

# FRICION IDENTIFICATION WITH GENETIC ALGORITHMS

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Identification of the function part of the analytical model of the movement of a mass along vertical guide way is made. Indeed, knowledge of friction parameters is necessary to detect wear or mechanical failure. Because of the non linearity the identification is made using Genetic Algorithms (GA). After many simulations, the proposed approach is tested with the recorded experimental data from a test bed. *Copyright © 2005 IFAC*

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## 1. INTRODUCTION

System identification is the process of deriving a mathematical model from observed data in accordance with a selected criterion. In the past few decades, many researchers have extensively studied various identification methods for linear systems (Ljung, 1999). Many friction models are parametric models. They try to represent the complexity of physic phenomena. However, friction is still one of the great unknowns in mechanical systems. Beside that, friction model is not linear (Olsson, Astrom, Canudas de Wit, 1998). Friction model identification can be made using various methods (Besançon-Voda Besançon, 1999) such as: non linear estimation, (Elhami, Brookfield, 1996), linear and non linear regression or dynamical network (Parlitz, Al-Bender, Fassois, Wong, 2004), neural networks (Domínguez, Michelin, Martinez, 1995). In this paper genetic algorithm (GA) are used in batch processing to find the best estimated values of the friction model parameters. GA are particularly effective for optimization of non linear function. The advantage of the use of GA to identify friction parameters is that velocity measure (or estimation) is not required. The paper is organized as follow: in section 2, the selected friction model is presented. The identification method is presented in section 3. Simulations are presented in section 4. Some practical results concerning an actual plant are given in section 5.

## 2. FRICTION MODEL

Friction is the tangential force between two surfaces in contact. It's a very complex physical phenomenon that varies with chemical and physical properties of materials. Indeed, dynamic, thermal and lubrication conditions strongly modify the friction force. Hence the construction of a general friction model is a difficult task (Richardson, Nolle, 1976).

The classical models of friction can be classified in two categories: static models and dynamic models (Canudas de Witt, Lischinsky, 1997). In this work static model is used.

Let  $y$  be the position of a mass moving along a surface and let  $v$  denote the velocity of this mass. The force friction has the form:

$$F(v) = (F_c + (F_s - F_c) \cdot e^{-\frac{v}{v_s}}) \cdot \text{sgn}(v) + F_v \cdot v \quad (1)$$

With:  $F_s$ : Stribeck friction,  $v_s$ : Stribeck velocity,  $F_c$ : Coulomb friction,  $F_v$ : viscous friction coefficient  
Force friction depends on 4 parameters:  $\alpha_1, \dots, \alpha_4$ .

$$F(v) = (\alpha_1 + \alpha_2 \cdot e^{-\frac{v}{\alpha_3}}) \cdot \text{sgn}(v) + \alpha_4 \cdot v \quad (2)$$

With the following interpretation of parameters:

$$\alpha_1 = F_c, \quad \alpha_2 = F_s - F_c, \quad \alpha_3 = v_s, \quad \alpha_4 = F_v.$$

The sign function is defined as:

$$\text{sgn}(v) = 1 \text{ for } v > 0, \quad 0 \text{ for } v = 0, \quad -1 \text{ for } v < 0$$

Figure 1 shows an example of force friction (N) respect to the linear velocity (m/s). This model is symmetric, but some authors use an asymmetrical friction model (Elhami, Brookfield, 1996).

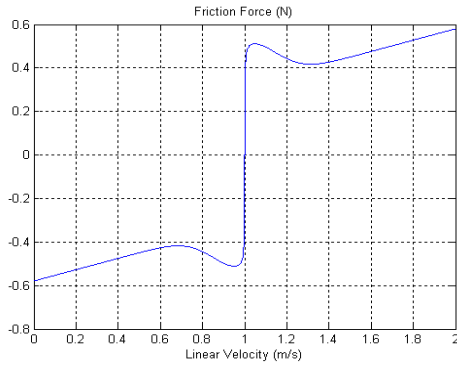


Fig. 1. Example of friction force

### 3. IDENTIFICATION WITH GA

#### 3.1 Identification with GA

The goal of identification is then to determine the numerical values of the parameters  $\alpha_1, \dots, \alpha_4$ . System identification is an experimental approach for determining the parameter of the model. Typically, identification includes four steps (Ljung 1999):

- a) apply an input and record the measures,
- b) choice a model,
- c) estimate the parameters,
- d) validate the identified parameters.

