IDENTIFICATION OF A STATE SPACE MODEL FOR A SYNCHRONOUS MACHINE

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Abstract: Application of a recursive subspace identification method to derive a state space model for a synchronous machine is described in this paper. Simulation studies show the effectiveness of such an algorithm to identify on-line a synchronous machine model over a wide range of operating conditions and disturbances. The model provides a foundation for further study on a MIMO adaptive power system stabilizer.

Keywords: Synchronous machines; State-space models; Recursive algorithms; MIMO

1. MOTIVATION

One aspect of power system stability is dynamic stability. The unstable situation in this case results in long-term low frequency oscillations. Power system stabilizer (PSS) is used to dampen such oscillations and it efficiently improves the stability of the power system.

In recent years adaptive PSS (APSS) has been developed to overcome inherent shortcomings of conventional PSSs. Most of them utilize the excitation control based on the single input single output model of a synchronous machine. As is well known, governor can also improve the power system stability by maintaining the generator output and frequency at predetermined values.

It is worth trying to coordinate excitation and governor controls together, that is, a multi-input multi-output (MIMO) APSS. To obtain such a controller, an accurate model of the plant is required. State space model is a better choice than transfer function model that is cumbersome with polynomial representations in the multivariable case. Thus, identification of a MIMO state space model is a key step in designing a model based APSS.

The paper is organized as follows. An ordinary MOESP (MIMO Output Error State Space) algorithm is introduced in Sec. 2. Recursive subspace identification based on PAST (Projection Approximation Subspace Tracking) approach is proposed in Sec. 3. Section 4 provides some discussion concerning identification of a simple power system consisting of a single machine connected to an infinite bus. In Sec. 5, simulation results of MIMO identification are presented. Finally, the conclusions are given in Sec. 6.

2. INTRODUCTION OF THE ORDINARY MOESP (OM) SCHEME

Mathematically, a state space model can be expressed by the following equations (Verhaegen and Dewilde, 1992):

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u}_k + \mathbf{w}_k \\ \mathbf{y}_k &= \mathbf{C}\mathbf{x}_k + \mathbf{D}\mathbf{u}_k + \mathbf{v}_k \end{aligned} \tag{1}$$

where $\mathbf{x}_k \in R^{n\times1}, \mathbf{u}_k \in R^{m\times1}, \mathbf{y}_k \in R^{l\times1}$ .The noise processes are the process noise $\mathbf{w}_k$ and the output measurement noise $\mathbf{v}_k$. They are assumed to be discrete time, zero mean, white noise with appropriate dimensions.

For subspace-based algorithms, the following matrix definitions are frequently used.

- Hankel matrix

The Hankel matrices are constructed from the input and output data. The following is an input Hankel matrix:

$$\mathbf{U}_{k,i,N} = \begin{bmatrix} \mathbf{u}_k & \mathbf{u}_{k+1} & \dots & \mathbf{u}_{k+N-1} \\ \mathbf{u}_{k+1} & \mathbf{u}_{k+2} & \dots & \mathbf{u}_{k+N} \\ \dots & \dots & \dots & \dots \\ \mathbf{u}_{k+i-1} & \mathbf{u}_{k+i} & \dots & \mathbf{u}_{k+i+N-2} \end{bmatrix} \tag{2}$$

where k is the starting time index, i is the number of the rows and N is the number of columns.

- Extended observability matrix Γ_i

$$\Gamma_i = \begin{bmatrix} \mathbf{C} & \mathbf{CA} & \ldots & \mathbf{CA}^{i-1} \end{bmatrix}^T \quad (3)$$

- Toeplitz matrix

$$\mathbf{H}_i = \begin{bmatrix} \mathbf{D} & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{CB} & \mathbf{D} & \ldots & \mathbf{0} \\ \ldots & \ldots & \ldots & \ldots \\ \mathbf{CA}^{i-2}\mathbf{B} & \mathbf{CA}^{i-3}\mathbf{B} & \ldots & \mathbf{D} \end{bmatrix} \quad (4)$$

- Matrix $\mathbf{X}_{k,N}$ of state vectors

$$\mathbf{X}_{k,N} = \begin{bmatrix} \mathbf{x}_k & \mathbf{x}_{k+1} & \ldots & \mathbf{x}_{k+N-1} \end{bmatrix} \quad (5)$$

