REACHABILITY ANALYSIS OF SWITCHED LINEAR SYSTEMS WITH SWITCHING/INPUT CONSTRAINTS

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Abstract: In this note, we address the reachability of switched linear systems with switching/input constraints. We prove that, under a mild assumption of the feasible switching signals, the reachability set is the reachable subspace of the unconstrained switched system. We also investigate the local reachability for switched systems with input constraints and present a complete criterion for a general class of systems.

Keywords: Switched linear systems, Switching signal, Reachability, Switching constraints, Input constraints, Directed graph

1. INTRODUCTION

A switched linear system is a hybrid dynamic system which switches at different time instants among a finite set of linear time-invariant subsystems. Switched linear systems have attracted increasingly more attention in the past few years. The literature grew exponentially and quite a few new ideas and powerful tools have been developed from various disciplines. The reader is referred to (Decarlo *et al.* 2000, Sun & Ge 2005) for surveys of recent development.

A fundamental pre-requisite for the design of feedback control systems is full knowledge about the structural properties of the switched systems under consideration. These properties are closely related to the concepts of controllability, observability and stability which are of fundamental importance in the literature of control. For controllability and reachability of switched linear systems, much work has been done (Stanford & Conner 1980, Ezzine & Haddad 1989, Sun & Zheng 2001, Sun, Ge, & Lee 2002, Yang 2002, Xie & Wang 2003). In particular, complete geometric and equivalent algebraic criteria have been presented in (Sun, Ge, & Lee 2002, Gurvits 2002). In these works, however, the switching signal and the control input are assumed to be designed in an arbitrary way without constraints. That is, we do not impose any restriction on the possible way of switching and control. In many practical situations, this is not the case. For example, in workshops, the order of the activated subsystems is pre-assigned rather than arbitrarily assigned. In this case, for instance, we must first active subsystem 1, then switch to subsystem 2, then subsystem 3, etc. This fixed sequence imposes a restriction on the switching signal. Another example is the control input which is subject to certain saturation that imposes a restriction on the control input.

In this note, we discuss several kinds of restrictions and their possible influences in the reachability of the systems. As both the switching signal and the system input are control variables, the constraints may impose on the switching law, or the control input, or both. Here, we focus on the following cases: the switching signal is constrained by a direct graph, and/or the control input is subject to saturations. Complete reachability criteria are presented for the constrained switched linear systems under mild assumptions.

2. PRELIMINARIES

Let $M = \{1, \cdots, m\}$ be an index set.

Consider the switched linear control system given by

$$\sum (A_i, B_i)_M : \qquad \dot{x}(t) = A_\sigma x(t) + B_\sigma u(t), \quad (1)$$

where $x \in \mathbf{R}^n$ is the state, $u \in \mathbf{R}^p$ is the piecewise continuous input, $\sigma \in M$ is the switching signal, and A_i and B_i are real constant matrices with compatible dimensions.

In the sequel, we briefly review some existing results which will be used in the later derivations.

Let $\phi(t; t_0, x_0, u, \sigma)$ denote the state trajectory at time t of switched system (1) starting from $x(t_0) = x_0$ with input u and switching signal σ.

Let $\mathcal{S}$ be the allowed set of switching signals, and $\mathcal{U}$ be the allowed set of inputs. The reachable set of system (1) under $\mathcal{S}$ and $\mathcal{U}$ is the set of states which are attainable from the origin in a finite time by appropriate choices of the inputs and switching signals in the allowed sets.

Definition 2.1. The reachable set of system (1) at time $T > 0$ starting from x under $\mathcal{S}$ and $\mathcal{U}$, denoted $R(x, T, \mathcal{U}, \mathcal{S})_M$, is the set

$$\{\phi(T; 0, x, u, \sigma) : u \in \mathcal{U}, \ \sigma \in \mathcal{S}\}.$$

System (1) is said to be (completely) T-reachable under $\mathcal{S}$ and $\mathcal{U}$, if

$$R(x, T, \mathcal{U}, \mathcal{S})_M = \mathbf{R}^n, \ \forall \ T > 0, \ x \in \mathbf{R}^n.$$

This notion of reachability implies the conventional controllability, reachability and small-time controllability/reachability.

Another concept is the local reachability. Local reachability means the ability to steer the system from an initial point to its nearby (local) states by means of the feasible switching signals and inputs.

