A DECENTRALIZED PROBABILISTIC FRAMEWORK FOR THE PATH PLANNING OF AUTONOMOUS VEHICLES

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Abstract: A three dimensional probabilistic approach for the path planning of uninhabited air vehicles (UAVs) is presented. The algorithm can be used in real time because of a low computational load in spite of the fact that it finds a path in three dimensions. The paths are locally optimal and are feasible for the UAV to follow by keeping the turn angle within a certain maximum limit. For this purpose, a relation has been derived that transforms the maximum turn angle into a maximum search angle. The UAVs are prevented from flying at very low altitudes to avoid crashing into the ground. Because of limited fuel, a compromise is made between risk and fuel consumption by limiting the height and search angle.

Keywords: UAV, Autonomy, Path Planning, Local Minimisation, Probabilistic Approach

1. INTRODUCTION

Uninhabited air vehicles (UAVs) are likely to become increasingly important in the 21st century (Pachter and Chandler, 1998; Bortoff, 1999; McLain, 1999; Chandler *et al.*, 2002). UAVs are advantageous over piloted counterparts in terms of manoeuvrability, low human risk, low cost and light weight. Examples of the wide use of UAVs for civil, military and commercial applications include weather and atmospheric monitoring, emergency communications, telecommunications, border patrol, and battlefield deployment, etc. The ever increasing interest in UAVs demands higher levels of autonomous behaviour. Among the many open issues in the development of autonomous, intelligent UAVs, flight path planning, or trajectory selection, is of crucial importance. To develop an algorithm for such a task is a challenging problem. The algorithm must compute a stealthy path which steers the vehicle away from potential dangers. The path selected should be optimal in a certain sense as well as feasible. The algorithm must be fast enough for real time use in an uncertain environment, and efficient in memory and computational demand so that it can be run on airborne processors.

The problem of path planning has been studied for decades in a variety of different contexts and tackled with various approaches. Path planners are generally divided into local and global planners. The former group work in on-line mode while most of the latter group work off-line. A global path planner needs to know everything about the system and its environment. A clear disadvantage of this is that a replanning is necessary each time the environment changes, which often happens in a real situation. There is a tendency therefore to design local path planners for vehicle autonomy (Borenstein and Koren, 1991; Elnagar and Base, 1993). A local path planner does not suffer from the above disadvantage, although such a planner would lead to the loss of global optimal-

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ity. In this paper, we present a method for path planning which is based on local optimisations and may take into account flight constraints such as turning angles. The algorithm can be used to select a flight path for each UAV in a group of UAVs on a collaborative mission. Each vehicle has its own processor and applies the algorithm to find its path with consideration of the other vehicles' positions and movements. This kind of algorithm is thus called decentralized. The formulation and example discussed below will, however, concern one vehicle only, due to the length limit of the paper.
This paper is organised as follows: Section 2 describes the problem and models for estimating risk. Section 3 addresses a local probabilistic minimisation approach and its application to path planning. An example is shown in Section 4 with simulation results and discussions. Section 5 concludes the paper.

2. PROBLEM FORMULATION AND RISK MODELLING

The problem under consideration is to find a "safe"path for a UAV to fly from a starting point P_0 to a target point P_T. Suppose that the UAV moves at a constant speed and let $p(x, y, z)$ be the risk at position (x, y, z). The path selected is a sequence of points in 3-dimensions (waypoints) obtained by minimising the cost function

$$J = \int_0^T p(x, y, z)dt \tag{1}$$

over all points (x, y, z). The overall UAV risk function is complex due to the influence of different factors and can be modelled adequately in a probabilistic framework as described below.

Probability of Hit

For each defence unit (radar and SAM) aiming at a UAV, there is a hit probability. Within a given range, this probability depends on the position of the UAV. This can be calculated by a function of height (h) and distance (d) of the UAV from SAM site and is given by (GARTEUR Action Group AG14, 2003)

$$p_k(h, d) = (1 - \mathtt{SS}(\mathtt{d}, \mathtt{R_{s,m,l}}, \mathtt{s_{k_1}})) \cdot \mathtt{SS}(\mathtt{d}, 0.1.\mathtt{R_{s,m,l}}, s_{k_2}) \cdot \mathtt{SS}(\arcsin(\mathtt{h}/\mathtt{d}), \gamma, \mathtt{s_{k_3}}) \tag{2}$$

where

$$\mathtt{SS}(\mathtt{x}, \mathtt{x_0}, \mathtt{s}) = \frac{1}{2}\left(\frac{\mathtt{x} - \mathtt{x_0}}{\sqrt{\mathtt{s}^2 + (\mathtt{x} - \mathtt{x_0})^2}}\right)$$

and γ is the lower coverage angle of the radar and s_{k_i} the softness of the step function SS. $R_{s,m,l}$ is the range of the missile which may be short, medium or long. The hit probability for long range

