PI AUTOTUNERS BASED ON BIASED RELAY IDENTIFICATION

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Abstract: The paper describes an automatic tuning procedure for a wide class of continuous-time systems without and with time delays. Every auto-tuning method has two basic steps. The first one is experimental and yields an estimated model. The second step consists in a design procedure of controllers. The developed auto-tuning procedure identifies a first order estimation model obtained by a biased relay feedback test. The consecutive design is based on a general solution of Diophantine equations and results in a PI-like controller. Moreover, the Diophantine equation approach gives a scalar tuning parameter which is adjusted according to the equalization of weighted moments.

Keywords: Autotuners, Feedback control, First-order systems, PI controllers, Polynomial methods, Relays.

1. INTRODUCTION

The conventional PID control has been predominant in most industrial applications for decades. The importance of the PID feedback is supported e.g. by IFAC Workshop PID'00 where many stimulating contributions were presented (Åström and Hägglund, 2000; Gorez and Klán, 2000; Ingimundarson and Hägglund, 2000). However, inadequate tuning of the controller parameters (Åström and Hägglund, 1995) may cause their poor performance. Hence, automatic tuning became a very desirable feature in industrial applications as well as in control producers. Nowadays, there are many different auto-tuning principles (Åström and Hägglund, 1984; Majhi and Atherton, 1998; Pecharromán and Pagola, 2000; de Arruda and Barros, 2001; Thyagarajan and Yu 2002).

An auto-tuning procedure consists of a process identification experiment plus a controller design method. The present day trend is a relay feedback test in two basic modifications. The traditional method was proposed by Åström and Hägglund (1984) is based on a symmetrical relay feedback test when a relay of magnitude "h" is inserted in the feedback loop. The period of the limit cycle is the ultimate period T_u and a limit cycle of amplitude "a" is generated by the process output. Then an approximate ultimate gain K_u can be calculated by:

$$K_u = \frac{4}{p} \cdot \frac{h}{a} \tag{1}$$

and consequently the well-known Ziegler-Nichols method can be used for the controller design.

Another asymmetrical limit cycle data test was proposed in Kaya and Atherton (2001) and it is also documented in Yu (1999).

The second step of auto-tuning principles generally includes a design method. The traditional approach utilizes the Ziegler-Nichols adjustment (Åström and Hägglund, 1984). However, these controller parameter settings suffer from some drawbacks, e.g. oscillatory transients or high overshoots in the controller output. Hence, a novel approach to PID controller tuning known as "equalization method" was proposed by Gorez and Klán (2000). Another modifications of the design settings can be found (Garcia and Castelo, 2000; Thyagarajan and Yu, 2002; Ingimundarson and Hägglund, 2000).

This contribution brings another auto-tuning method. The identification experiment is based on a biased relay test and the process is estimated by a first order (linear) model. Then a control design of a PI-like controller is performed by a solution of Diophantine equation in an appropriate ring. This approach enables to introduce a positive scalar parameter m>0 which is adjusted by the "equalization" principle.

The report is arranged in the following manner. In section 2 the identification method is introduced. The design principle is introduced in section 3. Section 4 summarizes the automatic tuning procedure and outlines the program implementation. In section 5 simulation examples are given and finally conclusions are drawn in section 6.

2. RELAY FEEDBACK IDENTIFICATION

A relay is usually used for on-off control of the process. Åström and Hägglund (1984) proposed a method for determining the ultimate frequency and gain called autotune variation. Fig.1 shows a process with a feedback with a symmetrical ideal relay which can be used for auto-tuning principles. The scheme in Fig.1 has two phases. During the phase 1, only the relay feedback is applied and the process is estimated through an oscillation test. A typical biased relay test is depicted in Fig.2. Then the estimation procedure is performed and PI (PID) controller parameters are designed. During the phase 2, the PI (PID) control loop is connected.