Definition 2.2. Switched linear system (1) is locally T-reachable at x_0 under $\mathcal{S}$ and $\mathcal{U}$, if x_0 is an interior point of set $R(x_0, T, \mathcal{U}, \mathcal{S})_M$.

Recall that the reachable set of matrix pair (A, B) is the minimal A-invariant subspace that contains the image space of B. This criterion has been extended to switched system (1) without constraints where the allowed set of switching signals is

$$\mathcal{S}_0 \stackrel{def}{=} \{\sigma : [0, \infty) \to M \ \text{ piecewise constant}\},$$

and the allowed set of inputs is

$$\mathcal{U}_0 \stackrel{def}{=} \{u : [0, \infty) \to \mathbf{R}^p \text{ piecewise continuous}\}.$$

In fact, let $\mathcal{V}(A_i, B_i)_M$ be the minimum subspace of $\mathbf{R}^n$ which is invariant under all A_i, $i \in M$ and contains all the image spaces of B_i, $i \in M$. This subspace can be obtained recursively by

$$\mathcal{V}_1 = \sum_{i \in M} \Im B_i,$$
$$\mathcal{V}_{j+1} = \mathcal{V}_j + \sum_{i \in M} \sum_{k=1}^{n-1} A_i^k \mathcal{V}_j, \ j = 1, 2, \cdots \quad (2)$$

and we have

$$\mathcal{V}(A_i, B_i)_M = \mathcal{V}_n = \sum_{i_0, \cdots, i_{n-1} \in M}^{j_1, \cdots, j_{n-1} = 0, \cdots, n-1} A_{i_{n-1}}^{j_{n-1}} \cdots A_{i_1}^{j_1} \Im B_{i_0}, \quad (3)$$

where $\Im B$ denotes the image space of B.

Lemma 2.1. (Sun, Ge, & Lee 2002) For the unconstrained switched linear system (1), the reachable set is precisely the subspace $\mathcal{V}(A_i, B_i)_M$.

We thus refer to $\mathcal{V}(A_i, B_i)_M$ as the reachable subspace of system (1).

In (Sun, Ge, & Lee 2002), a constructive procedure is provided to compute appropriate switching signal and control input that steer the switched system from an initial state x_0 at $t = 0$ to any target state x_f at $t = T$ in the reachable subspace. Here we briefly recall part of the procedure which will be used later.

First, let $l = \sum_{k=0}^{n-1} m(mn)^k - 1$, and define the cyclic index sequence

$$\begin{aligned} & i_0 = 1, \quad i_1 = 2, \quad \cdots, \quad i_{m-1} = m, \\ & i_m = 1, \quad i_{m+1} = 2, \quad \cdots, \quad i_{2m-1} = m, \\ & \cdots \\ & i_{l-m+1} = 1, \quad i_{l-m+1} = 2, \quad i_l = m, \end{aligned} \quad (4)$$

and fix a time sequence

$$0 = t_0 < t_1 < \cdots < t_l < t_{l+1} = T.$$

Then, we have

$$\begin{aligned} & e^{A_{i_l} h_l} \cdots e^{A_2 h_1} < A_1 | B_1 > + \cdots \\ & + e^{A_{i_l} h_l} < A_{i_{l-1}} | B_{i_{l-1}} > + < A_{i_l} | B_{i_l} > \\ & = \mathcal{V}(A_i, B_i)_M, \end{aligned} \quad (5)$$

where $< A|B >= \Im[B, AB, \cdots, A^{n-1}B]$ denotes the reachable subspace of pair (A, B), and $h_j = t_{j+1} - t_j$ for $j = 0, \cdots, l$.

Second, solve the linear equation

$$x_f - e^{A_l h_l} \cdots e^{A_1 h_0} x_0 =$$
$$e^{A_l h_l} \cdots e^{A_2 h_1} \int_{t_0}^{t_1} e^{A_1(t_1-\tau)} B_1 B_1^T e^{A_1^T(t_1-t)} d\tau a_1$$
$$+ \cdots +$$
$$\int_{t_l}^{t_{l+1}} e^{A_{i_l}(t_{l+1}-\tau)} B_{i_l} B_{i_l}^T e^{A_{i_l}^T(t_{l+1}-t)} d\tau a_{l+1} \qquad (6)$$

for $a_1, \cdots, a_{l+1} \in \mathbf{R}^n$. Let

$$W_t^k = \int_0^t e^{A_k(t-\tau)} B_k B_k^T e^{A_k^T(t-\tau)} d\tau,$$

and

$$a = [a_1^T, \cdots, a_{l+1}^T]^T.$$

(6) is equivalent to

$$x_f - e^{A_l h_l} \cdots e^{A_1 h_0} x_0 = [e^{A_{i_l} h_l} \cdots e^{A_2 h_1} W_{h_0}^1, \cdots, e^{A_{i_l} h_l} W_{h_{l-1}}^{i_{l-1}}, W_{h_l}^{i_l}] a. \qquad (7)$$

According to (Sun, Ge, & Lee 2002), this equation has at least has one solution.