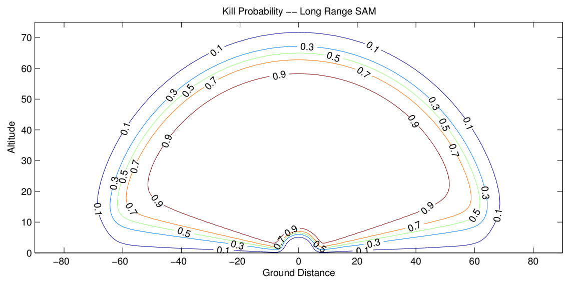

Fig. 1. Hit probability of long range SAM

SAM ($R = 65$ km) is shown in Figure 1 for $s_{k_1} = 12$, $s_{k_2} = 2.4$, $s_{k_3} = 0.1$, with numbers on different contours indicating the hit probability.

Probability of Destruction

If the UAV is within the reach of M SAM sites, the hit probability is increased by possible co-operation such as alternating radar transmission or choice of launch site. This effect is modelled by evaluating the hit probability of all covering SAM sites $p_k^j(h, d)$ (the worst case) and using the relation

$$p_{des}(h, d) = 1 - \prod_{j=1}^{M}(1 - p_k^j(h, d))$$

Probability of Crash

When a UAV flies at very low altitude, there is a possibility of crashing into ground objects like trees or hills. So in order to prevent all UAVs from flying around at zero altitude, a crash probability can be modelled as

$$p_{cr}(h) = 1 - \mathtt{SS}(\mathtt{h}, \mathtt{h_{cr}}, \mathtt{s_{cr}})$$

where h_{cr} is the nominal critical height and each UAV is forced to fly above this height for safety reasons. s_{cr} is the softness parameter of the above probability function which can be tuned according to the situation. This softness parameter is used to relax or strictly follow the critical height.

Probability of Survival

Due to small sizes, UAVs have limited fuel capacity. Typically, flight times of UAVs are 30 (min). The risk decreases with increasing height after a critical height but on the hand fuel decreases. So a compromise can be made between risk and fuel consumption by limiting the height.This effect can be modelled as survival probability p_{sur} similar to the crash probability.

Probability of Collision

When the mission involves a group of UAVs, risk of collision with other vehicles is a function of the distance of the vehicle from other UAVs and can be modelled as

$$p_{co}(D) = 1 - \text{softStep}(D, d_{co}, s_{co})$$

Where d_{co} is the safety distance to avoid collision.

Probability of Risk
The overall risk probability can be calculated due to all the above factors as

$$p(h,d) = 1 - [(1-p_{cr})\prod_j(1-p_k^j)] \; . \quad (3)$$

Note that the collision probability is not included in (3).

3. PROBABILISTIC LOCAL MINIMISATION ALGORITHM

The algorithm is based upon a search for a local minimum on a disc whose centre passes through the line of sight of the target from the current point and is also perpendicular to that line. The radius of the disc is decided from the maximum search angle, which in turn can be decided from the maximum turn angle. The disc is divided into sectors by a suitable number of lines all passing through its centre as shown in Figure 2 below. The search is carried out along these lines. The distance of the search disc from the current point is chosen according to the type and range of the search sensor mounted on the UAV and all points on the disc should be within the range of the search sensors. The maximum search angle could be different for different search lines in the disk. This is because the maximum climb rate may be different in different directions. To find the coordinates of each point on the disc, it is necessary to specify a known reference line lying in the disc and passing through its centre. The reference line (RL) is selected to be parallel to the horizontal plane. For the two dimensional case, the search is limited to the reference line only but in three dimension, we have to search the whole disc along various lines.
Consider such a disc having centre at a point $P_c(x_c, y_c, z_c)$, which is at a distance h from the current point $P_i(x_i, y_i, z_i)$ on the line of sight of the target having coordinates $P_T(x_T, y_T, z_T)$ as shown in Figure 2. For each line in the disc, the search is initialised from the point $P_c(x_c, y_c, z_c)$ and moves on either side of the line with equal small steps; it is limited by the maximum search angle for that line.The equation of the plane containing the disc and $P(x, y, z)$ representing any point on the plane is given by

