COMPARISON OF LINEAR DYNAMIC MODELS FOR AIR TRAFFIC FLOW MANAGEMENT

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Abstract: Traditionally, models used in air traffic flow management are based on simulating the trajectories of individual aircraft. While practical, this approach does not offer any insights into the dynamics of the traffic flow. Recently, two different approaches, (1) aggregate traffic model approach and (2) Eulerian model approach, have been presented to generate linear dynamic models that represent the behaviour of the air traffic flow. The dimension of the linear models is independent of the number of aircraft in the system and depends on the spatial discretization of the airspace. The resulting linear models can be used both for the analysis and synthesis of traffic flow management techniques.

Keywords: Air traffic control, Transportation, Linear Systems, Flow control, Modelling, Model approximation.

1. INTRODUCTION

Demand for air transportation has seen a six-fold increase in the past 30 years and estimates call for a strong average growth rate of 4.7% during the next 20 years (Airbus, 2002). This increase in demand will put a further strain on the airports and sectors within the National Airspace System (NAS). The United States Congress has recognized the impact of unmet increased demand and has established a Joint Planning Office for creating and developing a Next Generation Air Transportation System to transform NAS operations. There are more than 40,000 commercial flights operated in the U. S. airspace alone on a typical day at the present time. In order to ensure that this traffic moves smoothly and efficiently in the presence of disruptions caused by convective weather and airport conditions, innovative modeling and design methods are needed in traffic flow management (TFM).

Currently, air traffic flow prediction is done by propagating the trajectories of the proposed flights forward in time and using them to count the number of aircraft in a region of the airspace. The Center TRACON Automation System (CTAS) and the Future Automation Concepts Evaluation Tool (FACET) (Bilimoria, *et al*., 2001) use this physics-based modelling approach for demand forecasting. The accuracy of these predictions is impacted by departure and weather uncertainties (Muller and Chatterji, 2002; Evans, 2001). These trajectory-based models predict the behaviour of the NAS adequately for short durations of up to 20 minutes. With the short prediction accuracy, it is difficult, if not impossible, to make sound strategic decisions on air traffic management.

For instance, a strategic TFM decision may involve rerouting all aircraft originating from the west coast, heading to airports on the east coast, to deal with anticipated stormy weather conditions near Chicago over the next 4 hours. Strategic TFM is a hierarchical system consisting of large number of states, and operating over time scales extending from a few hours to 24 hours. As shown in Figure 1, the airspace in the United States is divided into 20 Centers in the continental United States plus one each in Alaska and Hawaii. The flow relationship between neighboring Centers is shown via links in Figure 1. For example, the figure shows that Kansas City Center (ZKC) receives and sends traffic to the Minneapolis Center (ZMP). Proper mixes of strategic and tactical flow controls initiated by the System Command Center and the 22 Control Centers accomplish TFM in the U. S. Some of the frequently used flow restrictions include ground stop, ground delay, metering (miles-in-trail and time based) and rerouting. Dispatchers and air traffic coordinators at airlines respond to these flow control actions by rescheduling and canceling flights, thus, changing flow patterns.

Since strategic TFM requires control of flows of aircraft rather than individual aircraft, an aggregate model of traffic flow that does not use trajectories of individual aircraft is desirable. Strategic TFM can be

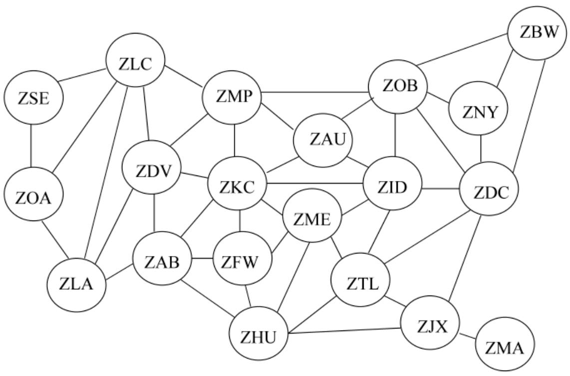

Figure 1: The Centers in Continental US Airspace.

substantially improved by the development of simpler, but more accurate models that allow the exploitation of different analysis and synthesis techniques from Systems Theory.

