ADAPTIVE TRACKING CONTROL VIA EXTREMUM-SEEKING METHOD

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Abstract: In this paper, an adaptive tracking controller is proposed using Extremum-seeking strategy. For unknown disturbances satisfying matching condition, the controller can adaptively seek for a proper magnitude level large enough for disturbance rejection without excessive control efforts. The proposed control strategy is robust to disturbances and uncertainties and is continuous in nature. As a result, there will be no chattering phenomenon and control energy will be largely saved comparing with a classic sliding mode output tracking controller. As an example, the proposed control strategy is applied to an adaptive vehicle following control to test it in a problem with high uncertainties.

Keywords: Extremum-seeking control, adaptive tracking control, vehicle following control

INTRODUCTION

This paper treats the problem of tracking an external reference signal with modeling uncertainties in system. Taking advantage of the Extremum-seeking methodology, the proposed method rejects the unknown disturbances adaptively with only necessary control efforts.

Extremum-seeking control (ESC) strategy is considered as an adaptive control methodology. In (S.K.Korovin and V.I.Utkin, 1972) and (Korovin and Utkin, 1974), a static optimization and nonlinear programming algorithm with sliding mode control was proposed by Korovin and Utkin. After that, pioneering work on Extremum-seeking control via sliding mode appeared in (Drakunov and Özgüner, 1992). Successful application of Extremum-seeking control dates back to Özgüner et al, (Drakunov *et al.*, 1995) where an Anti-lock Braking System was designed with ESC strategy via sliding mode. In (Haskara *et al.*, 2000), optimal set-point determination was analyzed in detail with a two time scale sliding mode optimization method. Extremum-seeking via sliding mode to system with time lags was studied in (Yu and Özgüner, 2002*a*) and (Yu and Özgüner, 2002*b*). In (Yu and Özgüner, 2003), Extremum-seeking strategy is developed with second order sliding mode control considering control smoothness.

Based on the Extremum-seeking strategy proposed in (Yu and Özgüner, 2002*b*), the main contribution of this paper is to extend the application of Extremum-seeking control to tracking problems. For accurate tracking, it is usually expected to have enough knowledge on the system model and reference signal as in adaptive control or to have enough control potential as in sliding mode control. Working in a highly uncertain environment, knowledge on system model may not always be sufficiently available and it is usually undesired to operating the control actuation system at its maximum level for long time. All those considerations make Extremum-seeking strategy a better substitution for classic sliding mode control in tracking problems. First, a tracking controller with Extremum-seeking strategy can adaptively seek for a magnitude level of control effort large enough to counteract all disturbances and uncertainties. Thus, it avoids using up all its control potential and saves control energy. Second, the proposed method is continuous in nature and avoids high feedback control gains such that there will be no chattering phenomenon accompanying in the control system designed.

The paper is organized as follows: In Section 1, the tracking control strategy is proposed for a nominal system and a lemma governing its stability is given. In Section 2, the proposed method is applied to an adaptive vehicle following control problem. Simulations are carried out in TruckSim. Robust and adaptive tracking performance has been obtained in this example. Conclusions are made in section 3.

1. A PROTOTYPE TRACKING PROBLEM

The typical structure of adaptive tracking control with Extremum-seeking strategy is shown in Figure 1. Due to the uncertainties in model, the gradient information of the user defined cost function $C(t,x,r) = \frac{e^2}{2}$ is unavailable directly in this problem. In order to extract the gradient information to minimize the cost function, the Extremum-seeking strategy exerts perturbation signal to obtain a projection of the gradient and and involve it in an online optimization problem. As a result $C(t,x,r) \to 0$ and $x \to r$ asymptotically.