For the special case of the absence of $\mathbf{w}_k$, $\mathbf{v}_k$, the data equation can be written in a condensed form:

$$\mathbf{Y}_{k,i,N} = \Gamma_i \mathbf{X}_{k,N} + \mathbf{H}_i \mathbf{U}_{k,i,N} \quad (6)$$

It is obvious from (6) that the output from the system contains two parts. One is the zero input response $\Gamma_i \mathbf{X}_{k,N}$, the other is the zero state response $\mathbf{H}_i \mathbf{U}_{k,i,N}$. Using the LQ factorization to get rid of the zero state response from the output, $span_{col}(\Gamma_i)$ can be estimated and hence the matrices $\mathbf{A}$ and $\mathbf{C}$.

$$\begin{bmatrix} \mathbf{U}_{k,i,N} \\ \mathbf{Y}_{k,i,N} \end{bmatrix} = \mathbf{LQ} = \begin{bmatrix} \mathbf{L}_{11} & \mathbf{0} \\ \mathbf{L}_{21} & \mathbf{L}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{Q}_1 \\ \mathbf{Q}_2 \end{bmatrix} \quad (7)$$

where $\mathbf{Q}$ is an orthogonal matrix.

$$\mathbf{U}_{k,i,N} = \mathbf{L}_{11}\mathbf{Q}_1$$

$$\mathbf{Y}_{k,i,N} = \mathbf{L}_{21}\mathbf{Q}_1 + \mathbf{L}_{22}\mathbf{Q}_2$$

$$\mathbf{Y}_{k,i,N}\mathbf{Q}_2^T = \mathbf{L}_{21}\mathbf{Q}_1\mathbf{Q}_2^T + \mathbf{L}_{22}\mathbf{Q}_2\mathbf{Q}_2^T = \mathbf{L}_{22} \quad (8)$$

From (6)

$$\begin{aligned} \mathbf{Y}_{k,i,N}\mathbf{Q}_2^T &= \Gamma_i \mathbf{X}_{k,N}\mathbf{Q}_2^T + \mathbf{H}_i \mathbf{U}_{k,i,N}\mathbf{Q}_2^T \\ &= \Gamma_i \mathbf{X}_{k,N}\mathbf{Q}_2^T + \mathbf{H}_i \mathbf{L}_{11}\mathbf{Q}_1\mathbf{Q}_2^T \\ &= \Gamma_i \mathbf{X}_{k,N}\mathbf{Q}_2^T \end{aligned} \quad (9)$$

Comparing (8) and (9)

$$\mathbf{L}_{22} = \Gamma_i \mathbf{X}_{k,N}\mathbf{Q}_2^T \quad (10)$$

Doing a SVD factorization of $\mathbf{L}_{22}$, a consistent estimate of $span_{col}(\Gamma_i)$ can be obtained.

$$\mathbf{L}_{22} = \mathbf{USV}_2^T \quad (11)$$

$$\hat{\Gamma}_i = \mathbf{U}(:,1:n) \quad (12)$$

It is easy to find the estimate of $\mathbf{A}$ and $\mathbf{C}$ from $\hat{\Gamma}_i$.

$$\hat{\mathbf{C}} = \hat{\Gamma}_i(1:l,:) \quad (13)$$

$\mathbf{A}$ can be estimated by solving the least squares problem. y is the Moore-Penrose pseudo-inverse.

$$\hat{\mathbf{A}} = \hat{\Gamma}_i(1:(i-1)l,:)^{y}\ \hat{\Gamma}_i(l+1:end,:) \quad (14)$$

Estimation of $\mathbf{B}$ and $\mathbf{D}$ becomes a least squares regression problem with known $\hat{\mathbf{A}}$ and $\hat{\mathbf{C}}$. Mathematically

$$\mathbf{y}(k) = \mathbf{CA}^k\mathbf{x}(0) + \sum_{t=0}^{k-1}\mathbf{CA}^{k-1-t}\mathbf{Bu}(t) + \mathbf{Du}(k) \quad (15)$$

This equation can be rewritten in the following form by using the Kronecker product $\otimes$ and stacking the columns of $\mathbf{B}$ and $\mathbf{D}$ by vec($\mathbf{B}$) and vec($\mathbf{C}$).

