# DISTURBANCE ESTIMATION AND CANCELLATION FOR LINEAR UNCERTAIN SYSTEMS

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Abstract: This paper considers a class of linear uncertain systems in which the uncertainty is an additive perturbation of a known (nominal) linear model. It is supposed that the uncertainty and/or disturbance is known to be bounded, but its bound is *unkown*. A novel, easy to implement, adaptive feedback control law is designed to estimate the bounded disturbance on-line. This information is then used to cancel the effect of the disturbance in the system. The main advantage is that, if further design objectives are to be realized (for example, with respect to a tracking problem), the controls can be designed on the information from the nominal model only and not on the uncertain model. 

Keywords: Adaptive control; disturbance rejection; disturbance estimation; uncertain linear systems.

## 1. INTRODUCTION

Many robust control problems are studied using a deterministic approach for robust control, which often assumes knowledge of an uncertainty/disturbance bound or bounding function. In practice, such *a priori* knowledge of the uncertainty and/or disturbance may be difficult or almost impossible to obtain for the specific application.

Although there have been numerous studies in the area of robust control, there are very few studies that consider the problem of estimating and cancelling uncertainty and/or disturbance without *a priori* knowledge of uncertainty and/or disturbance. Those studies can be classified into three classes. One class consists of methods that use inverse dynamics of the nominal model to estimate the disturbance, see (Nakao *et al.*, 1987) for example. A second class of methods utilise obsevers with Lyapunov min-max type controllers, for example see (Chen and Su, 2002). The final class involves those methods that use high gain disturbance observers, as studied in (Yim and Singh, 2003). In the traditional disturbance observer (see (Nakao *et al.*, 1987; Yamada *et al.*, 1996) and (Komada *et al.*, 1991), for example), it is shown that the disturbance and uncertainty can be estimated using inverse dynamics of the nominal system (i.e. the known linear system). In the studies on disturbance observers with Lyapunov min-max type controllers (see, for example, (Chen *et al.*, 2000; Lu and Chen, 1995; Chen and Su, 2002)), it is shown that the uncertainty/disturbance can be estimated using an observer-like system with Lyapunov min-max type controllers and an adaptive law. In the studies (Chen *et al.*, 2000; Lu and Chen, 1995), it is shown that, with *a priori* knowledge of the bound on the uncertainty/disturbance, it is possible to estimate the uncertainty/disturbance using a non-adaptive control law. In (Chen and Su, 2002), using an adaptive control law, it is shown that the uncertainty/disturbance can be estimated without *a priori* knowledge of the uncertainty/disturbance. In the studies utilizing high-gain disturbance observers, as in (Yim and Singh, 2003), the disturbance is treated as one of the states of the observer, and it is shown that the disturbance can be estimated using a constant high-gain observer.

In the context of adaptive control, there are some studies which try to stabilize unknown systems using a control input which is produced by an observer-like system (see (Mårtensson, 1985; Miller and Davison, 1989) and (Fu and Barmish, 1986), for example); however, only parametric uncertainty is considered. Other studies do not require *a priori* knowledge of a disturbance. Often, in these studies, the control gain is determined from the dynamics of a differential equation, as in (Ilchmann and Ryan, 2004). However, none of these methods provide any estimates for the disturbances.

In this work, an on-line uncertainty/disturbance estimation and cancellation method is proposed. The approach is based on the inverse problem of tracking and it does not require any *a priori* knowledge on the bounds of any disturbances. The basic idea for estimating the disturbances is to track the state of the real system, to be controlled, by using the output of an observer-like system. The real system is assumed to be modelled as an additive unknown perturbation to a known linear system, known as the *nominal model*. The observer-like system is designed to be a known linear system with the same system matrices as the nominal model for the real system. Since both systems have the same system matrices, if it were possible to track the real system by the observer-like system, then the control input, for tracking, will produce almost exactly the same signal as the disturbance. In addition to this basic method, a feedforward filter is introduced in order to estimate and cancel out the effect of the disturbance in the closed-loop system. The resulting robust control scheme and the controller itself are very simple and, hence, it is easy to implement in practice.