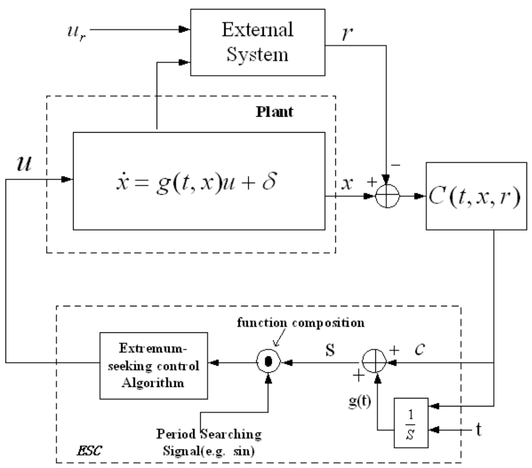

Fig. 1. Control Scheme of adaptive tracking control with ESC

Consider a nominal system:

$$\dot{x} = g(t,x)u + \delta \tag{1}$$

where $x \in \Re$ is the system's state and $u \in \Re$ is the control input. For general nonlinear SISO system with high order of relative degree, the nominal system could be obtained through feedback linearization and backstepping technology. The control gain $g(t,x)$ may be time varying uncertainly. In this paper, $|g(t,x)|$ is assumed to be bounded both from below and from above by positive constant γ and g_m, (i.e. $g_m \geq |g(t,x)| \geq \gamma,\ \forall t > 0,\ \forall x \in \Re$), and $g(t,x)$ is continuous almost everywhere except at finite time instants. δ accounts for all the disturbances and $|\delta|$ is bounded from above by a positive constant δ_m. The reference signal is given by:

$$r = f(t) \tag{2}$$

It is assumed that the absolute value of the first order derivative of r is bounded from above $|\dot{r}| \leq r_m$. The control objective is to design $u(t,x,r)$ such that the system state x tracks the reference signal $r(t)$ stably and asymptotically in the presence of uncertainty in $g(t,x)$ and unknown disturbance δ.

Let $e = x - r$. Define cost function:

$$C(t,x,r) = \frac{e^2}{2} = \frac{(x-r)^2}{2} \tag{3}$$

which is a measure of the difference between x and r. The control input u is designed as follows

$$s = C + h(t) \tag{4}$$

$$\dot{h}(t) = \begin{cases} \rho > 0 & if \quad C > \rho \\ C & if \quad C \leq \rho \end{cases}$$

$$u = -\kappa \sin\left(\frac{\pi}{\alpha}s\right) \tag{5}$$

where $h(t)$ is a monotonically increasing function of time and ρ, κ and α are positive constants. The control parameters at our disposal are κ,ρ and α and their values will be determined according to stability and performance conditions.

Lemma 1. For system (1) with control input (5), the state variable x converges to $r(t)$ asymptotically as $C(t,x,r)$ goes to zero in finite time if

$$\gamma\kappa > r_m + \delta_m + \sqrt{\frac{\rho}{2}} + \epsilon \tag{6}$$

where ϵ is a small value and accounts for moving rate of a manifold as shown later.

Proof: With Extremum-seeking control designed as (5), the following inequality:

$$|x - r| \geq \frac{\dot{h}}{|g(t,x)\kappa| - |\delta| - |\dot{r}|} \tag{7}$$

holds both in the region where $C(t, x, r) > \rho$ and in region where $C(t, x, r) \leq \rho$ if only the condition (6) is satisfied $\forall x \in \Re$ and $\forall t > 0$. Without loss of generality, assume initially that $x(0) - r(0) > 0$. The corresponding initial state of control variable $s(0) = \tilde{s}(0) + n\alpha$, for some value of integer n. By definition $n = 2j + (1 + \text{sgn}(x(0) - r(0)))/2$ and $j = \texttt{fix}(s(0)/2\alpha)$. The function $\texttt{fix}(z)$ rounds the element z to the nearest integer. n is even in the region where $x(t) - r(t) > 0$ and n is odd in region where $x(t) - r(t) < 0$. The value of n is determined according to the initial value of $x(0)$ and $s(0)$. Define manifold

$$
\begin{aligned}
M(t, x, r) &= \frac{\alpha}{\pi} \arcsin\left(\frac{\dot{h} + (x - r)(\dot{r} - \delta)}{(x - r)g\kappa}\right) \\
|\dot{M}(t, x, r)| &< \epsilon \\
-\alpha - M(t, x, r) \leq \tilde{s}(0) &\leq \alpha - M(t, x, r)
\end{aligned}
$$

Taking the first order derivative of s along the system trajectory:

$$
\begin{aligned}
\dot{s} &= \frac{\partial C}{\partial x}\dot{x} + \frac{\partial C}{\partial r}\dot{r} + \dot{h} \\
\dot{s} &= (x - r)[-g(t, x)\kappa \sin(\frac{\pi s}{\alpha}) + \delta - \dot{r}] + \dot{h}
\end{aligned}
$$