$$\mathbf{y}(k) = \mathbf{CA}^k\mathbf{x}(0) + \left[\sum_{t=0}^{k-1}\mathbf{u}(t)^T \otimes \mathbf{CA}^{k-1-t}\right]vec(\mathbf{B}) + \left[\mathbf{u}(k)^T \otimes \mathbf{I}_l\right]vec(\mathbf{D}) \quad (16)$$

Construct a vector with all output signals

$$\mathbf{Y}_{0,N,1} = \begin{bmatrix} \Gamma_i & \mathrm{E}_y & \mathrm{E}_u \end{bmatrix}\begin{bmatrix} \mathbf{x}_0 & vec(\mathbf{B}) & vec(\mathbf{D}) \end{bmatrix}^T \quad (17)$$

where $\mathbf{Y}_{0,N,1} = \begin{bmatrix} \mathbf{y}(0) & \mathbf{y}(1) & \ldots & \mathbf{y}(N-1) \end{bmatrix}^T$

$$\mathbf{E}_y = \begin{bmatrix} \mathbf{0} & \mathbf{u}(0)^T \otimes \mathbf{C} & \ldots & \sum_{t=0}^{N-2}\mathbf{u}(t)^T \otimes \mathbf{CA}^{N-2-t} \end{bmatrix}^T$$

$$\mathbf{E}_u = \begin{bmatrix} \mathbf{u}(0)^T \otimes \mathbf{I}_l & \mathbf{u}(1)^T \otimes \mathbf{I}_l & \ldots & \mathbf{u}(N-1)^T \otimes \mathbf{I}_l \end{bmatrix}^T$$

Γ_i is the extended observability matrix as defined in (3). Solving (17)

$$\begin{bmatrix} \mathbf{x}_0 & vec(\mathbf{B}) & vec(\mathbf{C}) \end{bmatrix}^T = \begin{bmatrix} \Gamma_i & \mathrm{E}_y & \mathrm{E}_u \end{bmatrix}^{y}\ \mathbf{Y}_{0,N,1} \quad (18)$$

For the more general model with noise $\mathbf{w}_k$ and $\mathbf{v}_k$, (6) becomes:

$$\mathbf{Y}_{k,i,N} = \Gamma_i \mathbf{X}_{k,N} + \mathbf{H}_i \mathbf{U}_{k,i,N} + \mathbf{G}_i \mathbf{W}_{k,i,N} + \mathbf{V}_{k,j,N} \quad (19)$$

where $\mathbf{W}_{k,i,N}$ and $\mathbf{V}_{k,i,N}$ are Hankel matrices containing the process noise and the output measurement noise, respectively. Matrix $\mathbf{G}_i$ shows the impact from $\mathbf{w}_k$ on the output.

Introduction of IV (Instrumental Variable) can help to get rid of the influence of noise without disturbing the column space of Γ_i due to the following properties.

1) $\mathrm{E}\left[\mathbf{e}(t)\mathrm{X}(t)^H\right] = \mathbf{0}$
2) $Rank(\mathbf{M}_{xx}) = Rank\{\mathrm{E}\left[\mathbf{x}(t)\mathrm{X}(t)^H\right]\} = n$

where $\mathbf{e}(t)$ represents signal that needs to be eliminated while $\mathbf{x}(t)$ is the useful signal. $\mathrm{X}(t) \in R^{p\times 1}, p \geq n$ is an IV vector.

The rest of the algorithm to estimate system matrices is the same as that used in the noise-free environment. Different source and construction of IV can be used to handle different noise model (Lovera *et al.*, 2000).

3. RECURSIVE SUBSPACE IDENTIFICATION

As mentioned before, it is not only necessary to derive a state space model, but also need to update the model on line to adapt to the system variations. A bottleneck to the recursive implementation is the update of the SVD factorization. The idea of applying subspace-tracking algorithms to the recursive subspace identification was introduced in (Gustafson, 1998) to overcome the difficulty. A successful subspace-tracking algorithm is the PAST approach (Yang, 1995). The basic idea is to treat the signal subspace-tracking problem as the solution of an unconstrained minimization task. Furthermore a projection approximation reduces the minimization to the well-known exponentially weighted least squares problem.