Third, suppose $a_0 = [a_{0,1}^T, \cdots, a_{0,l+1}^T]^T$ is a solution of equation (7). Define the control input as

$$u(t) = B_{i_k}^T e^{A_{i_k}^T(t_{k+1}-t)} a_{0,k+1}, \quad t_k \le t < t_{k+1}, \; k = 0, 1, \cdots, l \qquad (8)$$

and the switching signal as

$$\sigma(t) = i_k, \text{ for } t \in [t_k, t_{k+1}), \; k = 0, 1, \cdots, l. \; (9)$$

Then, we have $x_f = x(T; 0, x_0, u, \sigma)$.

For the development of the main results, we need the following lemma which was presented in (Sun, Ge, & Lee 2002).

Lemma 2.2. For any given matrices $A_k \in \mathbf{R}^{n\times n}$ and $B_k \in \mathbf{R}^{n\times p_k}$, $k = 1, 2$, inequality

$$\text{rank}[A_1 e^{A_2 t} B_1, B_2] \ge \text{ rank}[A_1 B_1, B_2] \quad (10)$$

holds for almost all $t \in \mathbf{R}$.

3. REACHABILITY UNDER RESTRICTED SWITCHING SIGNALS

In reality, the switching transition is usually governed by a logic-based switching device. The switching logic can be generated either by an automata or by a directed graph. Here, we assume that the switching transition sequence (i.e., the sequence of switching index) is governed by a directed graph.

Suppose G is a directed graph composed of the set M of nodes, and a set of directed arcs N, where $N \subseteq M \times M$. Thus, $G = (M, N)$ denotes the allowed switchings from one subsystem to others. That is, for any $k \in M$, the (possibly empty) set

$$N_k = \{i \in M \colon (k, i) \in N\}$$

defines the allowed subsystem indices following the kth subsystem. In other words, a switching index sequence (k, i) with $i \notin N_k$ is prohibited. Accordingly, if N is a strict subset of $M \times M$, then the directed graph impose a nontrivial restriction on the switching index sequence and hence on the switching signal. Let $\mathcal{S}_G$ denote the set of switching signals obeying the restriction, that is, including each switching signal where any two consecutive switching indices belongs to N.

Note that in the above scheme, we do not impose any restriction on the switching time sequence. That is, the duration between any consecutive switching instants are arbitrarily chosen by the designer.

Given any sequence $i_1, \cdots, i_l$, we say the sequence generates the set Ł if $i_j \in$ Ł for any $j = 1, \cdots, l$ and each element in Ł appears at least once in the sequence.

A directed graph $G = (M, N)$ is said to permit a loop sequence $\{k_1, \cdots, k_s, k_1\}$ if $(k_j, k_{j+1}) \in N$ for $j = 1, \cdots, s-1$ and $(k_s, k_1) \in N$.

Note that the loop $\{k_1, \cdots, k_s, k_1\}$ means that the cyclic index sequence

$$k_1, \cdots, k_s, k_1, \cdots, k_s, \cdots$$

is an allowed sequence under G.

The following theorem presents a sufficient condition for complete reachability of the constrained switched linear systems.

Theorem 3.1. Suppose that directed graph G permits a loop sequence which generates the set Ł. If switched system $\sum(A_i, B_i)_{Ł}$ is completely reachable, then we have

$$R(x, T, \mathcal{U}_0, \mathcal{S}_G)_M = \mathbf{R}^n, \; \forall \; T > 0, \; x \in \mathbf{R}^n (11)$$

which means that switched system $\sum(A_i, B_i)_M$ is completely reachable under graph G.

Proof. We consider a switching signal with the cyclic switching index sequence

$$k_1, \cdots, k_s, \cdots, k_1, \cdots, k_s. \qquad (12)$$

The switching time sequence $0, t_1, \cdots, t_l$ is to be designed later.

Note that this switching signal is in the allowed switching set $\mathcal{S}_G$.