Let $s(t) = \tilde{s}(t) + n\alpha$, then

$$
\dot{\tilde{s}} = (x - r)[-g(t, x)\kappa \sin(\frac{\pi \tilde{s}}{\alpha}) + \delta - \dot{r}] + \dot{h}
$$

by design, κ is large such that the dynamics of s and that of x can be decomposed into two time scale. In the fast time scale, manifold $M(t, x, r)$ is substituted by its instant value and is viewed as constant in the boundary-layer model of s. Let $S = \tilde{s} - M(t, x, r)$. Select the Lyapunov candidate function $V = S^2/2$ and take the derivative of function V along the system trajectory:

$$
\dot{V} \approx (\tilde{s} - M(t, x, r))\,\dot{\tilde{s}}
$$

If $M(t, x, r) < \tilde{s} \leq \alpha - M(t, x, r)$,

$$
(\tilde{s} - M(t, x, r)) > 0, \dot{\tilde{s}} < 0,\ Thus,\ \dot{V} < 0
$$

If $0 < \tilde{s} < M(t, x, r)$,

$$
(\tilde{s} - M(t, x, r)) > 0, \dot{\tilde{s}} > 0,\ Thus,\ \dot{V} < 0
$$

If $-\alpha < \tilde{s} \leq 0$,

$$
(\tilde{s} - M(t, x, r)) < 0, \dot{\tilde{s}} > 0,\ Thus,\ \dot{V} < 0
$$

If $-\alpha - M(t, x, r) < \tilde{s} \leq -\alpha$,

$$
(\tilde{s} - M(t, x, r)) > 0, \dot{\tilde{s}} < 0,\ Thus,\ \dot{V} < 0
$$

It is clear that for positive definite Lyapunov function V, its derivative along the system trajectory is negative definite. Thus, $S \to 0$ asymptotically. As a result

$$
\tilde{s}(t) \to \frac{\alpha}{\pi} \arcsin\left(\frac{\dot{h} + (x - r)(\dot{r} - \delta)}{(x - r)g\kappa}\right)
$$

where $0 < \frac{\alpha}{\pi} \arcsin\left(\frac{\dot{h} + (x - r)(\dot{r} - \delta)}{(x - r)g\kappa}\right) < \alpha/2$. Meanwhile,

$$
s(t) \to n\alpha + \frac{\alpha}{\pi} \arcsin\left(\frac{\dot{h} + (x - r)(\dot{r} - \delta)}{(x - r)g\kappa}\right)
$$

That is $C(t, x, r) - h(t) \to (n\alpha + M(t, x, r))$. After $s(t, x, r)$ reaches the manifold $n\alpha + M(t, x, r)$

$$
C(t, x, r) - h(t) - (n\alpha + M(t, x, r)) = 0 \quad (8)
$$

Note that $h(t)$ is an monotonously increasing function of time if $e \neq 0$ and $M(t, x, r)$ is bounded as $(n - 1)\alpha \leq (n\alpha + M(t, x, r)) \leq (n + 1)\alpha$. Thus, $C(t, x, r)$ is forced to decrease simultaneously as $h(t)$ increases according to the constrain equation (8). As $C(t, x, r) \to 0$, $x \to r$ asymptotically. Adaptive tracking is achieved consequently. Along the $\tilde{s}$ plane, the stable region is an non-symmetry interval defined by $(-\alpha - M(t, x, r), \alpha - M(t, x, r))$ with the convergence manifold at $M(t, x, r)$. The width of the stable region is 2α and the stability radius is defined to be the width from the upper bound of the region to the convergence manifold, i.e. $\alpha - 2M(t, x, r)$, since this is the narrowest region to keep stability.