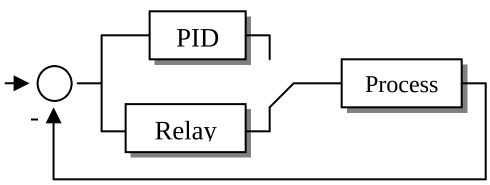

Fig. 1. Block diagram of a relay auto-tuner

For a large class of processes the relay gives an oscillation output the frequency of which is close to the ultimate frequency and the ultimate gain is approximated by relation (1). When a biased relay feedback is used the steady state gain of the process can be estimated (Ramirez, 1985) by:

$$K = \frac{\int_0^{iT_u} y(t)dt}{\int_0^{iT_u} u(t)dt}; \quad i = 1,2,3,... \qquad (2)$$

where T_u is a wave (ultimate) period shown in Fig.2. The goal of the identification for a PI or PID like controller design is to find a model in the form:

$$G(s) = \frac{K}{Ts+1} \cdot e^{-\Theta s} \qquad (3)$$

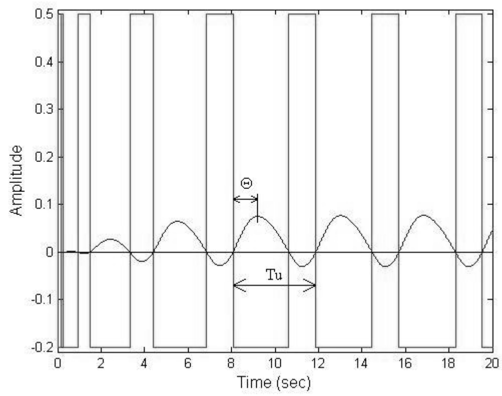

Fig. 2. Biased relay oscillation of stable processes

It is well known that many stable industrial processes can be adequately approximated by model (3). The gain K is given by relation (2) and time constant T is given according to Vyhlídal (2000):

$$T = \frac{1}{w_u} \cdot \tan(w_u \Theta) \qquad (4)$$

where Θ is the difference time between an extreme of y(t) to the preceding relay switch (see Fig.2) and ω_u is the ultimate frequency obtained through the ultimate period T_u from experimental data by:

$$w_u = \frac{2p}{T_u} \qquad (5)$$

For the PI controller design, only parameters K and T from (3) are used.

3. CONTROLLER DESIGN

3.1 Ziegler-Nichols setting

There are many methods for controller setting in the literature which can be found in standard autotuning principles (see e.g. Morilla, 2000; Åström and Hägglund, 1995; Yu, 1999; Kaya and Atherton, 2001). Among them, the Ziegler-Nichols method is probably utilized most frequently. Explicit relations were proposed in the form of a table where controller parameters are derived from the critical values K_u and T_u according to (1), (5).

Table 1 presents a revised modification (Åström and Hägglund, 1995). The control law is then supposed in the form of the ideal PID controller:

$$u(t) = K_P \cdot \left(e(t) + \frac{1}{T_I} \cdot \int e(t)dt + T_D \cdot \frac{de(t)}{dt} \right) \quad (6)$$

where K_P is the controller gain and T_I, T_D are the integral and derivative time, respectively.

However, this controller setting suffers from several drawbacks. Firstly, this method often results into oscillatory transients and into high overshoots. Secondly, the design approach is based on concepts as frequency response or dominant pole principles which are not familiarly known in practice. The advantage of Ziegler-Nichols consists in simplicity of the utilization. In the relay (often symmetric case) is the ultimate gain and frequency estimated and then Table 1 for PI or PID controller adjusting is used. Moreover, the structure of the controller can be easily modified by set point weighting (Åström et al., 1992). In the case of PI controller the modification takes the form:

$$u(t) = K_P \left[(bw - y) + \frac{1}{T_I} \int (w - y)(t)dt \right] \quad (7)$$

where w is the reference and $0<\beta\leq 1$ is the weighting factor.

Table 1 Ziegler-Nichols setting

	K_P	T_I	T_D
P	0.5 K_u	-	-
PI	0.45 K_u	0.85 T_u	-
PID	0.6 K_u	0.5 T_u	0.12 T_u

3.2 Equalization method

A novel approach to PID controller tuning for a wide class of processes was proposed in Gorez and Klán (2000). This approach is based on the equalization of the controlled output via constraints on weighted moments of their difference. The equalization method dramatically reduces output overshooting and improves whole control responses. The controller transfer function is according to (6) supposed in the form:

$$C(s) = K_P \frac{1 + T_I s + T_I T_D s^2}{T_I s} \quad (8)$$

The methodology then follows the idea that any change in the controller set point should be passed immediately with appropriate scaling to the controller output. This objective can be achieved if the process can be described by the transfer function

$$\tilde{G}(s) = K \frac{1 - T_o s}{1 + T_1 s + T_1 T_2 s^2} \quad (9)$$

where K is the static gain, T_1, T_2 are the time constants and T_0 is a time constant related to a nonminimum-phase behavior or a time-delay approximation. Then the controller parameters can be tuned through the choice $T_I = T_1$ and $T_D = T_2$ which yields the relation:

$$u = \frac{1}{K} \cdot \frac{1 + T_I s}{1 + \left(\frac{T_I}{K \cdot K_P} - T_0 \right) s} \quad (10)$$

where u, w are the Laplace transforms. Setting the controller gain to:

$$K_P = \frac{1}{K} \cdot \frac{T_I}{T_I + T_0} \quad (11)$$

gives $u(t) = \frac{1}{K} \cdot w(\infty)$. For processes with higher order dynamics this equality is not satisfied, however, the controller output can be tuned by "equalizing" of the steady-state value, for details see Gorez and Klán (2000). After some manipulations and derivations, the PI controller parameters can be tuned by simple relations:

$$K_P = \frac{1}{2K} \qquad T_I = 0.4 \cdot T_u \quad (12)$$

where T_u is the standard ultimate period.

3.3 Control design in R_{PS} ring

The growing role of algebraic methods is one of the features of modern control theory. Fields, ring and Diophantine equations became a powerful and effective tool especially for linear control design methods. Elegant control synthesis can be derived through the ring of proper and stable rational functions R_{PS} (see Prokop et al., 2002). The transfer function of linear (continuous-time) system is expressed as a ratio of two elements of R_{PS} which is given by:

$$G(s) = \frac{b(s)}{a(s)} = \frac{\frac{b(s)}{m(s)}}{\frac{a(s)}{m(s)}} = \frac{B(s)}{A(s)} \quad (13)$$

where a, b are polynomials (traditional in the Laplace representation) and m(s) is a stable polynomial with deg m = max {deg a, deg b}. A convenient form for the choice is:

$$m(s) = (s + m_0)^{\deg m} \qquad m_0 > 0 \quad (14)$$

The parameter $m_0 > 0$ ensures the stability of m(s) and it is a suitable “tuning knob” for the control behavior. As a consequence, fractions A, B in (13) belong to the ring R_{PS}.

A typical control problem can be formulated as follows: Find a controller such that the feedback control system is stable and some additional properties (reference tracking, disturbance rejection) are fulfilled.

A 1DOF control structure shown in Fig. 3 is described by the relations:

$$y=\frac{B}{A}(u+n)+v \qquad u=\frac{Q}{P}(w-y)$$
$$y=\frac{BQ}{AP+BQ}w+\frac{BP}{AP+BQ}n+\frac{AP}{AP+BQ}v \tag{15}$$

where e=w-y is the tracking error, w is the reference, n is a load disturbance and v is a output disturbance.

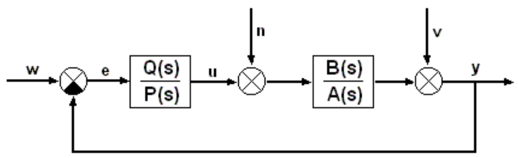

Fig. 3. One degree-of-freedom control structure

All stabilizing controllers are given as general solution of Diophantine equation:

$$AP+BQ=1 \tag{16}$$

which can be expressed by :

$$P=P_0+BZ \qquad Q=Q_0-AZ \tag{17}$$

where P_0 and Q_0 is any particular solution of (16) and Z is an arbitrary element of R_{PS}.

However, the final control aim is not restricted only to achieve stability but also asymptotic reference tracking, disturbance rejection and other specifications. The performance requirements are expressed through divisibility conditions in the appropriate ring. For asymptotic reference tracking, the denominator F_w of the reference must divide P in R_{PS}. This denominator for the step function is given by relation:

$$F_w=\frac{s}{s+m}; \quad m>0 \tag{18}$$

For the first order system (3), the control design is given as follows. The equation (16) takes the form:

$$(Ts+1)p_0+Kq_0=s+m \tag{19}$$

and the general solution of (17) is given by :

$$P=\frac{1}{T}+\frac{K}{s+m}\cdot Z$$
$$Q=\frac{Tm-1}{TK}-\frac{Ts+1}{s+m}\cdot Z \tag{20}$$

where Z is an arbitrary element of R_{PS}. The asymptotic tracking problem expressed by the divisibility of (18) is achieved by the choice $Z=-\frac{m}{TK}$. The final controller is then given in the PI form:

$$C(s)=\frac{Q}{P}=\frac{q_1 s+q_0}{s} \tag{21}$$

where m>0 is the tuning parameter, K and T are the parameters of system (3) and controller parameters q_0 and q_1 are given :

$$q_1=\frac{2Tm-1}{K} \qquad q_0=\frac{Tm^2}{K} \tag{22}$$

The parameter q_1 in (22) represents the controller gain and is tuned by the “equalization” principle. Comparing (12) and (22) gives the choice for the tuning parameter m > 0 by:

$$m=\frac{3}{4\cdot T} \tag{22}$$

The final PI parameters are then given by (22).

