat{\eta}^T]^T$, one can rewrite the above inequality as

$$\dot{V}(\overline{\eta}) \le -\overline{c}_1 \|\overline{\eta}\|^2 + \overline{c}_2 \|\overline{\eta}\| \qquad (45)$$

where $\overline{c}_1 = \min\{\mu_1 \lambda_{\min}(\mathbf{Q}) - \frac{1}{4}, e_1 - e_2^2\}$ and $\overline{c}_2 = \mu_0 \lambda_2 l_2$. The time derivative $\dot{V}$ is strictly negative for a sufficient large $\|\overline{\eta}\|$. Noticing that from Eqs. (39)-(41), and (43) and following a similar procedure of Li et al. (1995), the closed-loop system properties (**P1**), (**P2**) and (**P3**) then follow upon using standard arguments in the literature (Corless and Leitmann, 1981; Li et al., 1995) by taking

$$\bar{t}(\overline{d}, r) = \begin{cases} 0, & \text{if } r \le (r_2/r_1)^{1/2} \overline{d} \\ \dfrac{r_2 r^2 - r_1^2 r_2^{-1} \overline{d}^2}{\overline{c}_1 r_1 r_2^{-1} \overline{d}_2^2 - \overline{c}_2 r_1^{1/2} r_2^{-1/2} \overline{d}}, & \text{if otherwise} \end{cases},$$

$$d(r) = \begin{cases} (r_2/r_1)^{1/2} R, & \text{if } r \le R \\ (r_2/r_1)^{1/2} r, & \text{if } r > R \end{cases}, \quad \underline{d} = (r_2/r_1)^{1/2} R \quad \text{and} \quad \vartheta(\overline{d}) = R$$

where $r_1 = \min\{\mu_1 \lambda_{\min}(\mathbf{Q}), \mu_0 \sigma_1\}$, $r_2 = \max\{\mu_1 \lambda_{\max}(\mathbf{Q}), \mu_0 \sigma_2\}$, $R = \overline{c}_2 / \overline{c}_1$ and $\lambda_{\max}(\mathbf{Q})$ denotes the maximum eigenvalue of $\mathbf{Q}$. This completes the proof.

3. REGULATION CONTROL OF A NON-MINIMUM PHASE NONLINEAR UNCERTAIN INPUT-DELAY CHEMICAL PROCESS

The mathematical model of a nonisothermal Van de Vusse reactor in its deviation form is given by

$$\dot{\mathbf{x}}(t) = (\mathbf{f}(\mathbf{x}(t)) + \Delta\mathbf{f}(\mathbf{x}(t))) + (\mathbf{g}(\mathbf{x}(t)) + \Delta\mathbf{g}(\mathbf{x}(t))) u(t - \theta) \qquad (46a)$$

$$y(t) = h(\mathbf{x}(t)) \qquad (46b)$$

where

$$\mathbf{f}(\mathbf{x}(t)) = \begin{bmatrix} -k_1(x_3(t)+x_{3d})\cdot(x_1(t)+x_{1d}) - k_3(x_3(t)+x_{3d})\cdot(x_1(t)+x_{1d})^2 \\ + k_1(x_{3d})\cdot x_{1d} + k_3(x_{3d})\cdot x_{1d}^2 - x_1(t)\cdot u_d \\ k_1(x_3(t)+x_{3d})\cdot(x_1(t)+x_{1d}) - k_2(x_3(t)+x_{3d})\cdot(x_2(t)+x_{2d}) \\ - k_1(x_{3d})\cdot x_{1d} + k_2(x_{3d})\cdot x_{2d} - x_2(t)\cdot u_d \\ \frac{1}{\rho_s C_P}[-\Delta H_1 k_1(x_3(t)+x_{3d})\cdot(x_1(t)+x_{1d}) - \Delta H_2 k_2(x_3(t)+x_{3d}) \\ \cdot(x_2(t)+x_{2d}) - \Delta H_3 k_3(x_3(t)+x_{3d})\cdot(x_1(t)+x_{1d})^2 + \Delta H_1 k_1(x_{3d}) \\ \cdot x_{1d} + \Delta H_2 k_2(x_{3d})\cdot x_{2d} + \Delta H_3 k_3(x_{3d})\cdot x_{1d}^2] - x_3(t)\cdot u_d \end{bmatrix} \qquad (47)$$

$$\mathbf{g}(\mathbf{x}(t)) = \begin{bmatrix} x_{10} - x_{1d} - x_1(t) \\ -x_{2d} - x_2(t) \\ x_{30} - x_{3d} - x_3(t) \end{bmatrix} \qquad (48)$$

$$\Delta\mathbf{f}(\mathbf{x}(t)) = \begin{bmatrix} -e_f x_1(t) \\ 0 \\ 0 \end{bmatrix}, \quad \Delta\mathbf{g}(\mathbf{x}(t)) = e_g \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad (49)$$

$$h(\mathbf{x}(t)) = x_2(t) \qquad (50)$$

of which $0.1 \le e_f \le 0.3$ and $0.2 \le e_g \le 0.4$. Besides, the rate coefficients $k_i(T)$ are given by $k_i(T) = k_{i0}\exp(-E_i/RT)$. The values for the model parameter constants and operation conditions are listed as follows:

$k_{10} = 1.287 \cdot 10^{12}\ \mathrm{h}^{-1}$	$\Delta H_2 = -11\ \mathrm{kJ \cdot mol^{-1}}$
$k_{20} = 1.287 \cdot 10^{12}\ \mathrm{h}^{-1}$	$\Delta H_3 = -41.85\ \mathrm{kJ \cdot mol^{-1}}$
$k_{30} = 9.043 \cdot 10^{9}\ \mathrm{L(mol \cdot h)^{-1}}$	$\rho_s = 0.9342\ \mathrm{kg \cdot L^{-1}}$
$E_1/R = 9758.3\ \mathrm{K}$	$C_P = 3.01\ \mathrm{kJ(kg \cdot K)^{-1}}$
$E_2/R = 9758.3\ \mathrm{K}$	$\theta = 0.01\ \mathrm{hr}$
$E_3/R = 8560\ \mathrm{K}$	$x_{10} = 5\ \mathrm{gmol \cdot L^{-1}}$
$\Delta H_1 = 4.2\ \mathrm{kJ \cdot mol^{-1}}$	$x_{30} = 403.15\ \mathrm{K}$

The control objective is to maintain the process output x_2 as close as possible to the set point (steady-state value) by adjusting the dilution rate, u . In this case, the given steady-state values are $x_{1d} = 1.25$, $x_{2d} = 0.90$, $x_{3d} = 407.15$ and $u_d = 19.5218$. By linearizing this process model around the reference steady state, it exhibits locally asymptotically stable and locally non-minimum phase owing to no RHP pole (-96.518 and $-33.141 \pm 9.8118\,i$) and the presence of a RHP zero (-11.1673 and +122.6824). To construct a SEOM for the proposed SMC scheme, an auxiliary output is synthesized as

$$y_e = h_e(\hat{\mathbf{x}}) = \hat{x}_2 + \varepsilon_1 \Psi_1(\hat{\mathbf{x}}) + \varepsilon_2 \Psi_2(\hat{\mathbf{x}}) \qquad (51)$$

Let $z^0 = [-11.1673 \quad -122.6824]$ and $\Delta z = [0.2 \quad 2]$, and following the proposed searching algorithm, we have $\varepsilon_1 = -6.2415 \cdot 10^{-4}$ and $\varepsilon_2 = -2.5991 \cdot 10^{-3}$. Based on the auxiliary output y_s , it is easily verified $\rho = \kappa = w = 1$, which satisfy the condition of $w \ge \rho = \kappa$. With the values of $e_f = e_f^0 \pm \tau$ and $e_g = e_g^0 \pm \tau$, where $\tau = 0.1$, $e_f^0 = 0.2$ and $e_g^0 = 0.3$, we use the estimated maximum bound values of $\hat{f}_{\max} = 7$ and $\hat{b}_{\min} = 0.3$ for the sliding mode controller. The other parameters are set as $\lambda_s = 0.3$, $c_1 = 1.0$, $\hat{k}(0) = 1.0$, $\tilde{\gamma} = 1.0$ and $\beta = 0.01$. In order to verify the regulation ability, we suppose that the system states are perturbed to move away from their steady states to be $\mathbf{x}(0) = [-0.7 \quad -0.2 \quad 1]$ initially.

3.1 The presence of extra disturbances

The extra disturbances introduce significantly additional modeling errors, which leads the uncertainty vector $\Delta\mathbf{f}$ to be $\Delta\mathbf{f}(\mathbf{x}) = [-e_f x_1 \quad d_1 \quad d_2]^T$ where $d_1 = 0.5$ and $d_2 = 2$. For SMC design, $\hat{f}_{\max}$ should be increased to 10 in order to accommodate this extra disturbances. The simulation result is depicted in Fig. 2.

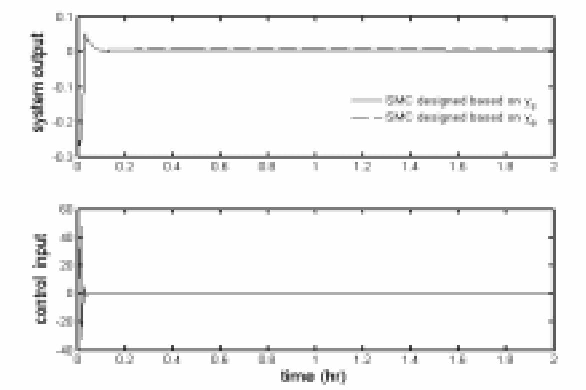

Fig. 2. Closed-loop system performance in face with the unmodeled side reaction, measuring error and extra disturbances.