As x converges towards r, the convergence manifold $M(t, x, r)$ and the upper bound of the stable region of $\tilde{s}$ move slowly toward α and 0 respectively as

$$
\frac{\dot{h}}{(x - r)g\kappa} \to 0
$$

Thus, the control variable $s(t, x, r)$ converges to a bounded moving manifold defined by

$$
s(t, x, r) \to n\alpha + \frac{\alpha}{\pi} \arcsin\left(\frac{(\dot{r} - \delta)}{g\kappa}\right)
$$

■

Remark 2. Although it is assumed that $x(0) - r(0) > 0$ in this proof, the results obtained are true with initial condition $x(0) - r(0) < 0$. The only difference is that when $x(0) - r(0) < 0$, definition for $\tilde{s}$ changes to be $s(t) = \tilde{s}(t) + (2j - 1)\alpha$ for some integer j.

2. ADAPTIVE VEHICLE FOLLOWING CONTROL

In automated vehicle control systems, adaptive vehicle following control adjusts both speed and inter-vehicle distance with respect to a leading vehicle or a moving obstacle for safety spacing purpose. This is a general problem that can be applied to agents like robots, cars and trucks for autonomous operation. Uncertainties both from surrounding environment and the agents themselves make such a problem challenging. In recent years, linear vehicle following control methods, like PID and pole placement control, have been studied as in (Zhang and et al., January 1999). The parameters of these linear controllers must be scheduled according to different vehicle type and loads change, etc. in order to achieve good tracking performance with changing operation conditions. Sliding mode has also been applied to vehicle following control (Lu and Hedrick, 2002), especially in the presence of large system uncertainties. For precise tracking performance, feed-forward information on disturbances like road loads, air forces is necessary. Otherwise, sliding mode controller needs to take large control magnitude to reject all sources of unknown disturbances undiscriminatingly even though some of them may be very small. In vehicle control system, both the throttle control system and braking control system can not sustain such a large control magnitude for long time considering fuel efficiency. The time lags embedded in the throttle and braking system will degrade the performance of sliding mode control and chattering phenomenon is unavoidable. As seen in the nominal tracking control system, the adaptive search characteristic of ESC enables the vehicle following controller to adaptively seek for a magnitude level of throttle or braking efforts that are necessary to reject disturbance without wasting even though the variations of disturbances are unknown. Furthermore, the control input is continuous and thus it will not incur chattering in operation.

In this paper, the leading vehicle's longitudinal dynamic is described as:

$$\dot{X}_l = V_l, \quad \dot{V}_l = a_l \tag{9}$$

where the leading vehicle's acceleration a_l is unknown and is viewed as bounded disturbance. Constant time headway policy is employed with vehicle following control design. Vehicle following control with constant time headway uses only information derived from sensors, and it maintains spacing proportional to V_f. By definition, the safety distance is:

$$d_s = \tau V_f + d_0 \tag{10}$$

where d_0 is the minimum spacing to be kept in between.

The speed control loop of the host vehicle is considered as a two-input (throttle angle command and brake torque command) one-output (vehicle speed) system. However, the system is still a SISO system since the throttle and brake will never work simultaneously. Assuming that the input scale of the throttle and brake pedal are $(0 \sim 1)$ with 1 as the maximum effort. The control gain R_t and R_b from these inputs to vehicle speed dynamic are usually unknown and are time varying due to the complexity of mechanical properties of powertrain and braking systems, like transmission etc. The longitudinal dynamics of the host vehicle are:

$$\begin{aligned} \dot{X}_f &= V_f \\ \dot{V}_f &= a_f \\ &= \frac{1}{M}(R_t(t,V_f)T_d - R_b(t,V_f)T_b \\ &\quad -R_l(V_f) - Mg\sin(\phi) + \sigma(t)) \\ &= g(t,V_f)T + \delta \end{aligned} \tag{11}$$

where T_d is the total traction/drive torque and T_b is the total braking torque applied on wheels. Since the traction and braking torques are proportional to the corresponding throttle and brake pedal input, they are substituted by variable T denoting throttle or pedal input, $0 \le T \le 1$, and proportional gains are combined together by an unknown term $g(t, V_f)$. Road loads as $R_l(V_f)$,$Mg\sin(\phi)$ and disturbance forces $\sigma(t)$ are clustered together by a unknown but bounded disturbance δ. Now, the control objective is to generate proper traction and braking input T such that the safety distance d_s is kept between the leading vehicle and the host vehicle with measurements of the leading vehicle's speed and inter-vehicle spacing. T denotes traction input when it is positive and it denotes braking input when negative. Define the error variables as:

$$\begin{aligned} &X_e = X_l - X_f,\ X_f(0) = 0,\ X_l(0) = L_l > 0 \\ &V_e = V_l - V_f,\ V_f(0) = V_{f0},\ V_l(0) = V_{l0} \\ &a_e = a_l - a_f \end{aligned}$$