There exist many methods to perform estimation (step c) when the output is linear in parameter (Pintelon, Schoukens, 2001). For example, estimation could be made using spectral analysis, linear regression, Kalman filter, etc. But the friction model is not linear in the parameters because of  $\alpha_3$ . The relation (2) shows that the model is not linear in velocity. Hence, parameter identification is treated as an optimization problem with the use of GA (Masseguerra, Zio, Torri, 2003). GA are different from conventional optimization methods because they start with multiple points, so they are more likely to obtain global solution. GA are search techniques based on the principle of natural selection. GA suppose that the potential solution of any problem is an individual and can be represented by a finite set of parameters. Here, GA are used to identify the friction of a system as is shown in figure 2. Friction parameters of the model are modified by GA in order to reduce the values of the output error.

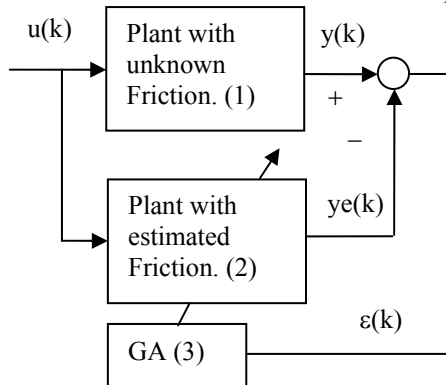


Fig. 2. Identification method

The following notations are used:

$u(k)$ : process input,  $y(k)$ : process measured output,

$ye(k)$ : estimated output by the model

$\varepsilon(k) = y(k) - ye(k)$  : output error.

In figure 2, the bloc (2) could be either simulated either could be represented by a data base recorded on the actual plant. To simulate the bloc (1), a Simulink© model with the friction parameter numerical values given by GA is used.

#### 3.2 Coding

As chromosomes, each parameter is coded in a binary form (8 bits or genes). GA work with the code of parameters and not with the parameters themselves. For example, considering the Coulomb friction  $\alpha_1 = F_c$ , suppose that the minimal value is  $\alpha_{1min} = 0$  N and the maximal value is  $\alpha_{1max} = 40$  N .

Noting E the value of the encoded parameter  $\alpha_1$ , the value of the parameter is given by:

$$\alpha_1 = \alpha_{1min} + \frac{\alpha_{1max} - \alpha_{1min}}{2^8 - 1} \cdot E \quad (3)$$

This is a key advantage of GA: we can incorporate specific knowledge in a straightforward fashion.

The 4 codes of the 4 parameters are collected to form an individual coded with 32 bits.

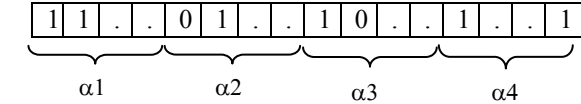


Fig. 3. Coding 4 parameters

#### 3.3 Initial Population

A key part of GA is derived from the fact that the initial population is build with several individuals chosen at random. For example, initial population has between 20 and 100 individuals. These individuals are called parents.

#### 3.4 Criteria

The goal is to reduce the output error:  $\varepsilon(k) = y(k) - ye(k)$ . In figure 2, the output error is computed with a Simulink© model having the parameters modified by GA at each iteration. The criterion I to minimize is defined as:

$$I = \frac{\sum_{k=1}^n \varepsilon^2(k)}{\sum_{k=1}^n y^2(k)} \quad (4)$$

Where  $n$  is the number of time points simulated or recorded at sampling time  $T_s$ . Because of non linearity, the sampling time has to be chosen very small. In this criterion, the denominator is build with the measured or simulated output of the machine. Knowing the friction parameter values, the calculus of the criteria I for a fixed window is called "evaluation". The length of the windows  $n$  is about 1500 to 2500 points, depending on the experiments.

