REPETITIVE-CONTROL SYSTEM FOR HARMONIC ELIMINATION IN THREE-PHASE VOLTAGE-SOURCE INVERTERS

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Abstract: This paper presents an algorithm based on a repetitive-control scheme to eliminate the low-frequency-harmonic contents of the output voltage in voltage-source inverters. The proposed controller can be used to minimise unexpected output-voltage harmonics due to practical implementation aspects of the pulse-width-modulation algorithm (switch dead time, for example). The controller can be implemented using the Park's transformation with a rotating reference frame for balanced three-phase voltage-source inverters. Simulation results are presented to illustrate the main contribution of this work.

Keywords: Active Filters, Harmonic Control, Voltage Inverter Switches

1. INTRODUCTION

Voltage-source inverters (VSIs) with pulse-width modulation (PWM) are frequently used in industrial applications. Traditional PWM schemes are tailored to control the fundamental frequency of the output voltage (Mohan *et al.* 1995). But practical limitations in VSI implementation (such as switch dead times) produce unexpected output-voltage distortion. More recently, the application of VSI in active-power filters requires accurate control of low-frequency output harmonics.

Open-loop schemes to control (or eliminate) low-frequency harmonics from the VSI output voltage have been proposed in the past (Patel and Hoft 1973). Closed-loop schemes have been proposed for active-power filters using an independent controller for each voltage harmonic to be controlled (see (Yuan *et al.* 2002)). This paper shows a controller based on a repetitive control scheme which is able to deal with several voltage harmonics simultaneously in three-phase VSIs. The control scheme is applicable under balanced and unbalanced conditions.

A simple model for voltage control in a single-phase VSI is presented in Section 2 together with the ideal transfer function of the proposed controller. The stability and robustness of the resulting closed-loop system is studied in Section 3. An alternative controller is proposed to ensure stability when differences exist between the VSI model and the actual plant. Section 4 explains the performance of the proposed controller if a balanced three-phase VSI can be considered, while Section 5 details the design procedure of the proposed controller. Finally, the main contributions are illustrated by simulation in Section 6.

2. VSI MODEL FOR OUTPUT-VOLTAGE CONTROL AND CONTROLLER PROPOSAL

A continuous-time model for a closed-loop-controlled single-phase VSI can be depicted as in Figure 1, where $C(s)$ is the controller and the ideal VSI is modelled by a pure delay $P_1(s) = e^{-t_0 s}$ (thus taking into account the PWM implementation in a microprocessor) and $P_2(s) = 1$. $V(s)$ is the reference voltage for the inverter, $U(s)$ is the control output and $Y(s)$ is the actual inverter output voltage. The disturbance $D(s)$ models unexpected output-voltage distortion due to, for example, switches dead time which often appears as low frequency unwanted output-voltage harmonics. The time delay t_0 is the sum of one-sample-period delay to calculate the control signals (see (Bellini *et al.* 1983) and (Séguier and Labrique 1993)) and half of the sampling period to model the discrete-time implementation ((Åström and Wittenmark 1997)). This model can easily be extended for three-phase applications.

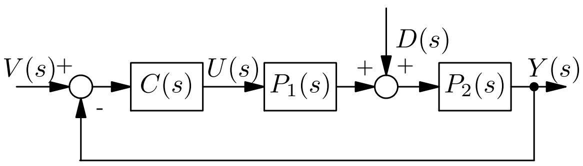

Fig. 1. Closed-loop control for a single-phase model of a voltage-source inverter

Recent publications propose controllers based on transfer functions such as:

$$C(s) = \frac{N(s)}{s^2 + \omega_h^2} \tag{1}$$

where ω_h is the frequency of the harmonic to be controlled in rad/s. This effectively introduces two open-loop poles at $s = \pm j\omega_h$ which guarantee elimination of any error signal $(V(s) - Y(s))$ of frequency ω_h in steady state, if the system is closed-loop stable. A controller of the form in (1) has to be implemented for each output-voltage harmonic to be controlled.

Let us consider an alternative controller based on the so called repetitive control, (Weiss and Häfele 1999), written as:

$$C(s) = \frac{M(s)}{1 - e^{-\frac{2\pi}{\omega_1}s}} \tag{2}$$

where $M(s)$ is a transfer function to be considered later in the paper and ω_1 is the fundamental frequency of the inverter output voltage.