Let $t_f > t_l$. Simple analysis exhibits that the reachable set at t_f via the switching signal is

$$\begin{aligned}\mathcal{R}(t_f) = & e^{A_{k_s} h_l} \cdots e^{A_{k_2} h_1} < A_{k_1} | B_{k_1} > + \cdots \\ & + e^{A_{k_s} h_l} < A_{k_{s-1}} | B_{k_{s-1}} > + < A_{k_s}, B_{k_s} >,\end{aligned}$$

where $h_j = t_{j+1} - t_j,\ j = 0, 1, \cdots, l-1$ and $h_l = t_f - t_l$.

Arrange a permutation of $\{i_1, \cdots, i_j\}$ of Ł such that $i_1, \cdots, i_j$ is a subsequence of $k_1, \cdots, k_s$. That is, there is a natural number sequence $1 \leq l_1 < l_2 < \cdots < l_j \leq s$, such that

$$i_\nu = k_{l_\nu},\ \nu = 1, \cdots, j.$$

It is clear that the cyclic index sequence

$$i_1, \cdots, i_j, \cdots, i_1, \cdots, i_j$$

is a subsequence of the index sequence (12). Denote the corresponding duration index subsequence of $h_0, \cdots, h_l$ to be $\tau_1, \cdots, \tau_\mu$. Applying Lemma 2.2 successfully, we have

$$\begin{aligned}& \dim(e^{A_{k_s} h_l} \cdots e^{A_{k_2} h_1} < A_{k_1} | B_{k_1} > + \cdots \\ & + e^{A_{k_s} h_l} < A_{k_{s-1}} | B_{k_{s-1}} > + < A_{k_s}, B_{k_s} >) \\ & \geq \dim(e^{A_{i_j} \tau_\mu} \cdots e^{A_{i_2} \tau_2} < A_{i_1} | B_{i_2} > + \cdots \\ & + e^{A_{i_j} \tau_\mu} < A_{i_{j-1}} | B_{i_{j-1}} > + < A_{i_j} | B_{i_j} >)\end{aligned}$$

for almost all $h_0, \cdots, h_l$. By (5), we have

$$\begin{aligned}& e^{A_{i_j} \tau_\mu} \cdots e^{A_{i_2} \tau_2} < A_{i_1} | B_{i_1} > + \cdots \\ & + e^{A_{i_j} \tau_\mu} < A_{i_{j-1}} | B_{i_{j-1}} > + < A_{i_j} | B_{i_j} > \\ & = \mathcal{V}(A_i, B_i)_{\text{Ł}} = \mathbf{R}^n.\end{aligned}$$

This means that complete reachability can be achieved via a single switching signal with index sequence (12). This fact clearly leads to the conclusion.

Corollary 3.1. Suppose that directed graph G permits a loop sequence that generates the set M. Then, for any $T > 0$ and $x \in \mathbf{R}^n$, we have

$$R(x, T, \mathcal{U}_0, \mathcal{S}_G)_M = \mathcal{V}(A_i, B_i)_M.$$

Proof. It is clear that the set $R(x, T, \mathcal{U}_0, \mathcal{S}_G)_M$ is a subset of $\mathcal{V}(A_i, B_i)_M$. On the other hand, from the proof of Theorem 3.1, we have

$$\dim \mathcal{R}_G(*) \geq \dim \mathcal{V}(A_i, B_i)_M,$$

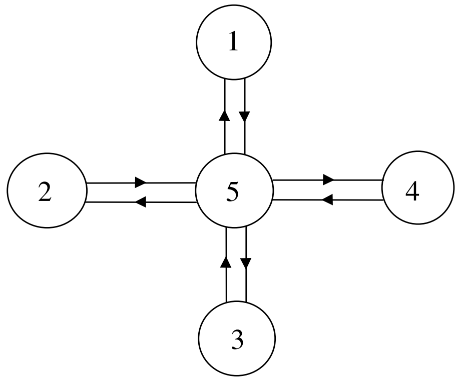

Fig. 1. The schematic of the directed graph with $m = 5$

where $\mathcal{R}_G(*) \subseteq \mathcal{R}_G(A_i, B_i)_M$ is the reachable set of switched system along an allowed switching signal under graph G. As

$$\mathcal{R}_G(*) \subseteq R(x, T, \mathcal{U}_0, \mathcal{S}_G)_M,$$

This means that the two sets $R(x, T, \mathcal{U}_0, \mathcal{S}_G)_M$ and $\mathcal{V}(A_i, B_i)_M$ coincide with each other.