### 3.5 Selection Reproduction

GA is based on three basic genetic rules in analogy to natural evolution: selection (or reproduction), crossover, mutation. Knowing the parents, the criterion  $I$  is valued for each parents. The individual that gives the greater criteria will be eliminated. But this can drive to a local minimum. Hence, only one parent is eliminated and is replaced by the parent that gives the smaller criteria.

### 3.6 Crossover

Crossover method is inspired by biological process. The aim is to combine parts of good parents to generate better children. The crossover procedure implements the exchange mechanism between two parents. For each parent, for each parameter coded, random integers  $L1$ ,  $L2$ ,  $L3$ , and  $L4$  are chosen between 1 and 7. These numbers are crossing point: they are generated using a selected "crossover rate". For example in figure 4 we have two parents, and two parameters coded with 5 bits (genes). Before crossing, suppose that  $L1=2$  and  $L2=4$  are determined by the crossover rate. For the first parameter, the first parent gives 2 bits for the "child" (1,1) and the following bits of the first parent are replaced by those (1, 1, 0) of the second parent. This procedure gives the "child" for the first parameter. The same procedure is applied for the others parameters. This is illustrated by the figure 4.

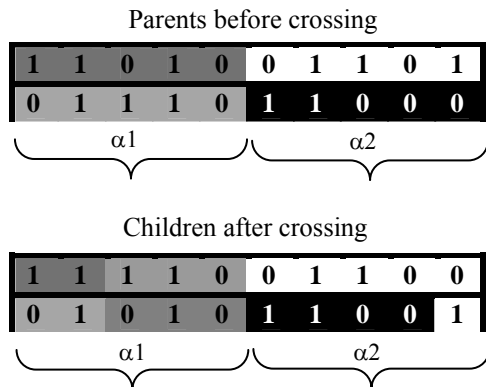


Fig. 4. Crossing procedure

### 3.7 Mutation

Mutation consists in changing the value of one or several bits (genes). Each bit has a mutation probability  $Pm$  very small, for example,  $Pm=0.05$ . Hence, mutation provides the random search capability for GA. It helps the GA avoid premature convergence and find the global optimal solution.

### 3.8 Implementation and comments

The flow chart is given in figure 5. At the first step, the data base is loaded. It contains the data recorded on the plant and data obtained by simulation.

At the second step parents are initialized at random, and each parent is valued. These values are used to simulate the plant (with Simulink©) and the criterion  $I$  is calculated (using (4)) over a window of length  $n \cdot Ts$ . If the criterion  $I$  is too big, the main loop begins.

The minimum of the selected parents is compared to the predefined value called  $Max$ . If this minimum is less or equal than  $Max$ , the corresponding individual (4 parameters  $\alpha_1, \dots, \alpha_4$ ) is put in the population. This is a slight improve of classical GA implementation: it allows keeping the best individual known at this iteration. In addition, selection, crossover and mutation are developed. This gives new population. If the minimum of the evaluation of this population is less than a previous  $optim$ , this minimum is kept as a new  $optim$ , and the integer  $mu$  is increased by one. This  $mu$  is a way to increase  $Pm$  and thus to introduce diversity into the population. Then local minima are avoided. The main loop is stopped when the criterion is smaller than  $Max$ .