$$a(x-x_c) + b(y-y_c) + c(z-z_c) = 0 \quad (4)$$

where

$$a = (x_T - x_i),\; b = (y_T - y_i),\; c = (z_T - z_i) \quad (5)$$

are the direction ratios of the line of sight of the target. Let $l, m, 0$ be the direction cosines of RL. We have

$$l^2 + m^2 = 1 \quad (6)$$

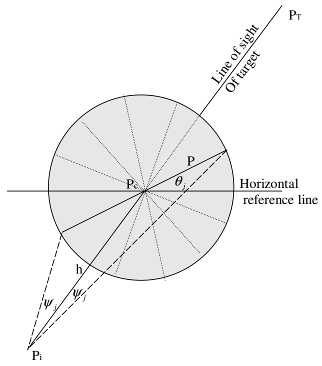

Fig. 2. Search Disk

Consider the j^{th} search line in the search disc which is an angle of θ_j from RL. Let $P(x, y, z)$ be the point on the line at a distance r from P_c and r is less than or equal to R_j which is the radius of the j^{th} search line. Then the equation of the j^{th} search line is

$$(x-x_c)l + (y-y_c)m = r\cos\theta_j \quad (7)$$

Since $P(x, y, z)$ and Pc are r distance apart

$$(x-x_c)^2 + (y-y_c)^2 + (z-z_c)^2 = r^2 \quad (8)$$

The reference line and the line of sight are perpendicular which implies

$$l \cdot a + m \cdot b = 0 \quad (9)$$

By solving Equations (6) and (9), we will get two sets of values representing two parallel unit vectors in the opposite direction. Select one set as

$$l = -\frac{b}{\sqrt{a^2+b^2}},\; m = \frac{a}{\sqrt{a^2+b^2}},\; n = 0$$

Hence Equation (7) becomes

$$-b(x-x_c) + a(y-y_c) = r\cos\theta_j\sqrt{a^2+b^2} \quad (10)$$

Furthermore, solving Equations (4), (10) and (8) for x, y and z, two points $P_L(x_L, y_L, z_L)$ and $P_R(x_R, y_R, z_R)$ will be obtained on either side of the search line at a distance r from the point P_c

$$\begin{aligned} x_L &= x_c - r dx \quad (x_R = x_c + r dx) \\ y_L &= y_c + r dy \quad (y_R = y_c - r dy) \\ z_L &= z_c + r dz \quad (z_R = z_c - r dz) \end{aligned} \quad (11)$$

where

$$\begin{aligned} dx &= \frac{b}{\sqrt{a^2+b^2}}\cos\theta_j + \frac{ac}{\sqrt{a^2+b^2}} \cdot \frac{\sin\theta_j}{\sqrt{a^2+b^2+c^2}} \\ dy &= \frac{a}{\sqrt{a^2+b^2}}\cos\theta_j - \frac{bc}{\sqrt{a^2+b^2}} \cdot \frac{\sin\theta_j}{\sqrt{a^2+b^2+c^2}} \end{aligned} \quad (12)$$

$$dz = \frac{\sqrt{a^2+b^2}}{\sqrt{a^2+b^2+c^2}} \sin\theta_j$$

When a local minimum point $P_m(x_m, y_m, z_m)$ is found after searching all the lines, the UAV will move in that direction by a distance h. From simulation, it has been observed that by moving exactly to the local minimum point along the chosen safe direction, the vehicle may find itself in a danger zone in the next iteration. Although the point is safe, it may be very near to the threat. So it is better to cover some shorter distance (in our case h) to have a safety margin. The next point of the path can be calculated as

$$\overrightarrow{OP}_{i+1} = \overrightarrow{OP}_i + h\hat{n}_{im}$$

where $\hat{n}_{im}$ is the unit vector in the direction of the local minimum $P_m(x_m, y_m, z_m)$ from point P_i and is calculated by

$$\hat{n}_{im} = \frac{[x_m - x_i, y_m - y_i, z_m - z_i]^T}{\sqrt{(x_m-x_i)^2 + (y_m-y_i)^2 + (z_m-z_i)^2}}$$