Recently, two different approaches, (1) aggregate traffic model approach (Sridhar, *et al*., 2004) and (2) Eulerian model approach (Menon, *et al*., 2002; Menon, *et al*., 2003; Menon, *et al*., 2004) have been presented to generate linear dynamic system models (LDSM) to represent the behaviour of the air traffic flow. The aggregate model uses flow relationship between adjacent Centers (Roy, *et al*., 2003). The LDSM in (Roy, *et al*., 2003) is built by counting the number of aircraft entering a Center from an adjacent Center, number of aircraft leaving a Center for a neighboring Center and the numbers of aircraft landing and taking off within a Center. Input to this model consists of the number of departures. Results presented in (Roy, *et al*., 2003), assuming that departures follow a Poisson distribution, show that the resulting numbers of aircraft in the Centers also fit a Poisson distribution. The main limitation of the However, not all aircraft in the airspace strictly follow the jet routes or Victor airways. This introduces the need for a more flexible modeling

results in (Roy, *et al*., 2003) is that modeling departures from Poisson distributions (albeit a different one for each major hub airport) ignores the fact that departure counts vary significantly during the day as banks of aircraft arrive and depart major hub airports. Aircraft counts in the Centers, forecast by LDSM, can be improved significantly by accounting for the nominal departure rates as a function of time and augmenting them by modeling departure uncertainty about these nominal rates.

The basic time-invariant LDSM proposed in (Roy, et al., 2003) has been extended to a time-varying system in (Sridhar, et al., 2004). Instead of a single state transition matrix, several state transition matrices (one for each hour) were used to cover the entire prediction period. State transition matrices were computed using historical air traffic data. The resulting model was then driven by average departure rates, also derived from historical air traffic data, to predict aircraft counts in the 23 airspace regions. These 23 regions consisted of 20 Centers in the continental United States, one each covering Hawaii and Alaska, and one for the international airspace. Uncertainty bounds around these nominal predictions were then obtained using the standard state covariance propagation model driven by the covariance of departure counts. Day-to-day variations about the average departure counts are assumed to be zero-mean Gaussian random variables. Results are presented for another day of traffic data (other than the four days used in LDSM) to show that these counts lie within the confines of the mean aircraft counts predicted by the LDSM and uncertainty bounds generated by the covariance propagation technique.

The development of an Eulerian (Prandtl and Tietjens, 1957) approach to modeling air traffic was discussed in recent research efforts (Menon, *et al*., 2002; Menon, *et al*., 2003). A computer-aided methodology for deriving Eulerian models of the airspace, and employing it for air traffic flow control is described in (Menon, *et al*., 2004). The approach uses FACET software as its foundation.

The Eulerian approach models the airspace in terms of line elements approximating airways, together with merge and diverge nodes. Since this modeling technique spatially aggregates the air traffic, the order of the airspace model depends only on the number of line elements used to represent the airways, and not on the number of aircraft operating in the airspace. Eulerian models are in the form of linear, time-varying difference equations.

The one-dimensional modeling methodology is an intuitive approach for deriving models of traffic flow networks formed by jet routes and Victor airways.

framework. This framework, first advanced in (Menon, *et al*., 2002), discretizes the airspace into surface elements (*SEL*s), within which the traffic flow is aggregated into eight different directions. This modeling provides adequate fidelity in en route airspace where the traffic flow is largely two dimensional. The traffic at all flight levels in Class A airspace (at or above 18,000 ft) is classified as belonging to any one of these eight directions, with inflows and outflows from airports and other external sources. Each surface element is connected to its eight neighbors, with the connection strengths being determined by the actual traffic flow patterns.

