# VIBRATION REDUCTION ABILITIES OF SOME JERK-CONTROLLED MOVEMENT LAWS FOR INDUSTRIAL MACHINES

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Abstract: Vibration-free positioning is a basic objective in industrial high-speed systems, i.e. systems for which axes are submitted to significant dynamical demands. The focus of this paper is on the pragmatic formalisation of the influence of some jerk-controlled movement laws on the residual vibrations. Analysis of limited-jerk, harmonic-jerk and minimum-jerk laws is conducted on a simplified axis drive model that accounts for axis control parameters and for predominant mode effects. Experimental measurements performed on industrial test-setups demonstrate the effectiveness of the proposed approach in estimating the evolution of the vibration level according to each movement law. 

Keywords: Jerk profiles, Residual vibrations, Movement law, Lumped constant models, Industrial machines

## 1. INTRODUCTION

One simple way to inhibit vibrations for industrial robots and CNC machine-tools is to employ mechanical components with high rigidity, but this is likely to increase the mass, with a resultant impairment of dynamic performances (Benning *et al.*, 1997). Another method, investigated in this paper, consists in acting directly on the movement law of each axis in order to take into account the estimated effects related to the dominating flexibilities.

The traditional movement laws with piecewise constant acceleration have discontinuities that regulators cannot follow, whatever the performances of the actuators. These discontinuities excite the structure in transitory stages and are responsible for a great part of the degradation of the dynamic behaviour. From the available parameters in modern CNCs, it is known that the maximum jerk value (per axis) can limit the oscillatory behaviour of the load (Bearee *et al.*, 2004). The jerk value represents the rate of change of acceleration, and for this reason it becomes possible to act on the smoothness degree of the movement. Thus, the use of a trajectory planning with piecewise constant jerk makes it possible to limit the level of the system oscillations, but on the other hand the theoretical movement time is inevitably increases (Yamamoto *et al.*, 1996). Confronted with this compromise, numerous works, mainly within the framework of robotics but also in the machine-tool field, deal with the optimisation of jerk-controlled trajectory (Jeon and Ha, 2000; Lee and Lin, 1998), and in particular deal with the realisation of minimum-jerk profiles (Hindle and


Fig. 1. Left: 3-axis Cartesian robot (max feedrate: 120 m.min$^{-1}$, max acceleration: 4 m.s$^{-2}$) ; Right: High dynamic machine-tool prototype (max feedrate: 100 m.min$^{-1}$, max acceleration: 20 m.s$^{-2}$, actuators: permanent magnet linear synchronous motors).

Singh, 2000; Piazzi and Visioli, 2000) supposed to reduce mechanical stresses and vibrations because of their similarity with the movement of human articulations (Harris, 2004).

The aim of this paper consists in concretely analysing the influence of some jerk-controlled movement laws on the vibratory behaviour of industrial positioning systems. The present study is limited to the case of Cartesian machines, for which movements are not coupled and can be carried out independently. After developing a simplified model of the axis vibratory error, the particular effects on residual vibrations of, respectively, limited-jerk, harmonic-jerk and minimum-jerk movement laws are derived and experimental validations are conducted on industrial test-setups.

## 2. DYNAMICAL AXIS DRIVE MODEL

### *2.1 Test-setup overview*

The experimental validations are carried out on two test-setup machines equipped with different architectures and dynamical feedforwarded. The robot was equipped with a real-time *dSPACE 1103* control card. The available measurements come from the actuator encoders of axis and a laser sensor directly measures the load position (the end-effector). The second test-setup is a prototype of high-speed machine (figure 1) whose structure is similar to that of a machine-tool. It is made of two orthogonal axes, driven by synchronous linear actuators and controlled by a *Num 1050* CNC unit.

### *2.2 Simplified dynamical error model*

This study aims at estimating the influence of the jerk profile on the dominating modes of vibration. Very often it is the fundamental mode that predominates. The dominating flexibility classically

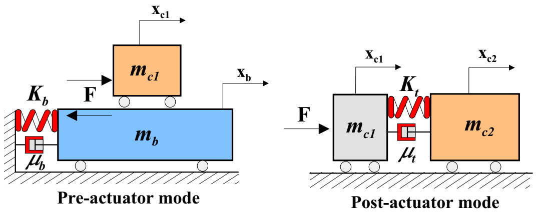


Fig. 2. Generic lumped constant models of pre-actuator and post-actuator modes.