The coordinates of the next point are

$$\begin{aligned} x_{i+1} &= x_i + \frac{h}{d_{im}}(x_m - x_i) \\ y_{i+1} &= y_i + \frac{h}{d_{im}}(y_m - y_i) \\ z_{i+1} &= z_i + \frac{h}{d_{im}}(z_m - z_i) \end{aligned} \qquad (13)$$

where

$$d_{im} = \sqrt{(x_m-x_i)^2 + (y_m-y_i)^2 + (z_m-z_i)^2}$$

The process of local minimisation for the case of the circular disc and the strong dependence of the maximum turn angle on the search angle can be seen in Figure 3.

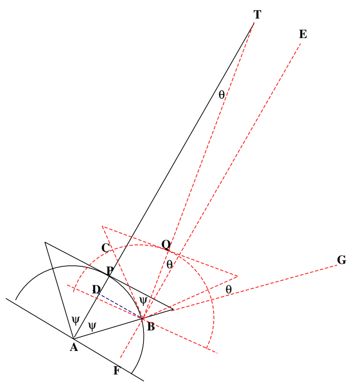

Fig. 3. Local minimisation and maximum turn angle

From Figure 3, the maximum turn angle β is

$$\beta = \angle CBG = 2\psi + \theta \qquad (14)$$

$$\tan\theta = \frac{BD}{AT - AD} = \frac{AB\sin\psi}{AT - AB\cos\psi} = \frac{h\sin\psi}{d - h\cos\psi}$$

$$\beta = 2\psi + \arctan\frac{h\sin\psi}{d - h\cos\psi} \qquad (15)$$

where h is the forward step size and d is the distance of the current point from the destination T. For constant h and ψ, the maximum turn angle depends on d only. When d is very large, then $\tan\theta \to 0 \Rightarrow \theta \to 0 \Rightarrow \beta \to 2\psi$. On the other hand β increases when the UAV approaches the targets.

The algorithm for path planning is described below. It requires information on the threats, starting point and target point. A few parameters should be chosen first including forward step size h and lateral step angle $\delta\psi$, disc search direction angles $[\theta_j]_{j=1}^{jmax}$, vector of maximum search angles along each search line $[\psi_j]_{j=1}^{jmax}$, risk threshold α, tolerance, etc..

ALGORITHM PROCEDURE:

Step 1 Initialise the path vector with the starting point and declare it the current point P_i and set the indices $i = 1$, $j = 1$, $d\psi = \delta\psi$.

Step 2 Find the distance $\sqrt{(x_T-x_i)^2 + (y_T-y_i)^2 + (z_T-z_i)^2}$ of the current point to the target. If this distance is less than the tolerance limit, go to Step 10. Otherwise go to the next step.

Step 3 Find the direction ratios a, b, c of the line of sight of the target using (5).

Step 4 Find a point P_c on the line of sight at a distance h from the current point P_i using (14) (P_m should be replaced by P_T) and calculate the risk value p_c at this point using the relation (3).

Step 5 If this value of the risk probability is less than the threshold α, then there is no need to worry about the safety of this point; add it to the path list, set $i = i + 1$, and go to Step 3. Otherwise declare it to be an expected point of the optimal path by assigning it to a point P_e and go to the next step.

Step 6 Set $r = h\tan d\psi$ and find two points P_L and P_R on the left and right side respectively of the point P_c at a search angle $\delta\psi$ along the j^{th} search line which makes an angle θ_j with the reference line using (11) and (13). Find the risk probabilities p_L and p_R at these points using the relation (3).

Step 7 If the risk p_e is less than or equal to the risks p_L and p_R, then go to Step 9. Otherwise go to the next step.