4. PROGRAM IMPLEMENTATION

A program system for design, tuning, simulation and comparison of introduced autotuning methods in Matlab-Simulink was developed. The Main menu of this program is in the Fig. 4.

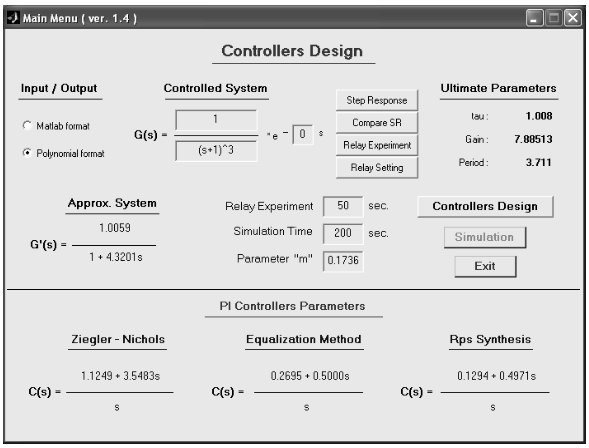

Fig. 4. Main menu of the program system

During the simulation routine a standard Simulink scheme is displayed (see Fig. 5) and the simulation of all three methods are performed. The simulation horizon can be prescribed in the middle of the main window and other simulation parameters can be specified in the Simulink environment. The end of the routine is chosen by pressing of “Exit” button.

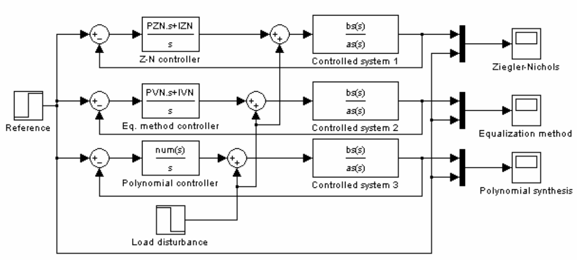

Fig. 5. Simulink scheme of control loop

In all simulation a change of the step reference is performed in the second third of the simulation horizon and a step change in the load is injected in the last third.

5. SIMULATION EXPERIMENTS

5.1 Example 1

The controlled system was a typical stable one (see Åström and Hägglund, 1995) with the transfer function:

$$G(s) = \frac{1}{(s+1)^3} \tag{24}$$

After the relay identification experiment (2), (4) the approximated transfer function was identified in the form:

$$\tilde{G}(s) = \frac{1}{2.9s+1} \tag{25}$$

Note, that according to model reduction method the approximated transfer function was:

$$\tilde{\tilde{G}}(s) = \frac{1}{3s+1} \tag{26}$$

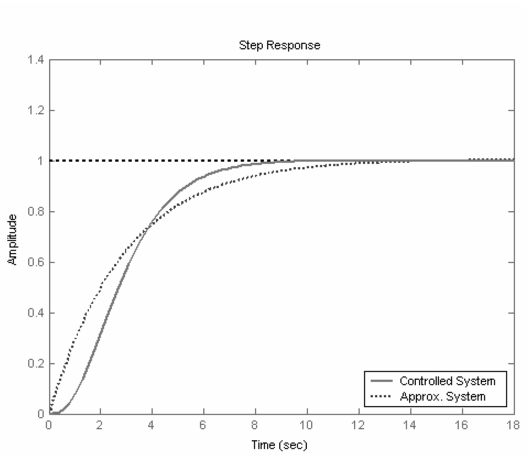

Fig. 6. Step responses of systems

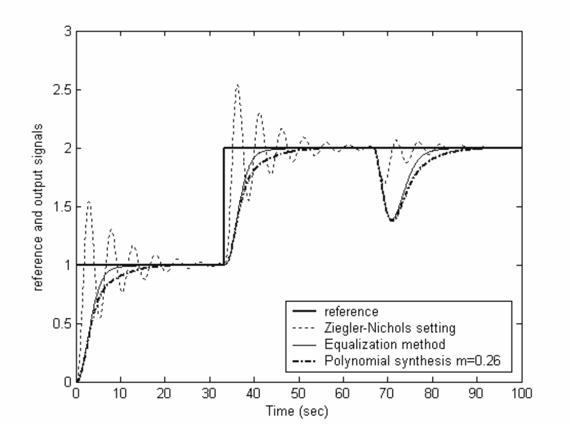

Fig. 7. Control responses of described controllers

The step responses of system (24) and approximated system (25) are shown in Fig. 6. Then three PI controllers were obtained by above mentioned methods. The control responses are pictured in Fig. 7.