This corollary provides an important information for the reachability of the switched linear system under the restricted switching logic. Indeed, according to the corollary, assume that the directed graph permits a loop sequence that generates the index set M, then the switched system possesses the complete reachability under the graph. The assumption is very mild and could be met in many practical situations. For example, in a workshop, there need m working procedures to produce a product. Suppose procedure sequence is cyclic among $1, \cdots, m-1$, but between any two procedures the mth procedure applies. This corresponds to the directed graph (with $m = 5$) depicted in Figure 1. It can be seen that a loop sequence is

$$1, m, 2, m, \cdots, m-1, m, 1$$

which generates the set $M = \{1, \cdots, m\}$. According to Corollary 3.1, the reachable set under the graph is exactly the full reachable subspace of the unconstrained switched system.

A special but very interesting case is that the subsystems are divided into different groups and transitions within a group are forbidden. That means that only transitions among different groups are allowed. A typical example is a production workshop with a set of procedures each of which could be implemented in several alternative ways. In this case, the switching is severely restricted as only transition between the groups (the procedures) is allowed. However, a loop sequence that generates the total index set always exists in this case, hence the reachable set coincides with the reachable subspace of the unconstrained system.

Corollary 3.1 could be further extended to more general cases. For example, for a switched system with a restriction on switching signals which is not necessarily described by a directed graph, we can prove that, the reachable set under the restriction coincides with the reachable subspace of the unconstrained system, provided that, there exists an allowed switching index sequence where each individual index appears sufficiently many times.

4. REACHABILITY UNDER RESTRICTED INPUTS

In this section, we consider the local reachability at the origin of the switched linear system with the input constraint

$$\dot{x}(t) = A_\sigma x(t) + B_\sigma u(t),\ u(t) \in U, \qquad (13)$$

where U is a set in $\mathbf{R}^n$ containing the origin as an interior.

Note that the system model reflects many practical situations where the control input is subject to hard constraints such as the saturation or force/enegy limitation. A typical example is that U is a bounded closed convex set. However, here we do not require that the set is convex or closed. The only intrinsic assumption is that the origin is an interior point of the set.

By means of the reachability criterion presented in Lemma 2.1, we are able to prove the following criterion.

Theorem 4.1. The constrained switched linear system (13) is locally reachable at the origin if and only if

$$\mathcal{V}(A_i, B_i)_M = \mathbf{R}^n. \qquad (14)$$

Proof. The necessity (only if) can be easily proven by contradiction. Indeed, the violation of (14) means that the unconstrained switched system is not locally reachable at the origin. This implies that the constrained switched system is not locally reachable at the origin.

To prove the sufficiency, we need to recall some formulas in Section 2.

Suppose the unconstrained switched system is completely reachable. Then, from any initial state the unconstrained system can be steered to the target state by means of the piecewise continuous control strategy (8), where the constant vectors satisfy (7), that is, given any $T > 0$, x_0 and x_f, there exist a cyclic index sequence

$$1, \cdots, m, \cdots, 1, \cdots, m$$

and a duration sequence $h_0, \cdots, h_l$, such that

$$x_f - e^{A_l h_l} \cdots e^{A_1 h_0} x_0 = [e^{A_{i_l} h_l} \cdots e^{A_2 h_1} W^1_{h_0}, \cdots, e^{A_{i_l} h_l} W^{i_{l-1}}_{h_{l-1}}, W^{i_l}_{h_l}] a,$$

where $a = [a_1^T, \cdots, a_{l+1}^T]^T$ and $\sum_{k=1}^l h_k < T$.

Let $x_0 = 0$ and define matrix

$$L = [e^{A_{i_l} h_l} \cdots e^{A_{i_1} h_1} W^1_{h_0}, \cdots, e^{A_{i_l} h_l} W^{i_{l-1}}_{h_{l-1}}, W^{i_l}_{h_l}].$$

The assumption (14) indicates that the linear operate

$$\Theta : \mathbf{R}^{(l+1)n} \to \mathbf{R}^n, \quad \Theta(a) \overset{def}{=} La.$$

is onto. This, together with the linearity of the operator, implies that, for any set W in $\mathbf{R}^{(l+1)n}$ containing the origin as an interior, the set

$$\Theta(W) = \{\Theta(a) : a \in W\}$$

also contains the origin as an interior in $\mathbf{R}^n$. On the other hand, from (8), it can be seen that the set of allowed constant vectors

$$\{a = [a_1^T, \cdots, a_{l+1}^T]^T : u(t) \in U, \forall\ t \in [0, \sum_{k=1}^l h_k]\}$$

contains the origin as an interior point in $\mathbf{R}^{(l+1)n}$.