The dynamic model for two vehicle platoon is:

$$\begin{aligned} \dot{X}_e &= V_e \\ \dot{V}_e &= -g(t,V_f)T + \delta \\ X_e(0) &= L_l \\ V_e(0) &= V_{l0} - V_{f0} \end{aligned} \tag{12}$$

Assumption 1. $\forall t > 0$, $X_f(t) < X_l(t)$ with collision happens if $X_f(t) = X_l(t)$. The following vehicle has acceleration and deceleration capability no less than the leading vehicle. There is no limit on leading vehicle's speed in our study even though it does exist in real life. The following vehicle, i.e. the host vehicle, can not exceed its maximum speed setpoint V_{set}.

The reference speed command is generated by:

$$V_{ref} = k_1 \tilde{X}_e + k_2 V_e + V_l$$
$$V_{ref} = \min(V_{set}, V_{ref})$$

where k_1, k_2 are positive constant determined according to performance requirements. If V_{ref} is less then V_{set}, it is desired for V_f to track the reference speed V_{ref} asymptotically. Once $V_f = V_{ref}$, consequently

$$\dot{X}_e = V_e, \quad \dot{V}_e = -k_1 \tilde{X}_e - k_2 V_e$$

such that

$$X_e \to d_s, \quad V_e \to 0$$

asymptotically. A tracking controller needs be designed such that $V_f \to V_{ref}$ asymptotically in the presence of uncertainties from both host vehicle's dynamics and from environment. Design adaptive speed control law according to the adaptive tracking strategy proposed in Section 1:

$$\dot{V}_f = g(t, V_f)T + \delta$$
$$T = -\kappa \sin\left(\frac{\pi}{\alpha} s\right)$$
$$s = C(t, V_f, V_{ref}) - h(t)$$
$$C(t, V_f, V_{ref}) = \frac{(V_f - V_{ref})^2}{2}$$
$$\dot{h}(t) = \begin{cases} \rho > 0 \ if & C > \rho \\ C \quad\ \ if & C \leq \rho \end{cases}$$

Asymptotical tracking of V_f to V_{ref} is achieved. As a result, adaptive vehicle following control is realized in this two car platoon problem.

Simulations are carried out in TruckSim with the proposed adaptive control system designed. TruckSim is a comprehensive software package for vehicle simulation. In the first simulation, the leading vehicle is originally at 30 meters ahead and is running constantly at $10m/s$. Figures 2–4 show the simulation results. It is clear that V_f converges to V_l after 20 seconds and $Xe \to d_s$, $V_e \to 0$ asymptotically. In the second simulation, the leading vehicle has the same original states as in the first simulation. It begins slowing down with a constant deceleration $-1m/s^2$ at $t = 50s$ until fully stopped. Simulation results shown in Figures 5–8 further consolidate the successful application of the proposed adaptive tracking control methodology in adaptive vehicle following control problem. Throttle and braking control inputs are continuous and are small when the magnitude of combined disturbances is small.

3. CONCLUSION

In this paper, a new adaptive tracking control strategy is proposed using Extremum-seeking control. Working in a highly uncertain environment, the proposed tracking control method is robust to disturbances and uncertainties. Furthermore, its adaptive seeking characteristic makes it applicable for systems with continuous control inputs and control energy can be largely saved. The proposed controller is simple in structure and easy to design and tune. Its application to adaptive vehicle following control problem has demonstrates great tracking quality in the presence of uncertainties.