The main strength of the LDSM described here is that all the tools available for analysis of linear dynamic systems can be applied to this model. The size of the linear models is independent of the number of aircraft in the system and depends on the number of control volumes used to represent the airspace.

The rest of the paper is organized as follows: The aggregate flow model is described in Section 2. Section 3 describes the Eulerian model. Section 4 compares the relative strengths of the two linear models. Finally, concluding remarks are given in Section 5.

2. AGGREGATE FLOW MODEL

A linear dynamic model for the air traffic in the NAS is developed in this section. This model can be used for predicting traffic count, which is the number of aircraft in a given Air Route Traffic Control Center, in the 22 Centers in the United States and one international region. The resulting traffic count forecast, which is a measure of future demand, can then be balanced against the available capacity using traffic flow management.

The number of arrivals (landings) and the number of aircraft leaving a Center in an interval of time, ΔT, are assumed to be proportional to the number of aircraft in the Center at the beginning of the interval. Following the notation in Figure 2 and using the principle of conservation of flow (analogous to the principle of mass balance in a control volume) in a Center, the number of aircraft in Center at the next instant of time, $k+1$, can be related to the number of aircraft in the Center at the current instant of time, k, via the difference in number of aircraft that came into the Center and the number of aircraft that that left the Center as follows.

$$x_i(k+1) = x_i(k) - \sum_{j=1}^{N} \beta_{ij} x_i(k) + \sum_{\substack{j=1 \\ j \neq i}}^{N} \beta_{ji} x_j(k) + d_i(k) \tag{1}$$

The fractions β_{ij} s' and β_{ji} s' are obtained as transition probabilities in (Roy, *et al.*, 2003). The departures within Center i are denoted by $d_i(k)$. For the purpose of modeling, these departures can be split into two components- a deterministic one and a stochastic one. The deterministic portion of the departures $u_i(k)$ can be computed from filed flight plans and from historical departure data. For example, $u_i(k)$ can be set to the average departure count derived from historical data.

The stochastic component of the departures, $w_i(k)$, can be modeled by assuming a suitable distribution such as, a Gaussian or a Poisson distribution. In such a model, $w_i(k)$, which can also be obtained from historical data, represents the expected variation around the deterministic component.

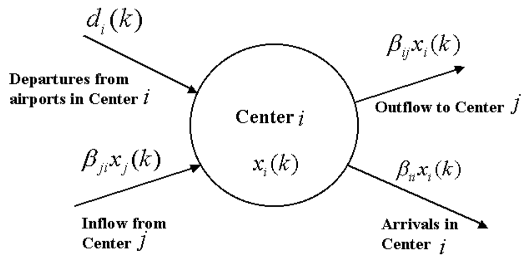

Figure 2: The components of aircraft flow contributing to the traffic count in a given Center.

The discrete system in Equation (1) can be rewritten in the standard State Space notation as:

$$\mathbf{x}(k+1) = \mathbf{A}(k)\mathbf{x}(k) + \mathbf{B}(k)\mathbf{u}(k) + \mathbf{C}(k)\mathbf{w}(k) \tag{2}$$

where,

- k denotes the time instant defined by $k\Delta T$, with ΔT being the sampling interval. In the earlier work in (Roy, *et al.*, 2003), it has been shown that a 10-minute sampling interval accurately approximates Center aircraft count.
- $\mathbf{x}(k) = [x_1(k), \ldots x_N(k)]$ is the state vector with the number of aircraft in the Centers at time k as its elements;
- $\mathbf{u}(k) = [u_1(k), \ldots u_N(k)]$ is the control vector with the number of aircraft departing (taking off) from the Centers as its elements;
- $\mathbf{w}(k) = [w_1(k), \ldots w_N(k)]$ is a vector for modeling departure uncertainties;
- $\mathbf{A}(k)$ is the state transition matrix that contains the information of how flights transition from one Center to the other Center.