## 2. PROBLEM STATEMENT AND PROPOSED METHOD OF SOLUTION

Consider the following system:

$$\dot{r}(t) = (A + \Delta A)r(t) + Bu(t) + w(t),\ \ t > t_0, \tag{1}$$

$$r(t_0) = r^0,\ \ t_0 \geq 0, \tag{2}$$

where $r(t) \in \mathbb{R}^n$ ($n \in \mathbb{N}$), the constant matrix $\Delta A$ and $w(t) \in \mathbb{R}^n$ represents parametric uncertainty and an external disturbance, respectively, $u(t) \in \mathbb{R}$ is the control input, and $A$ and $B$ are constant matrices of appropriate dimensions. When the parametric uncertainty $\Delta A$ and the disturbance $w(t)$ satisfy *matched conditions*, namely $\Delta A = BD$ and $w(t) = Bz(t)$, where $z(t)$ is now the external disturbance, (1) can be expressed in the form:

$$\dot{r}(t) = Ar(t) + B(u(t) + p(t)),$$

where $p(t) = Dr(t) + z(t)$, henceforth known as the *real system.* It is well known that, when $p(t)$ is bounded by some known function, the system can be controlled robustly. (see (Gutman, 1979; Leitmann, 1981; Corless and Leitmann, 1981; Gutman and Palmor, 1982) and the references therein). In practice, however, it may be difficult to obtain such a function for the specific class of systems. Thus, it is required to estimate the uncertainty/disturbance without *a priori* knowledge.

In this study, it is shown that such uncertainty/disturbance can be estimated by the inverse problem of tracking and the use of a feedforward filter, with an appropriately designed input to an 'observer-like' system. Consider the open-loop system:

$$\dot{r}(t) = Ar(t) + Bp(t), \tag{3}$$

where $p(t)$ represents the uncertainty/disturbance. Define an 'observer-like' system as follows:

$$\dot{x}(t) = Ax(t) + B\overline{u}(t), \tag{4}$$
$$x(t_0) = x^0,$$

where $x^0$ is specified. Suppose $\overline{u}(t)$ is an estimate of the *unknown* disturbance $p(t)$ such that tracking is almost perfectly achieved, i.e. $\overline{u}(t) \approx p(t)$ for $t$ sufficiently large. Then, it follows from (3) and (4) that $x(t) \approx r(t)$ for $t$ sufficiently large. In other words, if tracking $r(t)$ by $x(t)$ can be achieved, then $\overline{u}(t)$ is an estimate of the *unknown* disturbance $p(t)$.

Consider the closed-loop system described by:

$$\dot{r}(t) = Ar(t) + B(u(t) + p(t)), \tag{5}$$
$$\dot{x}(t) = Ax(t) + B\overline{u}(t), \tag{6}$$

Suppose it is desired to estimate the disturbance $p(t)$ without *a priori* knowledge of the input $u(t)$. If a tracking problem is considered, in which $x(t)$ follows $r(t)$ for system (5-6), then only $u(t) + p(t)$ can be estimated using $\overline{u}(t)$, not the disturbance $p(t)$. Thus, without prior knowledge of $u(t)$, it is impossible to estimate the disturbance by tracking. To overcome this problem, a feedforward filter and a modified reference signal are introduced. The feedforward filter is designed as follows:

$$\dot{x}_f(t) = Ax_f(t) + Bu(t), \tag{7}$$
$$x_f(t_0) = x_f^0,$$

where $x_f^0$ is specified. Define the modified reference signal as follows:

$$\overline{r}(t) := r(t) - x_f(t) \tag{8}$$

so that

$$\dot{\overline{r}}(t) = A\overline{r}(t) + Bp(t). \tag{9}$$

Note that (9) is independent of the control to the real system, namely $u(t)$. For estimation purposes, a tracking problem is considered for (9) together with the observer-like system (4). The structure of Equations (4) and (9) is exactly the same as that for the open-loop system (see (3-4)). Thus, tracking the state of the modified reference signal by an observer-like system enables one to estimate the uncertainty/disturbance for the closed-loop system, independent of the input $u(t)$.

## 3. DEFINITION OF AN ERROR SYSTEM

For analysis purposes, an error system is introduced and defined as follows:

$$e(t) := x(t) - \overline{r}(t)$$
$$\dot{e}(t) = Ae(t) + B(\overline{u}(t) - p(t)),$$

where $\overline{r}(t)$ is defined in (8). The input $\overline{u}(t)$, which is also the input to the observer-like system (4), is chosen to be a linear error feedback:

$$\overline{u}(t) = K(t)e(t), \tag{10}$$

where the feedback gain is *not* fixed, but **time-varying**. Thus,

$$\dot{e}(t) = (A + BK(t))e(t) - Bp(t) \tag{11}$$

with

$$e(t_0) = x^0 + x_f^0 - r^0.$$

## 4. ASSUMPTIONS

Some relevant assumptions are now introduced.