3.1 The PAST scheme

Consider a random vector $\mathbf{x} \in R^{m \times 1}$, and study the unconstrained criterion

$$J(\mathbf{W}(t)) = \mathrm{E}\left\|\mathbf{x}(t) - \mathbf{W}(t)\mathbf{W}(t)^H \mathbf{x}(t)\right\|^2 \quad (20)$$

with a full

rank $(=n)$ matrix $\mathbf{W}(t) \in R^{m \times n}, m > n$.

Applying the theorem

"The global minimum of $J(\mathbf{W}(t))$ is attained if and only if $\mathbf{W} = \mathbf{U}_s \mathbf{T}$, where $\mathbf{U}_s \in R^{m \times n}$ contains the n dominant eigenvectors of $\mathrm{E}\left[\mathbf{x}(t)\mathbf{x}(t)^T\right]$ and $\mathbf{T}$ is an arbitrary unitary matrix."

From (Yang, 1995), the columns of $\mathbf{W}(t)$ which minimize $J(\mathbf{W}(t))$ form an orthonormal basis of the signal subspace.

For a practical algorithm, considering forgetting factor l, replace (19) with

$$J(\mathbf{W}(t)) = \sum_{k=1}^{t} l^{t-k} \left\|\mathbf{x}(k) - \mathbf{W}(t)\mathbf{W}(t)^H \mathbf{x}(k)\right\|^2 \quad (21)$$

The key issue of the PAST is to replace $\mathbf{W}(t)^H \mathbf{x}(k)$ with $\mathbf{h}(k) = \mathbf{W}(k-1)^H \mathbf{x}(k)$. Hence the original fourth-order function of $\mathbf{W}(t)$ is reduced to a quadratic problem. $J(\mathbf{W}(t))$ is minimized by

$$\mathbf{W}(t) = \left(\sum_{k=1}^{t} l^{t-k}\mathbf{x}(k)\mathbf{h}(k)^H\right)\left(\sum_{k=1}^{t} l^{t-k}\mathbf{h}(k)\mathbf{h}(k)^H\right)^{-1} = \hat{\mathbf{M}}_{hx}(t)\hat{\mathbf{M}}_{hh}(t)^{-1} \quad (22)$$

3.2 The IV- PAST scheme

Using IV, similar results can be obtained as from the standard PAST.

$$J(\mathbf{W}(t)) = \sum_{k=1}^{t} l^{t-k}\left\|\mathbf{x}(k)\mathrm{X}(k) - \mathbf{W}(t)\mathbf{W}(t)^H \mathbf{x}(k)\mathrm{X}(k)\right\|^2 \approx \sum_{k=1}^{t} l^{t-k}\left\|\mathbf{x}(k)\mathrm{X}(k) - \mathbf{W}(t)\mathbf{h}(k)\mathrm{X}(k)\right\|^2 \quad (23)$$

$J(\mathbf{W}(t))$ is minimized by

$$\mathbf{W}(t) = \hat{\mathbf{M}}_{xx}(t)\hat{\mathbf{M}}_{hx}(t)^y \quad (24)$$

More details about the algorithmic steps of a recursive formulation of (23) are given in (Gustafson, 1998).

3.3 Recursive updating $span_{col}(\mathrm{G}_i)$ in the OM scheme

Recursive estimation of $\mathbf{A}$ is equal to recursive updating $span_{col}(\mathrm{G}_i)$. In the OM scheme a partial update of LQ factorization is required with the help of the method of Givens Rotations (Lovera *et al.*, 2000).

When a new data point arrives, the LQ must be updated as

$$\begin{bmatrix}\mathbf{U}_{k,i,N+1}\\ \mathbf{Y}_{k,i,N+1}\end{bmatrix} = \begin{bmatrix}\mathbf{L}_{11}(k) & \mathbf{0} & \mathrm{F}_{uf}(k+1)\\ \mathbf{L}_{21}(k) & \mathbf{L}_{22}(k) & \mathrm{F}_{yf}(k+1)\end{bmatrix}\begin{bmatrix}\mathbf{Q}_1(k) & \mathbf{0}\\ \mathbf{Q}_2(k) & \mathbf{0}\\ 0 & 1\end{bmatrix} \quad (25)$$

where the new set of I-O data vectors is

$$\mathrm{F}_{uf}(k+1) = \left[\mathbf{u}_{k+N}^T \quad \dots \quad \mathbf{u}_{k+i+N-1}^T\right]^T \quad (26)$$

$$\mathrm{F}_{yf}(k+1) = \left[\mathbf{y}_{k+N}^T \quad \dots \quad \mathbf{y}_{k+i+N-1}^T\right]^T \quad (27)$$