The system output $Y(s)$ can be written as:

$$Y(s) = F(s)V(s) + F_D(s)D(s) \tag{3}$$

with,

$$F(s) = \frac{M(s)e^{-t_0 s}}{1 - e^{-\frac{2\pi}{\omega_1}s} + M(s)e^{-t_0 s}} \tag{4}$$

$$F_D(s) = \frac{1 - e^{-\frac{2\pi}{\omega_1}s}}{1 - e^{-\frac{2\pi}{\omega_1}s} + M(s)e^{-t_0 s}} \tag{5}$$

Clearly, $F(j\omega_h) = 1$ and $F_D(j\omega_h) = 0$ for frequencies $\omega_h = h\omega_1$ with $h = 0, 1, 2 \ldots \infty$. Therefore, if the closed-loop system is stable, the error in steady state for a sinusoidal reference $v(t) = A\sin(\omega_h t)$ or a sinusoidal disturbance input $d(t) = B\sin(\omega_h t)$ is zero. This controller, effectively, places open-loop poles at $\pm jh\omega_1$.

3. STABILITY OF THE CLOSED-LOOP SYSTEM

Since t_0 is small compared to the inverter output voltage period ($\frac{2\pi}{\omega_1} > t_0$), $M(s)$ could be chosen as:

$$M(s) = e^{-\left(\frac{2\pi}{\omega_1} - t_0\right)s} \tag{6}$$

Substituting (6) in (4) and in (5), the system output yields:

$$Y(s) = e^{-\frac{2\pi}{\omega_1}s}V(s) + \left(1 - e^{-\frac{2\pi}{\omega_1}s}\right)D(s) \tag{7}$$

and, obviously, the closed-loop system is always stable.

Unfortunately, the inverter delay t_0 is not exactly known and the closed-loop system in Figure 1 will not be stable if a controller is used with (2) and (6) designed for an estimated $\hat{t}_0 \neq t_0$.

To tackle this problem, the controller $C(s)$ is proposed as:

$$C(s) = \frac{Q(s)e^{-\left(L - \hat{t}_0\right)s}}{1 - Q(s)e^{-Ls}} \tag{8}$$

where $Q(s)$ is the transfer function of a low-pass filter (Hara *et al.* 1988) and $\hat{t}_0$ is the estimated value for the inverter delay. The delay L is $L = \frac{2\pi}{\omega_1} - \beta$ and β is a design parameter smaller than the fundamental inverter output voltage period ($\frac{2\pi}{\omega_1} > \beta$).

The output system is:

$$\begin{aligned} Y(s) =& \frac{Q(s)e^{-Ls}e^{-\delta s}}{1 + Q(s)e^{-Ls}\left[e^{-\delta s} - 1\right]}V(s) + \\ &+ \frac{1 - Q(s)e^{-Ls}}{1 + Q(s)e^{-Ls}\left[e^{-\delta s} - 1\right]}D(s) \end{aligned} \tag{9}$$

with $\delta = t_0 - \hat{t}_0$.

The characteristic equation of the resulting closed-loop system is:

$$1 + \overbrace{Q(s)e^{-Ls}\left[e^{-\delta s} - 1\right]}^{G'(s)} = 0 \qquad (10)$$

In order to guarantee stability, the term $G'(s)$ in (10) must comply with the Nyquist criterion: if the number of unstable poles of the open-loop system $G'(s)$ is equal to zero ($P = 0$), then the number of counterclockwise encirclements of the point (-1,0) of the term $G'(j\omega)$ with $-\infty < \omega < \infty$ must be zero ($N = 0$).

Since all poles of $Q(s)$ are stable, which implies $P = 0$, then $N = 0$ to guarantee stability, and a sufficient condition for $Q(s)$ can be obtained making $|G'(j\omega)| = \left|Q(s)(e^{-\delta s} - 1)\right| < 1 \;\; \forall\omega$. This is fulfilled if:

$$2\left|\sin\left(\frac{\delta}{2}\omega\right)\right| |Q(j\omega)| < 1 \;\; \forall\omega \qquad (11)$$

Note that the condition (11) is independent of the delay value L of the controller $C(s)$ in (8).

Furthermore, if a Bessel filter is chosen for $Q(s)$ (Horowitz and Hill 1989) which can be approximated by a constant time delay (equal to β in equation (8), $Q(j\omega) \approx 1e^{-j\beta\omega}$) within its pass band, the closed-loop frequency response of the system will satisfy $F(j\omega_h) = 1$ and $F_D(j\omega_h) = 0$ while the approximation is valid. Therefore, the closed-loop system will show perfect reference tracking (and disturbance rejection) within the filter pass band.