*Assumption 1.* $(A, B)$ is a controllable pair

*Remark 1.* In view of Assumption 1, it is assumed, without loss of generality, that $A$ and $B$ are in controllable canonical form:

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}.$$

*Assumption 2.* All states of the real system (1-2) are available for control purposes.

*Assumption 3.* The norm of the external disturbance/uncertainty $p(t)$ is bounded by some unknown constant, that is

$$\|p(t)\| \leq \alpha, \ \forall t,$$

where $\alpha$ is unknown.

It is supposed that the time-varying feedback gain $K(t)$ is designed to satisfy the following conditions.

*Assumption 4.*
(a) $K(t)$ is bounded.
(b) The time-varying eigenvalues of $A + BK(t)$ are real, distinct, negative, decreasing and sufficiently smooth, namely $C^1$.

## 5. DIAGONALISATION OF THE ERROR SYSTEM

In view of conditions (a) and (b) of Assumption 4, $A + BK(t)$ can be diagonalised by a linear, nonsingular coordinate transformation (see (Mirsky, 1955) for details). Consider the following linear, time-varying transformation:

$$e(t) = M(t)\overline{e}(t),$$

where the column vectors of $M(t)$ are the time-varying eigenvectors of $A + BK(t)$. Using this transformation, the error system (11) can be expressed as:

$$\dot{\overline{e}}(t) = \Lambda(t)\overline{e}(t) - M^{-1}(t)\dot{M}(t)\overline{e}(t) - M^{-1}(t)Bp(t), \tag{12}$$

where $\Lambda(t)$ is the diagonal matrix $M^{-1}(t)(A + BK(t))M(t)$, whose elements are the eigenvalues of $A + BK(t)$. Corresponding to this transformation, the observer-like system and modified reference signal are represented as follows:

$$\dot{\overline{x}}(t) = \left(M^{-1}(t)AM(t) - M^{-1}(t)\dot{M}(t)\right)\overline{x}(t) + M^{-1}(t)B\overline{u}(t) \tag{13}$$

$$\dot{\tilde{r}}(t) = \left(M^{-1}(t)AM(t) - M^{-1}(t)\dot{M}(t)\right)\tilde{r}(t) + M^{-1}(t)Bp(t) \tag{14}$$

where $x(t) = M(t)\overline{x}(t)$ and $\overline{r}(t) = M(t)\tilde{r}(t)$.

*Remark 2.* If pole assignment is to be considered, this can be incorporated in the design procedure. For example, if a constant matrix $F$ is designed, then the real system has the form:

$$\dot{r}(t) = \overline{A}r(t) + B(p(t) + u(t)),$$

where $\overline{A} = A + BF$. Then, defining an observer-like system and a feedforward filter with respect to $\overline{A}$, the same relation as above is obtained. Therefore, the problem of robust pole assignment can be treated using the same formulation.

## 6. DESIGN OF THE CONTROLLER AND ADAPTIVE ALGORITHM

The feedback gains for the input to the observer-like system (4) are determined as follows:

$$K(t) = [k_1(t) \ k_2(t) \ldots k_n(t)], \tag{15}$$

where

$$k_n(t) = -a_{nn} - \sum_{i=1}^{n}(-\lambda_i(t))$$

$$k_{n-1}(t) = -a_{nn-1} - \sum_{i=j=1, i\neq j=1}^{i=j=n} (-\lambda_i(t))(-\lambda_j(t))$$

$$k_{n-2}(t) = -a_{nn-2} - \sum_{i=j=k=1, i\neq j\neq k=1}^{i=j=k=n} (-\lambda_i(t))(-\lambda_j(t))(-\lambda_k(t))$$

$$\vdots \qquad \vdots$$

$$k_1(t) = -a_{n1} - \prod_{i=1}^{n}(-\lambda_i(t)),$$

where $\lambda_i(t)$ are the eigenvalues of $A + BK(t)$ and are determined using the following algorithm.

Let $v(t) := \|\bar{e}(t)\|^2$, $\delta$ and $\epsilon_e$ are specified constants, and, for $\tau \geq 0$, $f(\cdot, \tau)$, $g(\cdot, \tau)$ are continuous decreasing functions, defined on $[\tau, \infty)$, that satisfy $f(\tau, \tau) = -\delta = g(\tau + h, \tau)$ and $f(\tau + h, \tau) = 0 = g(\tau, \tau)$ with $h > 0$.