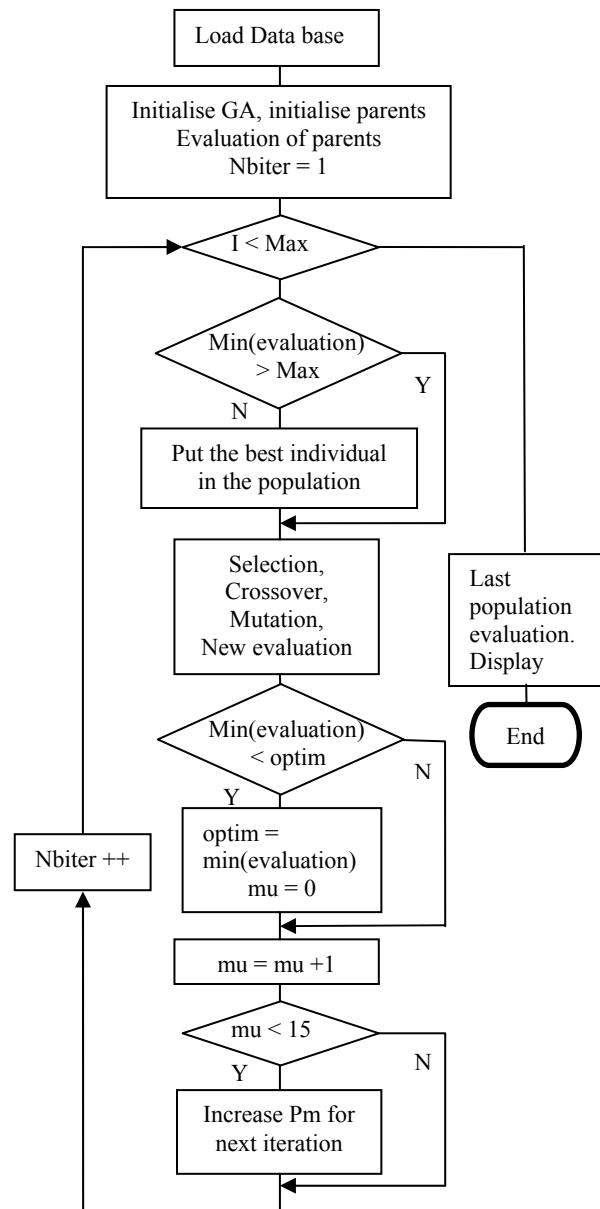


Fig. 5. Flow chart of GA identification

GA approach gives a good balance between complexity and efficiency. Unlike classical methods, GA do not require the knowledge of gradients or derivatives that are sensitive to noise sensors of actual process. GA automates the trial and error evaluation process. Therefore GA could be used for a very large variety of models. GA have some disadvantages: among them, the difficulty to find the exact global optimum. This usually requires a large initial population and a large number of iterations.

## 4. SIMULATION

### 4.1 Experimental set-up

At each extremity, a moving beam is linked to two belts driven by two brushless motors. The height of the vertical guide is 2 meters and the length of the beam is 1.8 meter. In this paper, only one extremity is moving up and down along rail guides.



Fig. 6. Plant process

Position of the extremity of the moving beam is obtained from a magnetostrictive sensor.

The measurement and control are carried out by a dSpace© real time computer. During the experiment, inputs (torques) and outputs (positions) are recorded with sampling time  $T_s = 0.01s$ .

### 4.2 Plant model

The mass  $M$  moves on a vertical guide way rail witch presents friction: it represents a part of the mass of the beam of fig 6. This mass is subject to gravity force and to the force exerted by a belt. This belt is engaged in two pulleys. One of these pulleys is driven by a brushless motor associated with a reducing gear: see figure 6.

Introduce the notations (see fig 7):

$R_c$ ,  $R_e$ ,  $R_m$ : pulley radius,  $M$ : mass in translation,  
 $C_r$ : input torque,  $C_m$ : torque provided by the motor,  
 $J$ : total inertia expressed with respect to the driving pulley axis,  
 $y$ : vertical position of the mass,  $g$ : gravity.