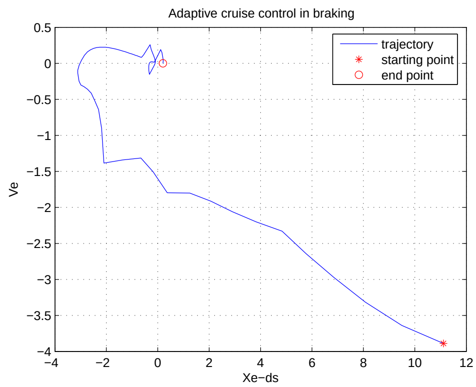

Fig. 2. Phase plane property of the adaptive vehicle following control process

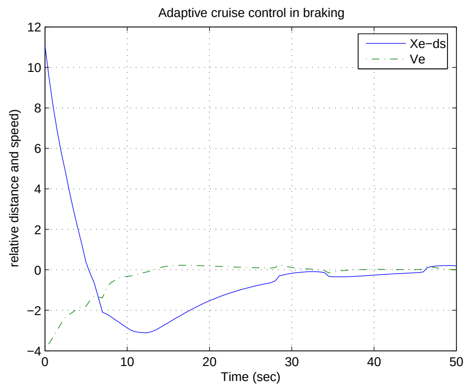

Fig. 3. Convergence process $Xe \to d_s$ and $Ve \to 0$

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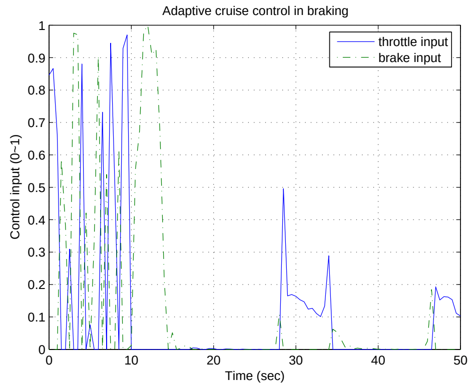

Fig. 4. Throttle and brake pedal inputs

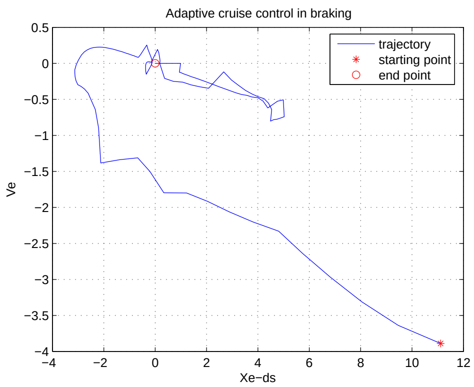

Fig. 5. Phase plane property of the adaptive vehicle following control process

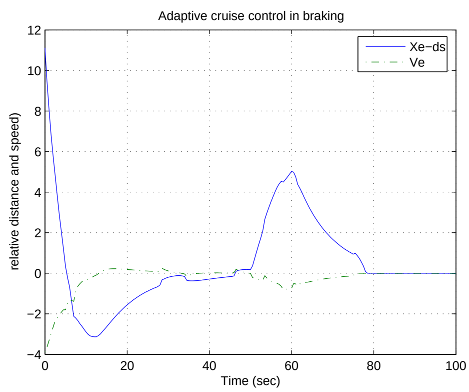

Fig. 6. Convergence process $Xe \to d_s$ and $Ve \to 0$

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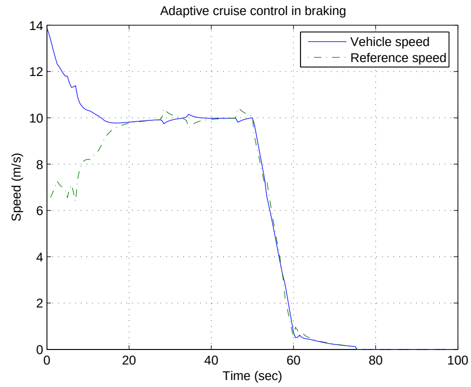

Fig. 7. Host vehicle speed vs. reference speed

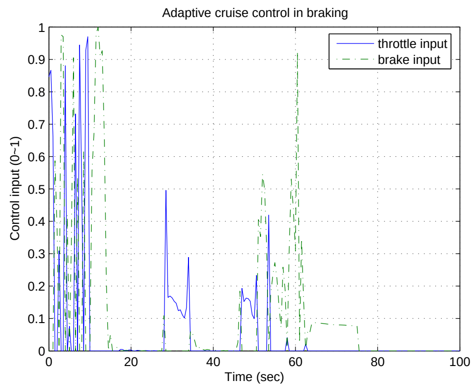

Fig. 8. Throttle and brake pedal inputs

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