Step 8 If the risk probability p_L (p_R) is less than the risk p_e and also it is less than or equal to the risk p_R (p_L), then P_L (P_R) is the minimal risk point among these points and there is a chance of getting a further low risk point on the left (right) side. Repeat the following:

$$P_e = P_L \quad (P_e = P_R)$$
$$d\psi = d\psi + \delta\psi, \quad r = h\tan d\psi$$

by updating P_L (P_R) each time using (11) and (13) until one of the following occurs:

- search angle exceeds its maximum value which means $d\psi > \psi$
- a local minimum is found which means risk $p_e <$ risk p_L (p_R)

Step 9 If $j < j_{max}$, then assign point P_e to P_j and the risk p_e to p_j and set $j = j + 1$, $d\psi = \delta\psi$ and go to step 6. Otherwise find the minimum risk point $P_m(x_m, y_m, z_m)$ on the disk by comparing the minimum risk p_j found from all search lines. Find a new $(i+1)^{th}$ safe path point using (14) and set $i = i+1$. Add this point to the path list and also set $j = 1$, $d\psi = \delta\psi$. Go to Step 2

Step 10 Output the result in the form of

- Optimal path which consists of way points.
- Total and average risk on this path

4. SIMULATION RESULTS

An example is discussed in this section. We consider 23 defence units (threats) located over a 300 km×300 km region. The threats are assumed to be medium range SAM units with range varying from 15 km to 35 km. For ease of demonstration, the algorithm will first be applied to a problem of finding a path in 2 dimensions with the height fixed at 10 km. The starting point of the UAV is at $(10, 100, 10)$ and the destination at $(250, 240, 10)$. Different risk contours are drawn around each SAM unit with the inner most having a risk of 0.9 while the outermost is at risk of 0.1. The parameters of maximum lateral search angle, lateral search angle step, path step, and stopping tolerance are 60°, 5.7°, 1 km, 2 km, respectively. Three risk thresholds, 0.01, 0.5 and 1, are chosen to generate 3 paths as shown in Figure 4.

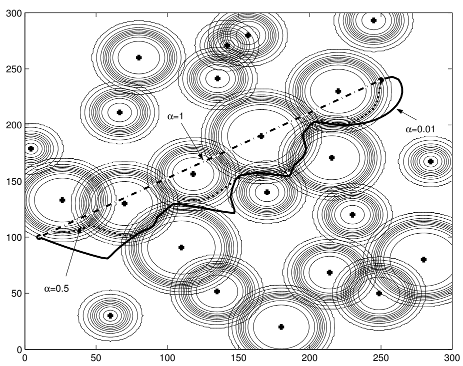

Fig. 4. Paths obtained for threshold, $\alpha = 0.01, 0.5, 1$

It is clear that when the value of risk threshold α increases, the path becomes shorter. For $\alpha = 1$, the path is a straight line. The target assignment feature of the algorithm can be explored by visiting more than one target in a specified order. Consider a UAV which starts at position $(10, 100, 10)$ and intends to visit a set of targets $(110, 270, 10)$, $(260, 200, 10)$, $(213, 38, 10)$, $(130, 160, 10)$. Call these points Targets $1, 2, 3, 4$, respectively. We are interested in finding a path to visit these targets and the effect on path and other parameters of taking different orders. The resultant paths for the first and fifth orders of target points (see Table 1) are shown in Figures 5 and 6. Distances and average risk estimations are summarised in Table 1. It is interesting to

Table 1. Data for different order of visit

Order	Targets				Distance (km)	Aver. Risk
1	1	2	3	4	924	19.48
2	4	1	2	3	767	23.52
3	1	4	3	2	838	20.45
4	2	1	4	3	914	16.02
5	3	4	1	2	845	16.85

note that in the first case when the vehicle is on its way to visit the second target, it is at one instant within the reaching range of three SAM units. This situation is difficult to avoid. In order to minimise the risk, the vehicle may have to fly with maximum speed to leave the zone quickly.