5.1 Example 2

Consider a stable system with time delay governed by the transfer function:

$$G(s) = \frac{10}{50s+1} \cdot e^{-3s} \tag{27}$$

First order system in (27) represents a wide class of frequent industrial processes. The approximated system was identified in the form:

$$\tilde{G}(s) = \frac{10}{44s+1} \tag{28}$$

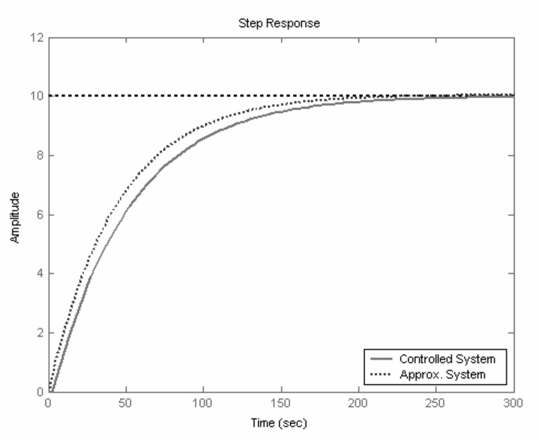

Fig. 8. Step responses of systems

The comparison of step responses (27) and (28) are shown in Fig. 8. Control responses of system (27) by all control principles are shown in Fig. 9. The influence of the control behavior by the tuning parameter m is pictured in Fig. 10. The tuning parameter influences reference tracking as well as load disturbance responses.

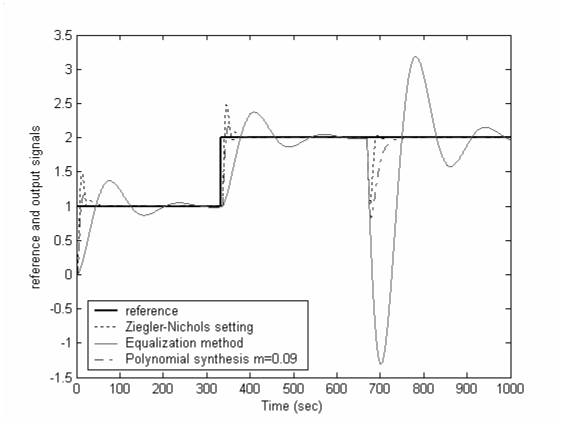

Fig. 9. Control responses of described controllers

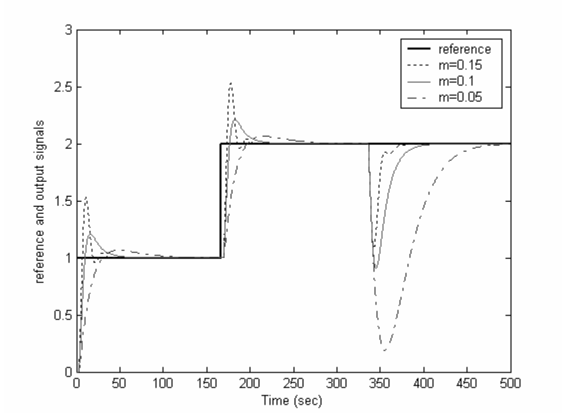

Fig. 10. Control responses of polynomial synthesis

6. CONCLUSION

The contribution is focused on autotuning methods using a relay identification experiments. A new method with a biased relay and polynomial control design is introduced. A first order system is identified and a PI like controller is generated through a simple Diophantine equation. The approach enables to introduce a single scalar parameter for further influencing of derived controllers. This parameter m>0 is tuned according to "equalization method" which results in the same controller gain (see Fig.9). For further purposes this parameter can be adjusted to fulfill additional requirements (see Fig.10). The developed method is compared with two classical principles known as Ziegler-Nichols (Åström and Hägglund, 1984) with symmetric relay and the Equalization method (Gorez and Klán, 2000). The program implementation of all approaches is performed in the Matlab and Simulink environment.

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