*Algorithm 1.* One of the eigenvalues, say $\lambda_1(t)$, is determined as follows. At $t = t_0$, $\lambda_1(t)$ is chosen to be $-\lambda_0$, where $\lambda_0 \in \mathbb{R}^+ := (0, \infty)$, and $\tau = t_0$. At $T > t$, $\dot{\lambda}_1(T)$ is determined as follows:

$$\dot{\lambda}_1(T) = \begin{cases} f(T, \tau), & \text{if } v(t) \leq \epsilon_e^2 \text{ and } \dot{\lambda}_1(t) = -\delta \\ g(T, \tau), & \text{if } v(t) > \epsilon_e^2 \text{ and if } \dot{\lambda}_1(t) = 0, \end{cases}$$

and $\tau = T$, if $v(t) \leq \epsilon_e^2$ and $\dot{\lambda}_1(t) = 0$, or if $v(t) > \epsilon_e^2$ and $\dot{\lambda}_1(t) = -\delta$.

The remaining eigenvalues, say $\lambda_2(t), \ldots, \lambda_n(t)$ are determined by

$$\lambda_i(t) = \kappa_i \lambda_1(t), \quad i = 2, \ldots, n,$$

with $\kappa_i > 0$ and $\kappa_i \neq 1$ for all $i$, and $\kappa_i \neq \kappa_j$ for $i \neq j$.

*Remark 3.* If, for $T > t$, the conditions of the algorithm do not hold, then the values of $\dot{\lambda}_1(t)$ and $\tau$ are not changed.

*Remark 4.* Examples of $f$ and $g$ satisfying the conditions of Algorithm 1 are:

$$f(t, \tau) = \begin{cases} -\frac{1}{2}\delta\left[1 + \cos\left(\pi(t - \tau)/h\right)\right], & t \in \mathcal{T}_1, \\ 0, & t \in \mathcal{T}_2, \end{cases}$$

$$g(t, \tau) = \begin{cases} -\frac{1}{2}\delta\left[1 - \cos\left(\pi(t - \tau)/h\right)\right], & t \in \mathcal{T}_1, \\ -\delta, & t \in \mathcal{T}_2, \end{cases}$$

where $\mathcal{T}_1 := [\tau, \tau + h]$, $\mathcal{T}_2 := (\tau + h, \infty)$ and $h > 0$ is a prescribed constant.

The basic idea of this adaptive algorithm is to decrease the eigenvalues of the error system (11) so that the norm of the transformed error state is sufficiently small. In this case the dynamics of the observer-like system and the real system will be almost the same for $t$ sufficiently large and, hence, the disturbance can be estimated using the control input to the observer-like system. To implement this idea, the following criteria are used:

(1) if the function $v$ has a value which is smaller than the prescribed constant $\epsilon_e$, then the eigenvalues are no longer decreased but are kept at some constant value, i.e $\dot{\lambda}(t) = 0$;
(2) if the value of the function $v$ is greater than $\epsilon_e$, then the eigenvalues are decreased, i.e $\dot{\lambda}(t) = -\delta$.

Note that the continuous functions $f$ and $g$ are introduced to ensure that $\dot{\lambda}$ is continuous.

## 7. PROPERTIES OF THE LINEAR COORDINATE TRANSFORMATION

The transformation $M(t)$, introduced in §5 can be chosen to be the Vandermonde (modal) matrix

$$M(t) = \begin{bmatrix} 1 & \cdots & 1 \\ \lambda_1(t) & \cdots & \lambda_n(t) \\ \vdots & \ddots & \vdots \\ \lambda_1^{n-1}(t) & \cdots & \lambda_n^{n-1}(t) \end{bmatrix}. \qquad (16)$$

For $M(t)$, defined in (16), the following properties hold.

*Lemma 5.* If $\lambda_i(\cdot)$ are determined by Algorithm 1, then $t \mapsto \left\|M^{-1}(t)B\right\|$ is a non-increasing function.