Using Givens Rotations matrix $\mathbf{P}(k+1)$ to annihilate $\mathrm{F}_{uf}(k+1)$.

$$\mathbf{P}(k+1) = \begin{bmatrix}\mathbf{P}_{11} & \mathbf{0} & \mathbf{P}_{12}\\ \mathbf{0} & \mathbf{I} & \mathbf{0}\\ \mathbf{P}_{21} & \mathbf{0} & \mathbf{P}_{22}\end{bmatrix} \quad (28)$$

$$\begin{bmatrix}\mathbf{L}_{11}(k) & \mathbf{0} & \mathrm{F}_{uf}(k+1)\\ \mathbf{L}_{21}(k) & \mathbf{L}_{22}(k) & \mathrm{F}_{yf}(k+1)\end{bmatrix}\mathbf{P}(k+1) = \begin{bmatrix}\mathbf{L}_{11}(k+1) & \mathbf{0} & \mathbf{0}\\ \mathbf{L}_{21}(k+1) & \mathbf{L}_{22}(k) & \bar{\mathrm{F}}_{yf}(k+1)\end{bmatrix} \quad (29)$$

$$\bar{\mathrm{F}}_{yf} = \mathrm{G}_i\left(\mathbf{X}_{k,N}\mathbf{Q}_1^T\mathbf{P}_{12} + \mathbf{x}_{k+N}\mathbf{P}_{22}\right) + \mathbf{E}_{k,i,N}\mathbf{Q}_1^T\mathbf{P}_{12} + \mathrm{F}_{ef}\mathbf{P}_{22} \quad (30)$$

$\mathbf{E}_{k,i,N}$ denotes a Hankel matrix that consists of noise in the past. F_{ef} represents current noise item at time instant $(k+1)$.

The new information contained in $\left[\mathrm{F}_{uf}(k+1)^T \ \mathrm{F}_{yf}(k+1)^T\right]^T$ is condensed into a rank one modification of $\mathbf{L}_{22}(k)$. Feeding $\bar{\mathrm{F}}_{yf}(k+1)$ directly to the PAST algorithm to update $span_{col}(\mathbf{R}_{22}(k))$ would result in a biased estimation due to the noise it contains. IV-PAST is used to rescue this problem. Previous I-O data are usually chosen as IV $\mathrm{X}(t) \in R^{p\times 1}, p \geq n$.

4. SYNCHRONOUS MACHINE IDENTIFICATION

The power system model consists of a synchronous machine connected to an infinite bus through a double circuit transmission line. The cylindrical rotor machine is simulated by seven first order differential equations in the d-q frame of reference (Anderson *et al.*, 1977). The AVR used is IEEE standard type ST1A. Electro hydraulic governor and steam turbine are also included in the model. System parameters are given in the Appendix.

The system configuration for identification is shown in Fig.1. For identification, the input signals are V_pss and V_gov that are connected to the AVR and governor summing junctions. Deviation of rotor speed and active power, dω and dPe, are chosen as the plant output signals.

Variance Accounted for (VAF) is used as a standard to measure the accuracy of identification.

$$VAF = \left(1 - \frac{\mathrm{var}iance(y_{measured} - y_{estimated})}{\mathrm{var}iance(y_{measured})}\right) \times 100\% \quad (31)$$

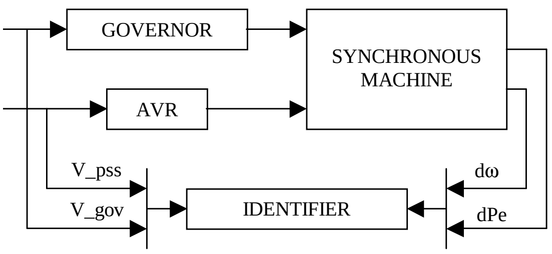

Fig. 1. System configuration.