The elements of the state transition matrix A are given by:

$$a_{ij} = \beta_{ij}; \quad i \neq j;\ i = 1, \cdots, N;\ j = 1, \cdots, N \tag{3}$$

where, $N = 23$ is the number of Centers including one for the international region. The off-diagonal terms $a_{ij}(k)$ represent the fraction of aircraft transitioning from Center i to the Center j at time k.

This quantity can be calculated from historical data and has been shown (Sridhar, *et al*., 2004) to be slowly varying over time.

The diagonal terms can be calculated as:

$$a_{ii} = 1 - \sum_{j=1}^{N} \beta_{ij} \qquad (4)$$

These terms represent the fraction of the aircraft that remained in the Center i during the k^{th} time step.

Numerical results in (Sridhar, *et al*., 2004) provide error bounds for the number of aircraft in the Center and show that a linear dynamic system with a few transition matrices and Gaussian departure distribution is adequate to represent traffic behavior at the Center level.

3. EULERIAN TRAFFIC FLOW MODEL

The Eulerian modeling process (Menon, *et al*., 2002; Menon, *et al*., 2003; Menon, *et al*., 2004) begins with the definition of a grid of surface elements (*SEL*s) covering the region of airspace being modeled. The surface element grid is defined by latitude-longitude tessellation on the surface of the earth in geocentric polar coordinates. Each surface element has equal angular dimensions in longitude and latitude as shown in Figure 3. However, due to the spherical nature of the airspace being modeled, surface elements far north or south of the equator will have smaller physical dimensions than those near the equator. One-degree latitude-longitude increments are generally employed in national-level traffic flow modeling (Menon, *et al*., 2003; Menon, *et al*., 2004). The eight different en route traffic flow directions within each surface element are indicated in Figure 4. In addition to these, the surface elements above airports will include one output stream for landing aircraft. The aircraft taking off from airports under a surface element are included in one of the eight en route traffic flow directions. Surface elements lying on the boundary of the airspace being modeled will have additional inputs representing traffic entering the system from un-modeled airspace (e.g., international flights).

Since the Eulerian model is discrete in space and time, a sample interval τ must also be specified. Although the spatial and temporal discretizations are based mainly on the level of detail desired in the model, due to the assumption that each surface element is connected only to eight of its neighbors, the sample time interval must be chosen so that no aircraft in a surface element travels beyond its immediate neighbors in a sample interval. Thus, the dimensions of the smallest surface element and the airspeed of the fastest aircraft in the airspace determine the acceptable sample interval.

As in (Menon, *et al*., 2002; Menon, *et al*., 2003), the air traffic flow pattern is modeled within each surface element using two sets of parameters. The first of these are the inertia parameters $a_{i,j,n,n}$, one for each of the eight streams representing the fraction of the aircraft that remained from the previous sample time. By definition, in any stream i, the fraction of aircraft that left the SEL in the previous sample interval is given by $(1- a_{i,j,n,n})$.

The second set of parameters are the flow divergence parameters $\beta_{i,j,m,n}$ representing the fraction of aircraft that switched streams within the SEL. Since the aircraft in a stream may stay in it, or switch to any of the other 7 en route streams, or land at an airport, for a given SEL there is a matrix of 9×8 = 72 flow divergence parameters. In order to satisfy the principle of conservation of aircraft in a surface element i,j , for each stream n, the divergence parameters to all the outputs must add up to unity, i.e.;

$$\sum_{m=1}^{9} \beta_{i,j,m,n} = 1 \qquad (5)$$

Note that one of the $\beta_{i,j,m,n}$ is not independent. By convention, let

$$\beta_{i,j,n,n} = 1 - \sum_{\substack{m=1 \\ m \neq n}}^{9} \beta_{i,j,m,n} \qquad (6)$$

It is assumed that an aircraft will nominally remain in the same stream, so the *default* values of the divergence parameters are:

$$\beta_{i,j,m,n} = \begin{cases} 1, & m = n \\ 0, & m \neq n \end{cases} \qquad (8)$$