**Sketch proof:** Noting that $M(t)$ can be expressed in the form $\Gamma(t)\Phi$, where

$$\Gamma(t) = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \lambda_1(t) & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & \lambda_1^{n-1}(t) \end{bmatrix}$$

and

$$\Phi = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & \kappa_2 & \cdots & \kappa_n \\ \vdots & \vdots & \cdots & \vdots \\ 1 & \kappa_2^{n-1} & \cdots & \kappa_n^{n-1} \end{bmatrix},$$

it can be shown that $t \mapsto \left\|M^{-1}(t)B\right\| = k_1|\lambda_1(t)|^{-(n-1)}$, where $k_1$ is a positive constant, which is non-increasing.

□

Let $\|\cdot\|_1$ denote the 1-norm of a matrix, defined by $\|A\|_1 := \sum_{i,j=1}|a_{ij}|$, where $A = [a_{ij}]$.

*Lemma 6.* If $\lambda_i(\cdot)$ are determined by Algorithm 1, then $t \mapsto \left\|M^{-1}(t)\dot{M}(t)\right\|_1$ is a non-increasing function.

**Sketch proof:** With $\Phi$ and $\Gamma$ defined in the proof of Lemma 5, $M^{-1}(t)\dot{M}(t)$ can be expressed in the form:

$$M^{-1}(t)\dot{M}(t) = \Phi^{-1}\Gamma^{-1}(t)\dot{\Gamma}(t)\Psi,$$

with

$$\Psi = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 1 & \kappa_2 & \cdots & \kappa_n \\ \vdots & \vdots & \cdots & \vdots \\ 1 & \kappa_2^{n-1} & \cdots & \kappa_n^{n-1} \end{bmatrix}.$$

Hence, there is a positive constant $k_2$ such that $\left\| M^{-1}(t)\dot{M}(t) \right\|_1 = k_2|\lambda_1(t)|^{-1}|\dot{\lambda}_1(t)|$. Since $\dot{\lambda}_1(\cdot)$ is bounded, the result follows.

□

## 8. MAIN RESULTS

Let $\mathbb{B}_n(r)$ denote an open ball in $\mathbb{R}^n$, centered on the origin with radius $r > 0$, and let $\overline{\mathbb{B}}_n(r)$ denote its closure. Using the results of Lemmas 5 and 6 the following lemmas are obtained (unfortunately, due to restrictions on space, sketch proofs are not provided).

*Lemma 7.* Suppose Assumptions 1-3 hold, $\epsilon_e > 0$ is given and $\lambda_i(\cdot)$ satisfy the conditions given in Algorithm 1, then, under the dynamics of (11), there exists $t^* > t_0$ such that $\overline{e}(t) \in \overline{\mathbb{B}}_n(\epsilon_e)$ and $\lambda_i(t)$ are finite for all $t \geq t^*$.

*Lemma 8.* Suppose Assumptions 1-3 hold and $\lambda_i(\cdot)$ satisfy the conditions given in Algorithm 1, then, under the dynamics of (11), $\overline{e}(\cdot)$ is uniformly bounded.

It follows from Lemma 7 that, using Algorithm 1, all trajectories of (12) reach $\overline{\mathbb{B}}_n(\epsilon_e)$. Also, by Lemmas 7 and 8, all internal signals, consisting of the transformed error state, the eigenvalues of the error system, and the feedback gain for the input to the observer-like system, are uniformly bounded. Therefore, $\|\overline{e}(t)\| \leq \epsilon_e$, for $t$ sufficiently large, implies $\overline{x}(t) \approx \tilde{r}(t)$ for $t$ sufficiently large. In view of (13) and (14), $\overline{x}(t) \approx \tilde{r}(t)$ for $t$ sufficiently large implies $\overline{u}(t) \approx p(t)$ for $t$ sufficiently large. Thus, the unknown disturbance is estimated by $\overline{u}(t)$.

*Theorem 9.* Consider the single-input system (5), with unknown uncertainty/disturbance. If Assumptions 1-3 hold, then, utilizing the observer-like system (6), the feedforward filter (7), the modified reference signal (9), and the feedback control defined by (10), (15) and Algorithm 1, $\overline{u}(t) \approx p(t)$ for $t$ sufficiently large, where $p(t)$ is the matched disturbance/uncertainty in system (5) and $\overline{u}(t)$ is the control input to the observer-like system (6).

*Remark 10.* If the 'estimated' disturbance signal with opposite sign, namely $-\overline{u}(t) \approx -p(t)$, is fed back to system (5), then the effect of the matched disturbance/uncertainty in system (5) will be cancelled out.

The method, outlined in this paper, can be extended, with suitable modification, to multi-input/multi-output linear systems. Also, some applications, based on this method, are presented in (Kim, 2004).