5.1 Selection of the model order n

Actually power system is a high order non-linear system. However, for control purposes, it is not necessary to identify a detailed model representing the whole system. For on-line identification the model is required to just stress the desirable features of the system, damping the low frequency oscillations of the mechanical mode of the interconnected system in the present case. A model that can represent this kind of oscillation accurately is required. A third order model can usually provide a pair of complex poles that represent the dominant oscillation frequency of the system and a single real pole that represents the free damping part of the system response. Therefore the model order n is selected as 3.

5.2 Scaling output data

Better identification results are obtained from a well-conditioned problem than an ill conditioned one. The results will be more sensitive to noise and perturbations for an ill-conditioned estimation problem.

dPe is nearly 100 times larger than dω. After scaling, the two output signals are brought to the same order of magnitude. Simulation studies show using scaled data leads to more accurate result than just using original data especially in the case of disturbance.

5.3 Observer for state estimation

Although state estimate is not a part of identification, it plays an important role for output prediction as well as the model parameters. The state space model is identified directly from the I-O data. Therefore, the states in the model have no physical meanings. An observer can be established to estimate all the states based on the following equations (Franklin *et al.*, 1998).

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) \quad (32)$$

$$\hat{\mathbf{x}}(k+1) = \mathbf{A}\hat{\mathbf{x}}(k) + \mathbf{B}u(k) + \mathbf{K}\mathbf{C}[\mathbf{x}(k) - \hat{\mathbf{x}}(k)] \quad (33)$$

Subtracting (32) from (33) yields the error vector

$$\mathbf{e}(k+1) = (\mathbf{A} - \mathbf{K}\mathbf{C})\mathbf{e}(k) \quad (34)$$

The solution to this homogeneous first-order vector difference equation is given by

$$\mathbf{e}(k) = (\mathbf{A} - \mathbf{K}\mathbf{C})^k \mathbf{e}(0) \quad (35)$$

If the matrix ($\mathbf{A} - \mathbf{K}\mathbf{C}$) has eigenvalues inside the unit circle, then $(\mathbf{A} - \mathbf{K}\mathbf{C})^k$ approaches the zero matrix as the time index k gets large.

Based on the pole placement algorithm, the eigenvalues of ($\mathbf{A} - \mathbf{K}\mathbf{C}$) are placed close to the origin. Therefore, $(\mathbf{A} - \mathbf{K}\mathbf{C})^k$ approaches zero in about n steps, where n is the system order.

5. SIMULATION RESULTS

In the simulation studies, the synchronous machine is operating in steady state at Pe = 0.7 p.u. , 0.85 power factor lag. The infinite bus voltage is 1.0 p.u. Using a sampling frequency of 20Hz and a variable forgetting factor (Park,1991) to improve fast tracking ability of the algorithm, identification results were obtained under steady state as well as under various disturbances.

A model from off line identification was used as the initial parameters for recursive subspace identification. Consider the following situations, where all disturbances occur at 70s:

Case 1: Steady state, Fig.2.

Case 2: Reference terminal voltage increased by 5%, Fig.3.

Case 3: Reference torque increased by 10%, Fig.4.

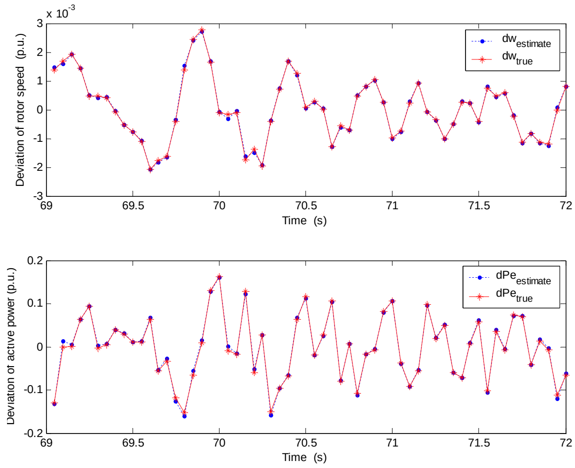

Fig. 2. Identifier response to a steady state.