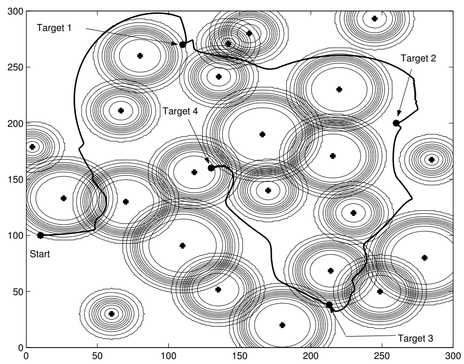

Fig. 5. Minimum risk path for order 1

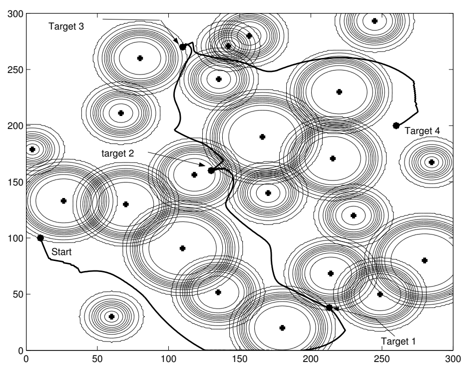

Fig. 6. Minimum risk path for order 5

For the 3-D case, we consider a multiple-target visit with all targets at the same altitude (10 km). Two search lines are used. One line on the search disk is parallel to the horizontal plane. The other one is perpendicular to the first one while remaining on the disk plane. The 5^{th} order of target visit in Table 1 is selected. The full version of the algorithm (i.e., 3-D) is applied. There is no restriction on the maximum flight altitude of the UAV. The path obtained is shown in Figure 7. Before the UAV reaches the first target point,

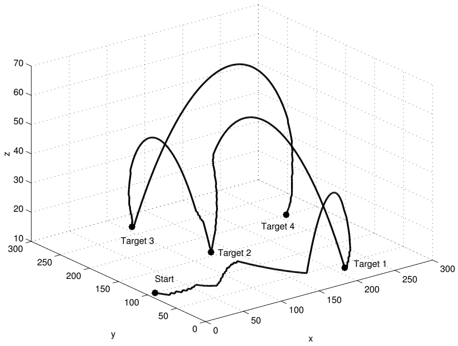

Fig. 7. 3-D Path for all targets without restriction on maximum height of UAV

it remains close to the horizontal plane of 10 km high. When it is near the first target, it starts climbing due to the risk from neighbouring threats. During the rest of the flight, the UAV always tries to fly high to escape from the dangers. It comes down only to reach the 10 km high target points. This choice is of course difficult to make in a real situation,due to limited fuel, climb rate and flight time constraints. For this reason, either height restriction or/and fuel consumption must be included in the cost function. With a maximum altitude set at 30 km, the path as shown in Figure 8 is obtained.

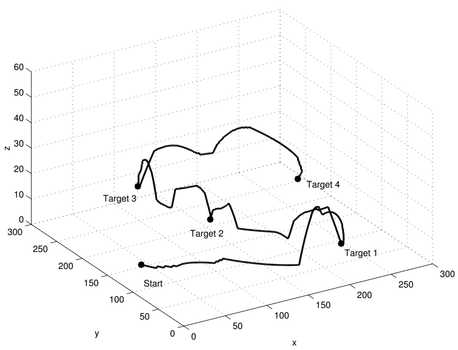

Fig. 8. Path obtained in $3D$ for all targets considering cost due to height

5. CONCLUSION AND FURTHER RESEARCH

A real time three dimensional probabilistic approach for the path planning of autonomous vehicles has been presented. The approach finds a safe path but also takes account of real world constraints. The problems of low fuel consumption, low altitude crashes have all been addressed. The paths are locally optimal and feasible for the UAV to follow by keeping the turn angle within some certain maximum limit. A relation has been derived that transforms the maximum turn angle into the maximum search angle. The algorithm is applied in decentralized mode, that is, each vehicle has its own processor and applies the algorithm to find its path with consideration of the collision with other vehicles. UAVs are prevented from flying at very low altitudes because of the danger of crashing into ground objects. Since each UAV has limited fuel, a compromise has been made between risk and fuel consumption by limiting the height and search angle.

One important observation of the proposed algorithm is that while evaluating the stealthy path for the UAV, every time vehicle achieved its target, which shows the convergence of the algorithm. This property can be utilised to explore the coordinated rendezvous aspect of the algorithm.

The algorithm produced good results despite its locally optimal nature. Optimality of the algorithm can be improved further by using it in combination with a globally optimal approach e.g., with the Voronoi diagrams approach.

6. ACKNOWLEDGEMENTS

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