## 9. ILLUSTRATIVE EXAMPLE

Here, the system to be examined is a third order single-input system given by

$$\dot{r}(t) = Ar(t) + B(u(t) + d(t)),$$

where $r(t) = [r_1(t)\ r_2(t)\ r_3(t)]^{\mathrm{T}}$, $u(t)$ is the control input to the system and $d(t)$ denotes matched uncertainty/disturbance. The system matrix and input matrix are given by

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -15 & -10 & -5 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$

and the dynamics are governed by the initial condition: $r(0) = \begin{bmatrix} 3 & -2 & 0 \end{bmatrix}^{\mathrm{T}}$. For the adaptive algorithm, the following values are chosen: $\lambda_1(0) = -1.0$, $\dot{\lambda}_1(0) = 0$, $\epsilon_e = 1.0 \times 10^{-3}$, $\delta = 2$, $\kappa_2 = 0.5$, $\kappa_3 = 1.5$ and $h = 0.01$. For simulation purposes, the 'unknown' disturbance is chosen to be

$$d(t) = (2 + \sin(t))s(t) + 10r_1(t) + 5r_2(t) + r_3(t),$$

where

$$s(t) = \begin{cases} 0, & 0 \leq t < 5, \\ \frac{1}{2}(t-5), & 5 \leq t < 10, \\ 2.5, & 10 \leq t < 20, \\ \frac{1}{2}(t-15), & 20 \leq t < 25, \\ 5, & t \geq 25. \end{cases}$$

The simulations are performed using MATLAB®. History of the disturbance in the closed-loop system (10) is shown in Figure (1), whilst the closed-loop responses for system (5) with $u(t) = -\overline{u}(t)$ are shown in Figure 2. The difference between the true and estimated disturbance is shown in Figure 3, and the feedback gains of the observer-like system (4), which are bounded, are illustrated in Figure 4. Finally, the estimated disturbance is shown in Figure 5.

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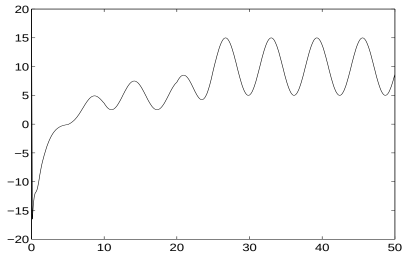


Fig. 1. History of the disturbance $p(t)$.

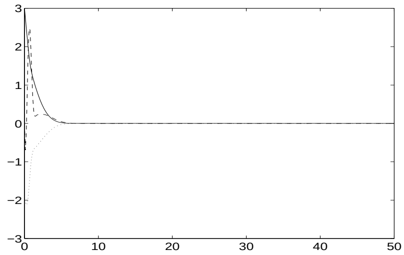


Fig. 2. Closed-loop responses: $r_1(t)$ $(-)$, $r_2(t)$ $(\cdots)$, $r_3(t)$ (- -).

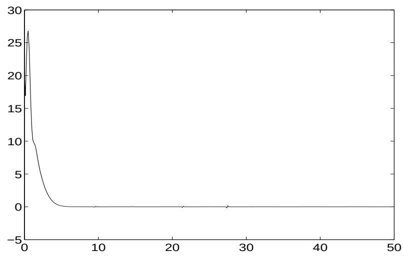


Fig. 3. Estimation error in the disturbance: $d(t) - \overline{u}(t)$.

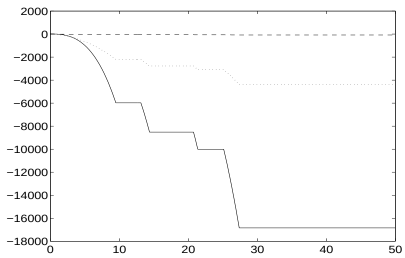


Fig. 4. Feedback gains of the observer-like system: $k_1(t)$ $(-)$, $k_2(t)$ $(\cdots)$, $k_3(t)$ (- -).

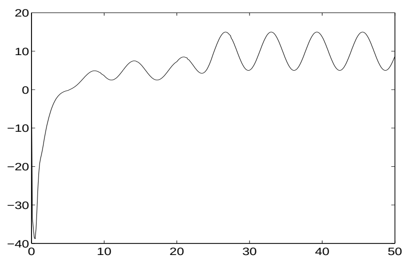


Fig. 5. Estimated disturbance $\overline{u}(t)$.


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