Obviously, only a limited number of harmonics will be controlled as described because the magnitude and the phase characteristics of the filter will deteriorate as frequency increases.

4. CONTROL SYSTEM FOR BALANCED THREE-PHASE INVERTERS

The system depicted in Figure 1 can be extended for three-phase inverters, where the output system for each phase (A, B and C) can be calculated as:

$$Y_A(s) = e^{-t_0 s}U_A(s) + D_A(s) \qquad (12)$$
$$Y_B(s) = e^{-t_0 s}U_B(s) + D_B(s) \qquad (13)$$
$$Y_C(s) = e^{-t_0 s}U_C(s) + D_C(s) \qquad (14)$$

A controller $C(s)$ has to be designed for each phase of the inverter, splitting the MIMO system described by (12)-(14) into three independent SISO systems. Under unbalanced conditions, this is the most popular alternative to control the inverter output voltage. However, under balanced conditions, an alternative solution to design the control system is proposed in this work.

Given a set of three-phase variables, a linear transformation can be defined that transforms the variables to an orthogonal $0 - ds - qs$ coordinate system as:

$$\begin{bmatrix} x_0 \\ x_{ds} \\ x_{qs} \end{bmatrix} = \mathbf{P} \begin{bmatrix} x_A \\ x_B \\ x_C \end{bmatrix} \qquad (15)$$

where x_A, x_B and x_C is the set of three-phase variables and x_0, x_{ds} and x_{qs} are the variables in the new coordinate system.

This transformation is the known Park's transformation and it is carried out by using a stationary reference frame with the matrix $\mathbf{P}$:

$$\mathbf{P} = \begin{bmatrix} k_1 & k_1 & k_1 \\ k_2 & k_2\cos\left(-\frac{2\pi}{3}\right) & k_2\cos\left(-\frac{4\pi}{3}\right) \\ 0 & -k_2\sin\left(-\frac{2\pi}{3}\right) & -k_2\sin\left(-\frac{4\pi}{3}\right) \end{bmatrix} \qquad (16)$$

If the real power must remain invariant, $\mathbf{P^t P} = \mathbf{I}$. Hence (Kundur 1994):

$$k_1 = \frac{1}{\sqrt{3}}, \; k_2 = \sqrt{\frac{2}{3}} \qquad (17)$$

If $x_A + x_B + x_C = 0$, the homopolar component x_0 is equal to zero and the three-phase system is reduced to a two-axis reference frame ($ds - qs$).

The Park's transformation can also be done using a reference frame $d - q$ rotating synchronously with the fundamental harmonic ω_1 (Krause 1986):

$$\begin{bmatrix} x_d \\ x_q \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_{ds} \\ x_{qs} \end{bmatrix} \qquad (18)$$

with $\theta = \int \omega_1 dt$.

If the three-phase system is balanced, the harmonics of the inverter output voltage are: 1st, 5th, 7th, 11th, 13th, 17th, 19th, etc. Furthermore using the reference frame rotating at the fundamental frequency ω_1, the fundamental harmonic is transformed into a d.c. signal, and the 5th and 7th harmonics are transformed into a 6th harmonic, the 11th and 13th harmonics are transformed into a 12th harmonic, and the 17th and 19th harmonics are transformed into a 18th harmonic in that frame (Mattavelli and Fasolo 2000).

Note that if the inverter system is found to be balanced, the homopolar component is zero, and the equivalent system described by (12)-(14) can be written using a synchronous reference frame as:

$$\begin{bmatrix} Y_d(s) \\ Y_q(s) \end{bmatrix} = \begin{bmatrix} e^{-t_0 s} & 0 \\ 0 & e^{-t_0 s} \end{bmatrix} \begin{bmatrix} U_d(s) \\ U_q(s) \end{bmatrix} + \begin{bmatrix} D_d(s) \\ D_q(s) \end{bmatrix} \quad (19)$$

Since the inverter-output voltage will contain only a 6th harmonic and its multiples, using this frame, it is possible to propose the following controller:

$$C_d(s) = C_q(s) = \frac{Q(s)e^{-\left(L'-\hat{t}_0\right)s}}{1 - Q(s)e^{-L's}} \quad (20)$$

with $L' = \frac{2\pi}{6\omega_1} - \beta$.

The condition to guarantee stability of the closed-loop system is again (11).

The closed-loop system using (20) is six times faster than that using (8).