The plant is represented by the following equation:

$$\ddot{y}_1 = \frac{R_c}{J_e} \cdot \rho \cdot C_m - \frac{M \cdot R_c^2}{J_e} \cdot g - \frac{R_c^2}{J_e} \times F(\dot{y}) \quad (5)$$

With,  $\rho = \frac{R_c}{R_m}$  and  $J_e = M \times R_c^2 + J$

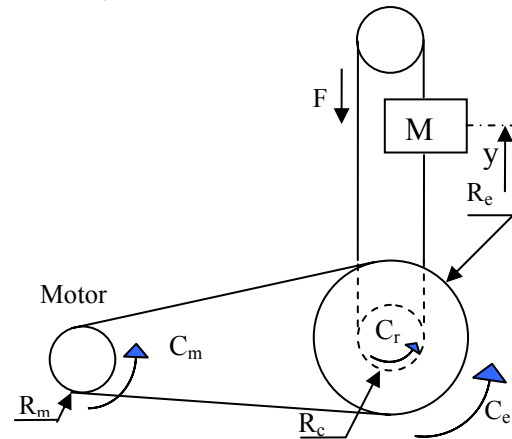


Fig. 7. Plant

### 4.3 Simulink model

Figure 8 shows the Simulink© model. The input is a torque (N·m) given by the motor and the output is the position (m) of the mass.

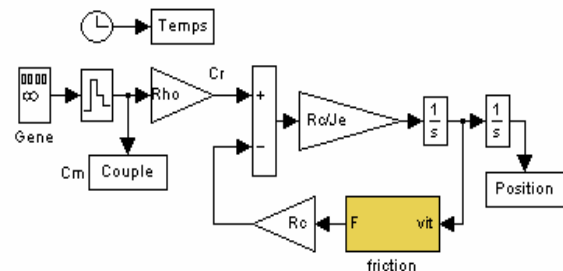


Fig. 8. Simulink model

The following values have been chosen because they are next to the plant friction value:  $F_c = 90 \text{ N}$ ,  $F_s = 60 \text{ N}$ ,  $F_v = 180 \text{ N/(m/s)}$ ,  $v_s = 0.1 \text{ m/s}$ .

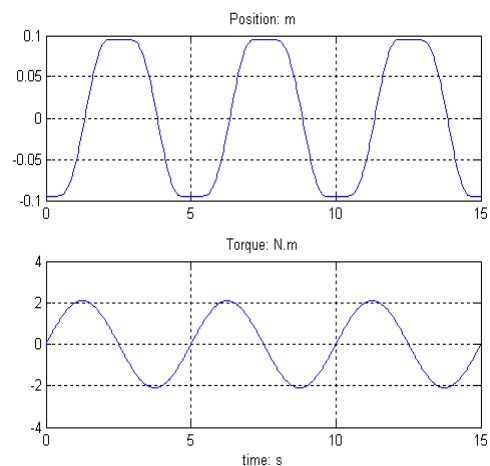


Fig. 9. Simulation results

For a sinusoidal torque with amplitude 2.1 N and frequency 0.2 Hz, the simulation is given in Fig 9.

Because of the non linearity of the friction, the output is periodic, but it is not sinusoidal. Each simulated position and torque are recorded in a data base used by GA to obtain parameters  $F_c$ ,  $F_s$ ,  $F_v$  and  $v_j$ . Sampling time value is  $T_s=0.01s$  and  $n=1500$ .

#### 4.4 Numerical values

For these simulations, each parameter  $\alpha$ ,  $\alpha_{\max} = 1.5 \cdot \alpha$  and  $\alpha_{\min} = 0.5 \cdot \alpha$  where  $\alpha$  is the parameter nominal value. After many simulations the following design has been selected:

- Initial population with 50 individuals,
- Mutation probability 0.03,
- Crossover rate = 0.5.

Figure 10 shows the evolution of the best criteria (top). Note that different parameter values could correspond to very close values of the criteria. When the optimum parameters are obtained, the output error (see fig 2) is plotted versus time.