Figure 5 illustrates the model of a stream in a surface element. The dynamics of the air traffic flow in a SEL can be described using the inertia parameters and the divergence parameters, through the principle of conservation of aircraft. For instance, the difference equation describing the air traffic flow in the easterly stream in the surface element i, j can be derived as (Menon, *et al*., 2002; Menon, *et al*., 2003):

$$\begin{aligned} x_{(i,j,3)}(k+1) = {} & a_{(i,j,3,3)} \sum_{m=1}^{8} \beta_{(i,j,m,3)}(k) x_{(i,j,m)}(k) \\ & + \tau u_{(i,j,3)}(k) + \tau y_{(i,j-1,3)}(k) \qquad (9) \\ & + \tau q_{(i,j,3)}^{depart}(k) + \tau q_{(i,j,3)}^{exo}(k) \end{aligned}$$

In this equation, $x(k)$ denotes the number of aircraft in the stream at the sample instant k, $u(k)$ are the aircraft flow rates held back in the stream through metering flow control actions, $y(.)$ is the air traffic flow rate from the neighboring SEL, q^{depart} is the air traffic flow rate joining the stream from airports under the SEL and q^{exo} is the air traffic flow rate entering the airspace. The control variables in this equation are the air traffic flow rates $u(k)$ due to metering actions, and the departure traffic flow rates q^{depart} from the airports under the SEL.

The en route output equations for a surface element can be written as:

$$y_{(i,j,m)}(k) = \left(\frac{1 - a_{(i,j,m,m)}(k)}{\tau}\right) \sum_{n=1}^{8} \beta_{(i,j,m,n)}(k) x_{(i,j,n)}(k) - u_{(i,j,m)}(k), \quad m = 1,2,\ldots,8 \tag{10}$$

Moreover, the landing air traffic flow rate into the airports under the SEL are given by:

$$y_{(i,j,m)}(k) = \frac{1}{\tau} \sum_{n=1}^{8} \beta_{(i,j,m,n)}(k) x_{(i,j,n)}(k), \quad m = 9 \tag{11}$$

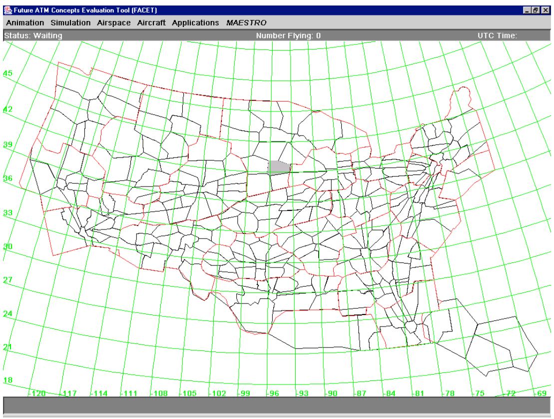

Figure 3: Latitude-longitude tessellation used in Eulerian flow modeling

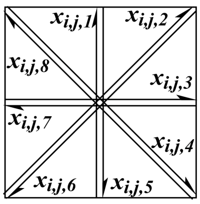

Figure 4:. Traffic flow directions in a Surface Element

Multiple surface elements are required to model realistic airspaces. In the present work, the numbering convention of the surface elements (i, j) is that the index j is increasing from left to right, in the easterly direction, and i is increasing from bottom to top, in the northerly direction. Air traffic flow models of several SELs can be combined to form the overall Eulerian model of the airspace, and can expressed in a compact form as:

$$x(k+1) = A(k)x(k) + Bu(k) + B_d q^{depart}(k) + B_e q^{exo}(k) \tag{12}$$

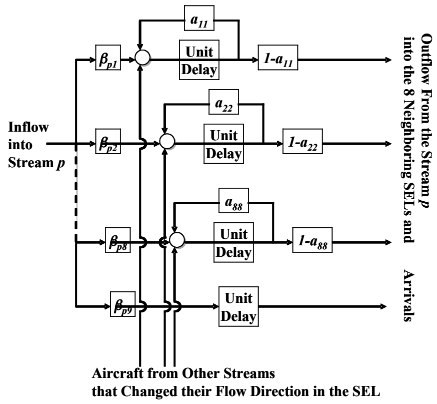

Figure 5: Eulerian model of an air traffic stream in a Surface Element

The departure traffic may be subdivided according to those airports where they will be controlled by a ground delay program, and where they will not. It is assumed that external traffic q^{exo} cannot be controlled directly. If the controlled inputs are combined into a vector $v(k)$, and all other inputs are collected together into a disturbance vector $w(k)$, the dynamic equation for the airspace is of the form:

$$x(k+1) = A(k)x(k) + B_1 v(k) + B_2 w(k) \tag{13}$$

The state vector $x(k)$ can be initialized using traffic data and then propagated forward in time. These equations can be used to facilitate analysis and synthesis of flow control strategies.

Typically, not all states are of interest for analysis or for flow control. An output equation can be formulated to isolate the variables of interest as:

$$y(k) = C(k)x(k) + D_1 v(k) \tag{14}$$

The Eulerian air traffic flow model consists of the time-varying difference equation for the state vector, and the time-varying algebraic equation for the output vector. These equations can be formulated for surface elements in any desired region of the NAS, and combined together to form a basis for analysis and flow-control system design. Eulerian models are then derived by examining traffic flows over a specified sample time interval into and in between the surface elements. These models are then used for analysis and flow control system design. It has been shown in (Menon, *et al.*, 2002; Menon, *et al.*, 2003) that the Eulerian models can be used to carry out a variety of analyses on the air traffic flow, such as controllability, reachability and model decentralization. Automatic derivation of Eulerian models from air traffic data has also been discussed (Menon, *et al.*, 2004).

An important application of the Eulerian models is in development of quantitative decision support tools for air traffic flow control. Recent research (Menon, *et al.*, 2004) has explored the application of the model-predictive control technique (MPC) to the air traffic flow control problem.

4. COMPARISON OF THE MODELS

Both models describe traffic flow as a system of time-varying linear difference equations. As such the main strength of the models described here is that all the tools available for analysis of linear dynamic systems can be applied to these models. The size of the linear models is independent of the number of aircraft in the system and depends on the number of spatial elements representing the airspace. Table1 compares the overall features of the two modelling approaches. The aggregation approach (Sridhar, *et al*., 2004) provides a lower order, fixed-resolution model of the airspace, while the Eulerian approach provides a flexible resolution model.

Table 1: Comparison between the two models

Model Characteristics	Aggregate Model	Eulerian Model
Spatial element	Center	Eight-connected SEL
Number of neighbors (Traffic Flow Directions)	Dictated by the airspace connectivity	Up to 8 neighbors
Inputs	Departure from airports in the Center	Departure from airports in the SEL, metering controls
States	Number of aircraft in the Center	Number of aircraft each stream
Transition matrix	Fixed-order, 23×23	8 × (Number of SEL)
Outputs	Number of aircraft in the ARTCC	Number of aircraft in each stream
Flexibility of model resolution	Fixed spatial resolution	Variable resolution
Model type	IIR	FIR
Weather	Center-level	SEL level
Flow Control	Center-level	SEL level

Each of the model may be useful in different aspects of strategic flow control decision making. Moreover, although there are differences in the way they approach the airspace modeling problem, they can each be used to verify the accuracy of the other model.

5. CONCULDING REMARKS

This paper discussed two different models for describing aircraft traffic flow. The first approach uses the airspace layout in terms of Air Route Traffic Control Centers as the basic modelling element, while the second approach represents the airspace using eight-connected latitude-longitude grid. Both approaches produce systems of time varying linear discrete-time dynamic equations. These can be used to carry out air traffic flow analysis and to design strategic flow control algorithms. Future research will focus on deriving decision support tools for strategic flow control using these models.

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