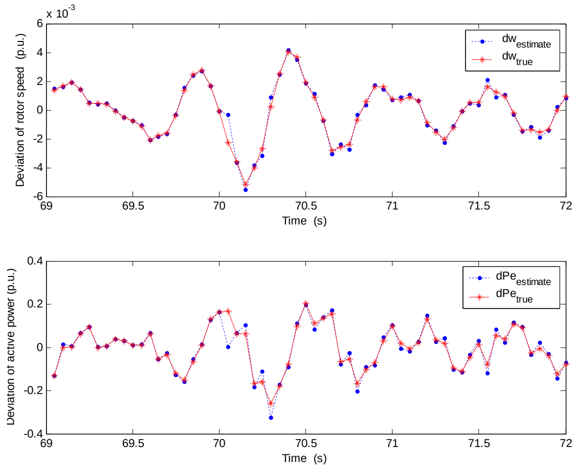

Fig. 3. Identifier response to a 5% increase in reference terminal voltage.

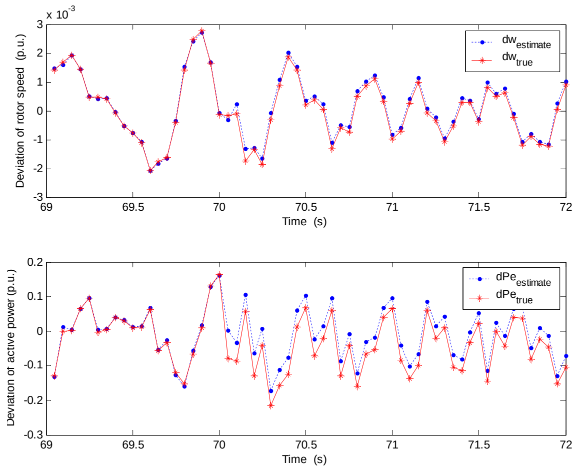

Fig. 4. Identifier response to a 10% increase in reference torque.

Case 4: Disconnection of one transmission line, Fig.5.

Case 5: A three phase to ground fault is applied at the middle of one transmission line and cleared 50ms later by opening the breakers at both ends, Fig.6.

Case 6: The same three phase to ground fault, using original output data without scaling, Fig.7.

Case 7: The same three phase to ground fault, using fixed forgetting factor instead, Fig.8.

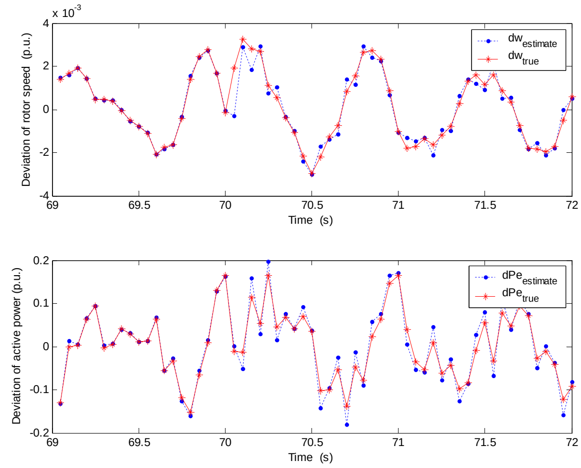

Fig. 5. Identifier response to disconnecting one transmission line.

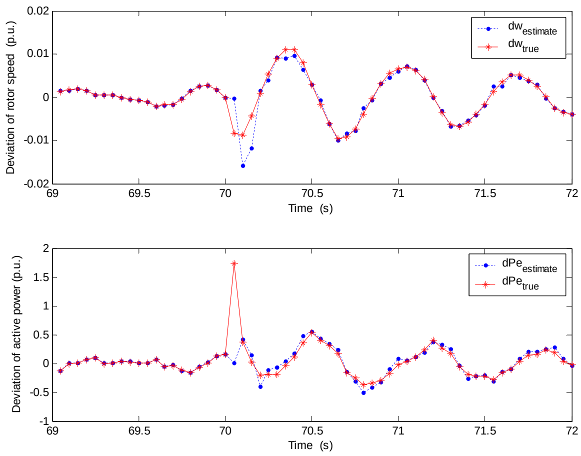

Fig. 6. Identifier response to a three phase to ground fault.