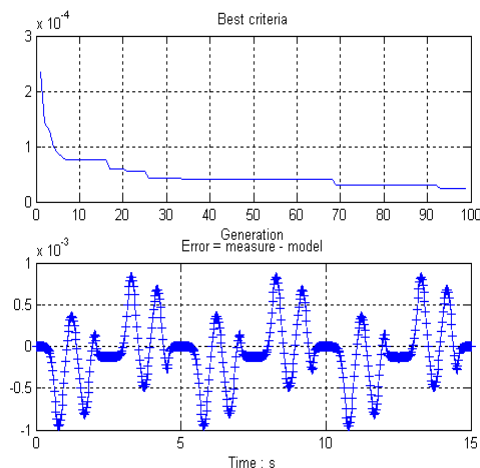


Fig. 10. Best criteria and error

The optimum parameters obtained are:  $F_c=93N$ ,  $F_s=69N$ ,  $v_s=0.12m/s$  and  $F_v=172N$ . The output error is less than one millimeter; the precision is quite good. This shows the efficiency of the identification with GA.

Figure 11 shows the parameters evolution versus the generation iterations. Due to random search algorithm, this evolution is rather chaotic.

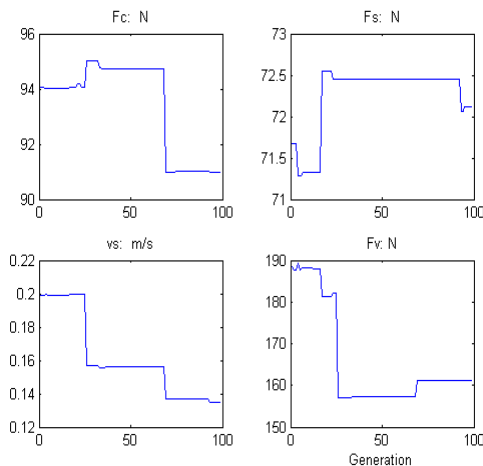


Fig. 11. Parameters evolution

#### 4.5 Input choice

When the model is linear in parameters, it is important to note that the input should be informative enough with respect to the model (Ljung 1999).

To obtain precise identification, the inputs have to be chosen very carefully. But, due to non linearity, is difficult to select a “good” input. Roughly speaking, the input have to produce  $\max(F_v, v)$  close to the  $F_c$  value. However, the output is constrained by physical limits of the machine. Input (torque) amplitude is chosen to obtain displacements about  $\pm 0.2$  m. GA are running with numerical values of §4.4. Different inputs have been tested and some comparisons are given in table 1. The symbol ++ means precise identification, and -- stands for very bad identification.

Table 1

Form	Freq (Hz)	Criteria I	quality
Sin	0.1	$9 \cdot 10^{-5}$	++
sin	0.2	$4 \cdot 10^{-5}$	++
sin	0.5	$8 \cdot 10^{-6}$	++
square	0.05	$7 \cdot 10^{-5}$	--
square	0.1	$3 \cdot 10^{-4}$	--
square	0.2	$2 \cdot 10^{-3}$	--
random		2.3	--

Some other forms have been tested; among them saw tooth, piecewise constant sequence of variable length, superposition of two squares at different frequency. But the identified parameters are not good. In conclusion, for the experiments on the system, the selected form of the input is a sine.

#### 4.6 Noise

Output process is corrupted by noise. Assume that this noise is zero mean Gaussian random signal with variance  $\sigma^2=10^{-6} m^2$ . Results of figure 12 have to be compared with figure 10. The output error reflects the noise influence. Identified parameters are very similar to those obtained without noise.