The above analysis exhibits that, for the switched linear system with the input constraint, the reachable set $R(0, \sum_{k=0}^l h_k, \mathcal{U}, \mathcal{S}_0)_M$ at the origin contains the origin as an interior point.

Finally, note that, for any $T_1 \le T_2$, we have

$$R(0, T_1, \mathcal{U}, \mathcal{S}_0)_M \subseteq R(0, T_2, \mathcal{U}, \mathcal{S}_0)_M. \qquad (15)$$

Indeed, suppose $x \in R(0, T_1, \mathcal{U}, \mathcal{S}_0)_M$, then, there is a switching signal $\sigma(\cdot)$ and input $u(\cdot)$ defined on $[0, T_1]$, with $u \in \mathcal{U}$, such that

$$x = \phi(0, T_1, u, \sigma).$$

Now, define another switching signal σ' and input u' on $[0, T_2]$ by

$$\sigma'(t) = \begin{cases} \sigma(0) & t \in [0, T_2 - T_1), \\ \sigma(t + T_1 - T_2) & \text{otherwise}, \end{cases}$$

and

$$u'(t) = \begin{cases} 0 & t \in [0, T_2 - T_1), \\ u(t + T_1 - T_2) & \text{otherwise}. \end{cases}$$

It can be seen that

$$\phi(0, T_2, u', \sigma') = \phi(0, T_1, u, \sigma) = x.$$

As a result, (15) holds.

The above reasonings shows that, for any $T > 0$, the set $R(0, T, \mathcal{U}, \mathcal{S}_0)_M$ at the origin contains the origin as an interior point.

The theorem asserts that the constrained switched system is locally reachable at the origin if the unconstrained switched system is completely reachable. Note that, in the proof of the theorem, the design of input and the design of the switching signal is decoupled in the following sense. In the proof, what we need is an index sequence $i_0, \cdots, i_l$, and a duration sequence $h_0, \cdots, h_l$, such that

$$\begin{aligned}
&e^{A_{i_l} h_l} \cdots e^{A_{i_1} h_1} < A_{i_0} | B_{i_0} > + \cdots \\
&+ e^{A_{i_l} h_l} < A_{i_{l-1}} | B_{i_{l-1}} > + < A_{i_l} | B_{i_l} > \\
&= \mathcal{V}(A_i, B_i)_M.
\end{aligned}$$

Any sequence $i_0, \cdots, i_l$ with this property suffices the purpose of proving the theorem. This observation, together with the proof of Theorem 3.1, indicates that the more general conclusion can be made.

Theorem 4.2. Suppose that directed graph G permits a loop sequence which generates the set M, and U is a set in $\mathbf{R}^n$ containing the origin as an interior. Then, the constrained switched system

$$\dot{x}(t) = A_\sigma x(t) + B_\sigma u(t), \ u(t) \in U, \ \sigma \in \mathcal{S}_G$$

is locally reachable at the origin, if and only if the unconstrained switched system is completely reachable.

5. CONCLUDING REMARKS

In this note, several reachability criteria have been presented for switched linear systems under certain switching/input constraints. We proved that, for a wide class of switching logics, the reachable sets under the switching constraints are in fact the reachable subspaces of the unconstrained switched systems. This means that, the reachability keeps unaffected even if the switching rules are severely restricted in a certain sense. In addition, if the control input is subject to some constraint, the switched system is still locally reachable provided that the unconstrained switched system is completely reachable.

Note that there are other kinds of switching/input constraints which we did not discuss in the note. For example, if the transition of the switching depends the on-line state variable, the reachability analysis may be very involved (Bemporad, Ferrari-Trecate, & Morari 2000). As for input constraints, a typical example is the force-free switched system where no input imposes on the system at all. For such systems, the only design variable is the switching signal. As the origin is always an equilibrium of the system under any switching signal, the origin itself forms an invariant set of the system. As an implication, the force-free system is not locally reachable at the origin. However, by means of the switching signal, it is still possible that the system is locally reachable at other states away from the origin (Xu & Antsaklis 1999, Cheng & Chen 2003). This is a very interesting open subject for further investigation.

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