5. CONTROLLER DESIGN AND PERFORMANCE

The following case study has been considered: the expected time delay of the inverter was chosen as being $\hat{t}_0 = \frac{3}{2}t_s$ and $\delta = 0.2\hat{t}_0$, where t_s is the sampling period for the control system, and it was made equal to 6300^{-1} s (see Section 2 for an explanation). The fundamental frequency ω_1 was chosen as being equal to 100π rad/s.

A second order Bessel filter is proposed whose transfer function is:

$$Q(s) = \frac{3\omega_B^2}{s^2 + 3\omega_B s + 3\omega_B^2} \quad (21)$$

where if $\omega_B < 7711.2\pi$ rad/s, to satisfy (10). The chosen value is $\omega_B = 7500\pi$ rad/s and Figure 2 shows that $\left|Q(s)\left(e^{-\delta s} - 1\right)\right| < 1$ (0 dB).

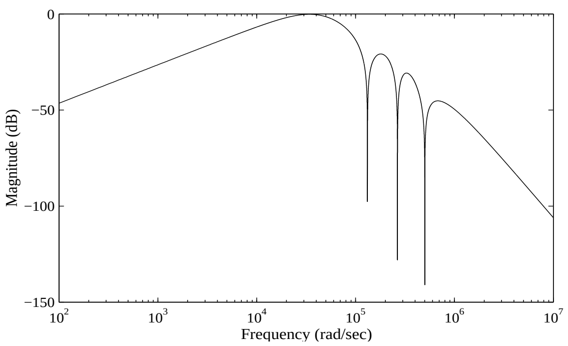

Fig. 2. Amplitude in dB of the term $Q(s)\left(e^{-\delta s} - 1\right)$.

The filter $Q(s)$ has a phase lag proportional to frequency (equivalent to a constant time delay equal to $\tau = 4.24 \cdot 10^{-5}$ s) up to $10 \cdot 10^3$ rad/s. The amplitude of the filter $Q(s)$ begins to decrease at, approximately, 5000 rad/s (about 16 times the fundamental frequency).

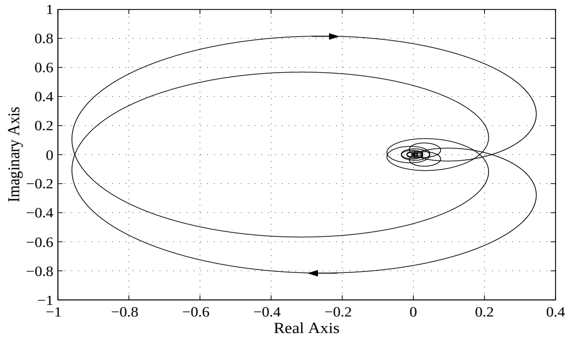

Fig. 3. Nyquist diagram of the term $Q(s)e^{-Ls}\left(e^{-\delta s} - 1\right)$, with $L = \frac{2\pi}{\omega_1} - \beta$.

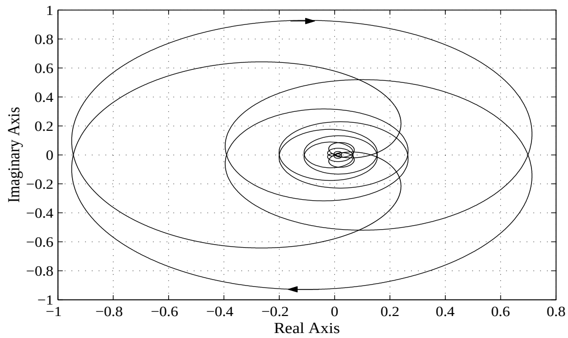

Fig. 4. Nyquist diagram of the term $Q(s)e^{-L's}\left(e^{-\delta s} - 1\right)$, with $L' = \frac{2\pi}{6\omega_1} - \beta$.

Figure 3 shows the Nyquist diagram of the term $Q(s)e^{-Ls}\left(e^{-\delta s} - 1\right)$ while Figure 4 shows the Nyquist diagram of $Q(s)e^{-L's}\left(e^{-\delta s} - 1\right)$. In both cases, the number of counterclockwise encirclements of the point $(-1, 0)$ is $N = 0$, therefore both closed-loop systems are stable.

In this case study, 16 harmonics of ω_1 will be eliminated if β is chosen to be equal to τ, while higher-order harmonics will only be attenuated.