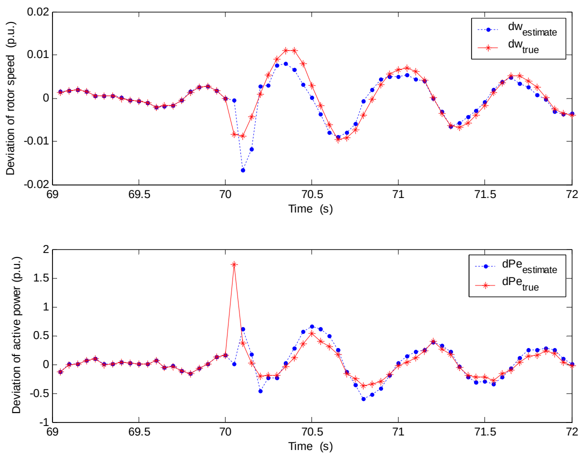

Fig. 7. Identifier response to a three phase to ground fault without scaling output data.

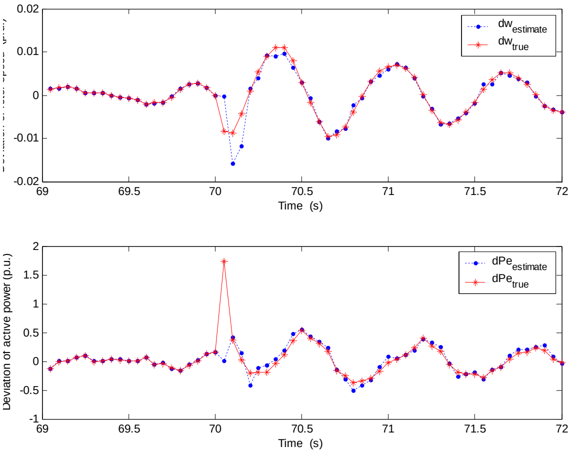

Fig. 8. Identifier response to a three phase to ground fault with a fixed forgetting factor

Table 1 Summary of identification results under different cases

Case	Vaf1 (%) dω	Vaf1 (%) dPe	Vaf2 (%) dω	Vaf2 (%) dPe	Scaled output	VFF
1	99.8	99.1	99.8	99.5	√	√
2	99.8	99.1	95.8	91.9	√	√
3	99.8	99.1	99.6	97.2	√	√
4	99.8	99.1	93.2	84.4	√	√
5	99.8	99.1	97.4	89.5	√	√
6	99.3	98.2	85.4	51.3	×	√
7	99.8	99.1	96.8	88.3	√	×

*: Vaf1 is the value of VAF before disturbance
Vaf2 is the value of VAF after 0.5s of disturbance
VFF means Variable Forgetting factor

It is obvious from Table 1 that a lower order linear state space model can be used to accurately represent the synchronous machine behaviors under steady state. The recursive subspace algorithm also performed well under small and large disturbances. Using variable forgetting factor can improve the identification results to some extent. Proper scaling of the output data makes the results more attractive.

6. CONCLUSIONS

Use of recursive subspace identification for a state space model of a synchronous machine is presented in this paper. The approach is based on IV ideas and on the use of subspace tracking for the update of the SVD. It has been tested with synchronous machine operating in steady state and under various disturbances. Accuracy of identification can be increased by incorporating properly scaled I-O data and by using a variable forgetting factor. The proposed approach provides a reasonable model for designing a MIMO APSS.

7. APPENDIX

Generator (Cylindrical rotor machine) Parameters In p.u.

$r_a = 1.1e-3 \quad x_d = 1.70 \quad x_q = 1.64$

$r_D = 1.3e-2 \quad x_D = 1.61$

$r_Q = 5.4e-2 \quad x_Q = 1.59$

$r_f = 7.4e-4 \quad x_f = 1.65$

$x_{md} = 1.55 \quad x_{mq} = 1.49 \quad H = 2.37(s)$

Transmission Line Parameters In p.u.

$r_e = 0.01 \quad x_e = 0.2$

AVR Parameters

$K_R = 1.0 \quad T_R = 0.001$

$T_C = 1 \quad T_B = 10 \quad K_C = 0.08$

$K_A = 190 \quad T_A = 0.001$

$V_{\max} = 7.8 \quad V_{\min} = -6.7$

Electro Hydraulic Governor parameters

$K_g = 25.0 \quad T_g = 0.1$

$C_{\max} = 0.5 \quad C_{\min} = -0.5$

$P_{\max} = 1.0 \quad P_{\min} = 0.0$

Steam Turbine parameters

$T_{CH} = 0.2 \quad T_{RH} = 4.0 \quad T_{CO} = 0.4$

$F_{HP} = 0.3 \quad F_{IP} = 0.4 \quad F_{LP} = 0.3$

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