From this figure, it is clear to observe that the SMC control system simply using y_e results in a small offset on the steady state because the design of y_e does not consider the influence of uncertainties. In contrast, the proposed SMC designed on the basis of y_s is capable of driving the process output gradually to achieve zero steady state offset performance.

3.2 Parameter uncertainties

In this case, we assume that the process's kinetic parameters k_{10}, k_{20} and k_{30} have +25% as well as ρ_s and C_P have -25% variation from their nominal values after time of 0.5 hr while these parameter values in the model remain unchanged. In designing SMC, the value of $\hat{f}_{\max}$ is set as 10. The closed-loop system performance is shown in Fig. 3, demonstrating that the proposed scheme is robust despite of the presence of the parameter variations.

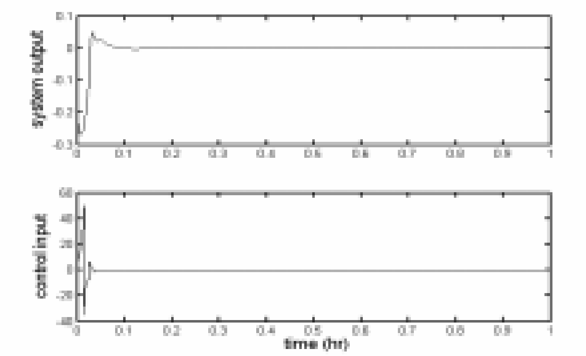

Fig. 3. Closed-loop system performance in face with parameter uncertainties.

4. CONCLUSIONS

This work has presented a systematic and robust SMC scheme for the regulation control of uncertain chemical processes in the presence of simultaneously the non-minimum phase behaviour and input-delay. A new algorithm has been proposed such that the designed auxiliary output is statically equivalent to the actual output and makes the resultant system minimum phase despite the influence of the process uncertainties. With the incorporation of the constructed SEOM as well as a time-advanced nonlinear predictor, a predictor-based SMC scheme can be easily established. The effectiveness of the proposed approach has been illustrated through the control of a Van de Vusse reactor.

ACKNOWLEDGEMENT

This work was supported by the National Science Council of Taiwan (ROC) under Grant NSC 93-2214-E-035-003.

REFERENCE

Camacho, O., R. Rojas and W. Garcia (1999). Variable structure control applied to chemical processes with inverse response. *ISA Transactions*, **38**, 55-72.

Chen, C.T. and C.S. Dai (2001). Robust controller design for a class of nonlinear uncertain chemical processes. *Journal of Process Control*, **11**, 469-482.

Corless, M.J. and G. Leitmann, (1981). Continuous state feedback guaranteeing uniform ultimate boundedness for uncertain dynamic systems. *IEEE Trans. Auto. Contr.*, **AC-26,** 1139-1144.

Herrmann, G., S.K. Spurgeon and C. Edwards (2003). A model-based sliding mode control methodology applied to the HDA-plant. *Journal of Process Control*, **13**, 129-138.

Hu, K.J., V.R. Basker and O.D. Crisalle (1998). Sliding mode control of uncertain input-delay systems. *Proceedings of the American Control Conference Philadelphia, Pennsylvania*, 564-568.

Kravaris, C., M. Niemiec, R. Berber and C.B. Brosilow (1998). Nonlinear model-based control of nonminimum-phase processes. In R. Berber & C. Kravaris (Eds.), *Nonlinear Model Based Process Control*. Dordrecht: Kluwer, 115-141.

Li, Z.H., T.Y. Chai and C. Wen (1995). Systematic design of robust controllers for nonlinear uncertain systems. *Int. J. Contr.*, **62,** 871-892.

Li, X. and S. Yurkovich (2001). Sliding mode control of delayed systems with application to engine idle speed control. *IEEE Transactions on Control Systems Technology*, **9**, 802-810.

Roh, Y.H. and J.H. Oh (2000). Sliding mode control with uncertainty adaptation for uncertain input-delay systems. *International Journal of Control*, **73**, 1255-1260.

Shyu, K.K. and J.J. Yan (1993). Robust stability of uncertain time-delay and its stabilizing by variable structure control. *Int. J. Control*, **57**, 237-246.

Spurgeon, S.K. and X.Y. Lu (1997). Output tracking using dynamic sliding mode techniques. *International Journal of Robust and Nonlinear Control*, **7**, 407-427.

Utkin, V.I. (1992). *Sliding modes in control and optimization*. Berlin: Springer.