6. SIMULATION RESULTS

A control system like the one depicted in Figure 1 has been simulated for a balanced three-phase VSI with the parameters of the case study presented in Section 5. The response to disturbances of frequencies $5\omega_1$, $7\omega_1$, $11\omega_1$, $13\omega_1$, $17\omega_1$ and $19\omega_1$ has been investigated. The amplitude of every disturbance input is 10 V. The fundamental frequency has been set at $\omega_1 = 100\pi$ rad/s (the fundamental period is 20 ms).

The controller has been implemented using Park's transformation in two cases: with a stationary reference frame and a delay $L = \frac{2\pi}{\omega_1} - \beta$ for the repetitive control, and with a reference frame rotating synchronously with ω_1 and a delay $L' = \frac{2\pi}{6\omega_1} - \beta$ for the repetitive control.

Figures 5 and 6 show the results obtained when the controller is implemented using a stationary reference frame and the delay L.

Figures 5(a) and 5(b) show the y_{ds} and y_{qs} components, respectively, of the output voltage for sinusoidal disturbances and zero reference inputs. The closed-loop system cancels all the disturbances. A clear attenuation is already in 40 ms ($2\frac{2\pi}{\omega_1}$ s), approximately.

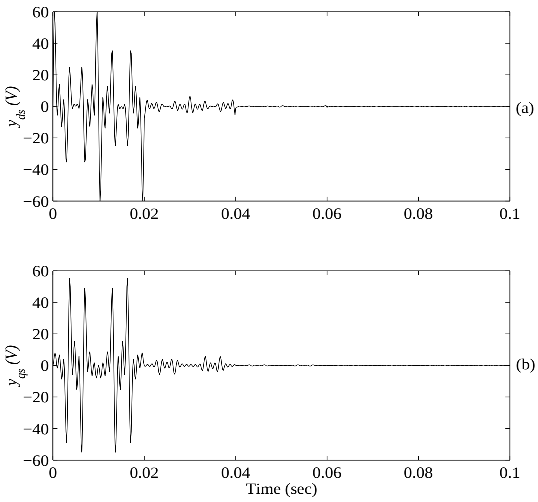

Fig. 5. Response of the system for sinusoidal disturbances using a stationary reference frame and the delay L in the controller: (a) y_{ds} voltage and (b) y_{qs} voltage.

Figure 6(a) shows the reference inputs v_{ds} and v_{qs} (their amplitudes were set at 50 V), while Figure 6(b) shows the y_{ds} and y_{qs} components of the output voltage: after, approximately, 40 ms the disturbances are eliminated and the outputs are equal to the reference inputs.

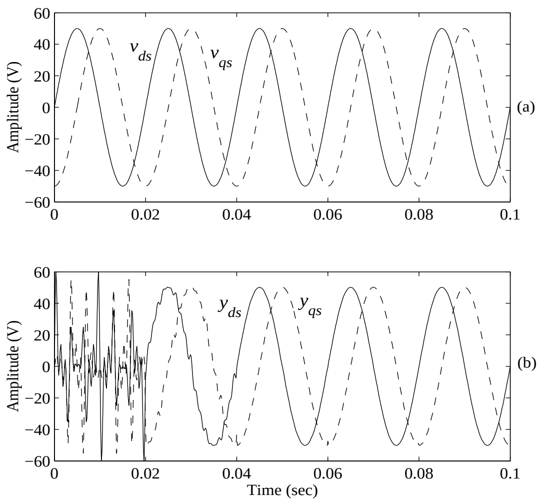

Fig. 6. Response of the system for sinusoidal references and disturbances using a stationary reference frame and the delay L in the controller: (a) references v_{ds} and v_{qs}, and (b) output voltage y_{ds} and y_{qs}.

Figures 7 and 8 show the results obtained when the controller is implemented using a reference frame rotating synchronously with ω_1 and the delay L'.

Figures 7(a) and 7(b) show the y_d and y_q components, respectively, in the rotating reference frame of the output voltage for sinusoidal disturbances and zero reference inputs. The disturbances are eliminated six times faster than in the case of the controller implemented using the stationary reference frame and the delay L (compare Figures 5 and 7).

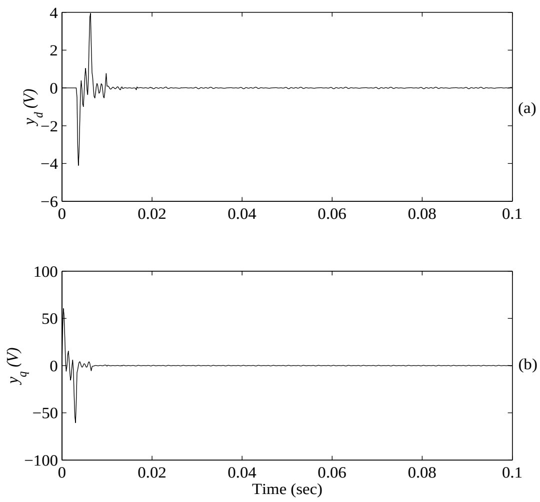

Fig. 7. Response of the system for sinusoidal disturbances using a rotating reference frame and the delay L' in the controller: (a) y_d voltage and (b) y_q voltage.