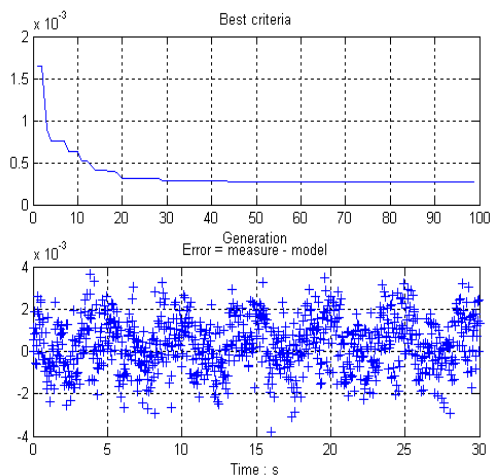


Fig. 12. Simulation with noise

## 5. EXPERIMENTS

The input is selected using the simulation results: the torque is a sine. Before identification, is necessary to filter input and output in order to reduce noise effects. Spectral estimation of the displacement (Fast Fourier transform) shows that a cut frequency  $f=2\text{Hz}$  is a good choice. To eliminate gravity effects, torque and displacement have been centered. In figure 13 filtered input (torque,  $\text{N}\cdot\text{m}$ ) and output (position,  $\text{m}$ ) are presented. As the figure shows, the position is not very symmetrical.

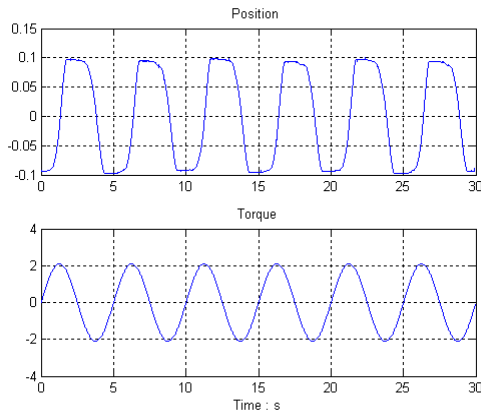


Fig. 13. Filtered position and torque

From this data base, a window of 2000 points is selected. Identification of friction parameters have been made using model (5) with the hypothesis that the inertia  $J$  is constant, which is a very strong hypothesis. For the GA, the same design parameters as in §4.4 were used. Each identified parameter is coded with 8 bits. Because of noise and model uncertainty, coding with 10 bits or more doesn't improve the accuracy on identified parameters. Many inputs were tested (periodic or not): same conclusions for simulations and for experiments are drawn, thus sinusoidal input is the best here.

### Results

The optimum parameters given by GA are:  $F_c = 91\text{N}$ ,  $F_s = 69\text{N}$ ,  $v_j = 0.1\text{m/s}$  and  $F_v = 150\text{N/(m/s)}$ . Figure 14 shows the measured and the simulated output with the "best" identified parameters.

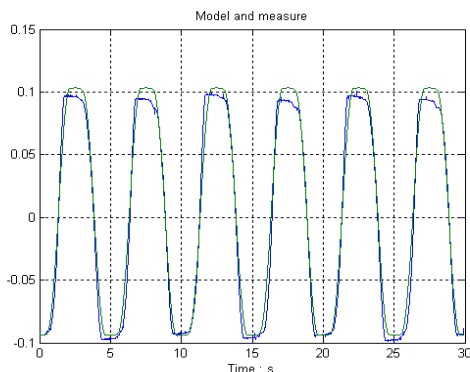


Fig. 14. Measured and estimated position with optimum parameters

Many experiments have been made and the conclusion is that friction is not very repetitive in this test bed. So, in order to control the position, it seems necessary to use adaptive control.

## 6. CONCLUSION

This paper has described a practical use of GA for identification of friction of a mass moving along guide ways. The need of a "good input" is pointed out. Identification based on GA gives good results. Indeed, the criterion chosen is only depending on the measured quantities and it is not depending on the velocity. The performance of the method was demonstrated by applying it to a simulated system and to an experimental test bed.

Friction identification is a very important task because of the knowledge of the parameters describing friction could be use for many purpose as for example:

- Non linear control for precision machine,
- Surface damage detection,
- Wear detection,
- Lubricant modification detection, etc.

Friction modeling is and will still be a challenge in the future.

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