Figure 8(a) shows the time response of the output-voltage components y_d and y_q when the reference inputs are $v_d = 30$ V and $v_q = -40$ V ($\mid v_{dq} \mid = 50$ V), the references are plotted (- -). Note that the disturbances are cancellated and the outputs are equal to the inputs in 7 ms, approximately. The transformation of the y_d and y_q components in a stationary reference frame is shown in Figure 8(b). The amplitude of both the components, y_{ds} and y_{qs}, is 50 V. The time response is six times faster than the one shown in Figure 6.

Finally, Figure 9(a) and 9(b) show the y_{ds} and y_{qs} components, respectively, of the output voltage for a sinusoidal disturbance of frequency $29\omega_1$ and amplitude 25 V. The reference inputs were set at zero, and the controller has been implemented using the stationary reference frame and the delay L. In this case, the closed-loop system does not eliminate the disturbance completely and this is only attenuated.

7. CONCLUSIONS

This work has investigated a repetitive controller to track sinusoidal references or to reject sinu-

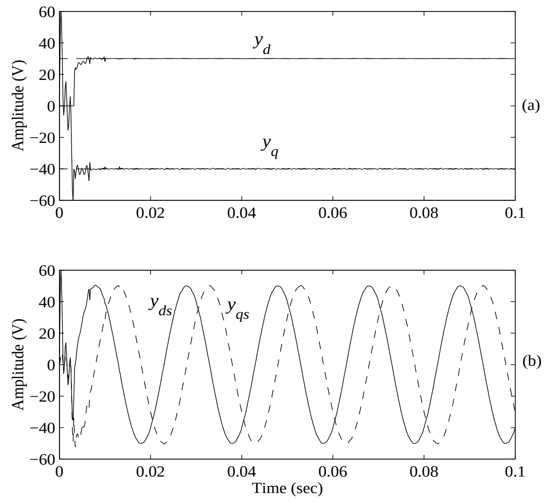

Fig. 8. Response of the system for sinusoidal references and disturbances using a rotating reference frame and the delay L' in the controller: (a) components y_d and y_q, and (b) components y_{ds} and y_{qs}.

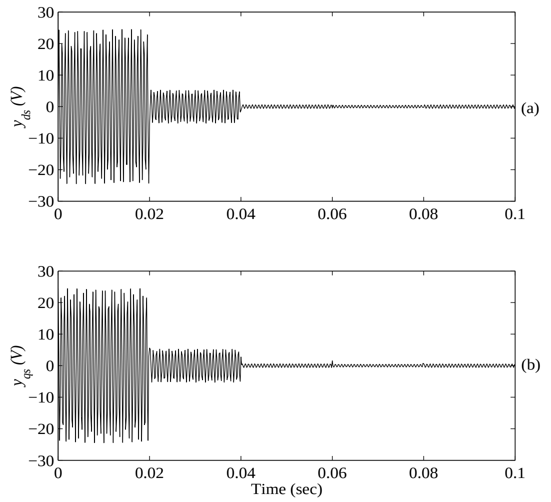

Fig. 9. Response of the system for sinusoidal disturbance of frequency $29\omega_1$ using a stationary reference frame and the delay L in the controller: (a) y_{ds} voltage and (b) y_{qs} voltage.

soidal disturbances in the output voltage of a three-phase VSI. This type of controller may be used to compensate the effects of switches dead times, which appear as unexpected low-frequency output voltage harmonics that can be modelled as disturbances.

The closed-loop stability has been studied. It was demonstrated that the use of a second order Bessel filter ensures stability when there are differences between the VSI model and the actual plant.

The Bessel filter limits the number of harmonics that can be eliminated with a single controller. The rest of the harmonics are only attenuated.

The controller can be implemented using Park's transformation in a stationary reference frame and a delay equal to the fundamental period. This scheme is valid under balanced conditions and under unbalanced conditions. If the three-phase VSI works under balanced conditions, faster closed-loop response can be obtained.

The performance of this type of controllers has been illustrated using simulation results.

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