originates from devices constituting mechanical transmission such as ball-screw or belt-pulley. The structural modes, which could reasonably be neglect before, sometimes become very sensitive. The influence of the structural modes is even further reinforced in the case of the linear actuators, since the parasitic signals are directly transmitted to the structure. Further, two families of dominating flexibilities are to be distinguished, (a) the mechanical drive mode (or post-actuator mode) and (b) the structural mode (or pre-actuator). Our aim is not to derive a complex mathematical model for close description of a particular system; instead one can achieve a much simpler, linear, physically explicable and generic model by following physical modelling techniques, as detailed in (Ellis, 2000; Bopearatchy and Hatanwala, 1990). This kind of model, described in figure 2 for the two families of modes, is very often sufficient to describe the influence of vibratory modes. Without loss of generalities, we only focus in this paper on the case of a predominant post-actuator mode, but the methodology, as well as the main results obtained, are extensible to the case of a base mode. According to the figure 2 notations, the equations of motion derived for the post-actuator mode lead to the following transfer functions expressed in continuous time domain

$$\frac{V_{c1}(s)}{F(s)} = \frac{1+\frac{2\zeta_n}{\omega_n}s+\frac{1}{\omega_n^2}s^2}{m_{tot}s\left(1+\frac{2\zeta_n}{\omega_n}s+\frac{1}{\omega_n^2(1+r)}s^2\right)};$$

$$\frac{V_{c2}(s)}{V_{c1}(s)} = \frac{1+\frac{2\zeta_n}{\omega_n}s}{1+\frac{2\zeta_n}{\omega_n}s+\frac{1}{\omega_n^2}s^2} \qquad (1)$$

with $m_{tot} = m_{c1} + m_{c2}, r = m_{c2}/m_{c1}, \omega_n = \sqrt{K_t/m_{c2}}, \zeta_n = \mu_t/(2\sqrt{K_t m_{c2}})$.

An industrial control is classically made up of a cascaded current, speed and position loops, regulated by PI controllers, as depicted in figure 3. A speed feeforward acts directly on the speed loops to compensate for the tracking error in permanent speed stages. In the following, one considers *ideal* speed and current loops, i.e. the load velocity is assumed to be the same as the reference velocity at all times. Posing $x_{ref}$ the reference movement and $x_{c2}$ the effective load displacement, the position transfer function between the expected and effective load positions takes the following form

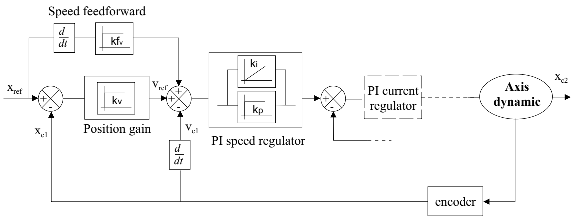


Fig. 3. Simplified structure model of an industrial axis drive control.

$$\frac{x_{c2}(s)}{x_{ref}(s)} = \frac{1+\frac{kf_v}{k_v}s}{1+\frac{1}{k_v}s}\,\frac{1+\frac{2\zeta_n}{\omega_n}s}{1+\frac{2\zeta_n}{\omega_n}s+\frac{1}{\omega_n^2}s^2} \qquad (2)$$

with $k_v$ and $kf_v$ the position and feedforward gains respectively.

The presence of discontinuities in the movement planning will tend to excite the flexible modes of the system and, consequently, will degrade the dynamical precision. The effective load displacement then differs from the ideal movement. The dynamical error of the movement is defined as

$$\varepsilon(t) = x_{ref}(t) - x_{c2}(t) \qquad (3)$$

Two types of errors constituting the dynamical error can be distinguished:

- *Aperiodical terms* representing the variations related to the tracking characteristics of the control structure, which are noted $\varepsilon_{ap}(t)$,
- *oscillatory periodical terms* related to the oscillating behaviour of the system, which are noted $\varepsilon_{vib}(t)$.

According to equation 2, the transfer functions governing the dynamical error evolution of the preceding axis model is given by

$$\frac{\varepsilon(s)}{x_{ref}(s)} = \frac{\frac{1-kf_v}{k_v}(s+\frac{2\zeta_n}{\omega_n}s^2)+\frac{1}{\omega_n^2}s^2+\frac{1}{k_v\omega_n^2}s^3}{(1+\frac{1}{k_v}s)(1+\frac{2\zeta_n}{\omega_n}s+\frac{1}{\omega_n^2}s^2)} \qquad (4)$$

In the next development, we are interested in the maximum vibratory error evolution related to the movement law used. Since our goal is not to calculate the amplitude of oscillation precisely, but to describe its parametric evolution according to the control and profile parameters, the damping of the predominating mode will be assumed to be null.

## 3. LIMITED-JERK MOVEMENT LAW

The limited-jerk movement law described in figure 4(b) is a succession of jerk steps. Posing $\mathbf{J} = [J, -J, -J, J, -J, J, J, -J]$ the vector of jerk step amplitude and $\mathbf{T} = [0, T_j, T_a, T_j, T_v, T_j, T_a, T_j]$ the vector of each jerk step time, the Laplace transform of the limited-jerk profile can be written as

$$x_{ref}(s) = \frac{1}{s^3}\sum_{i=1}^{n}\frac{J_i}{s}\,\mathrm{e}^{-\sum_{k=1}^{i}T_k s} \qquad (5)$$

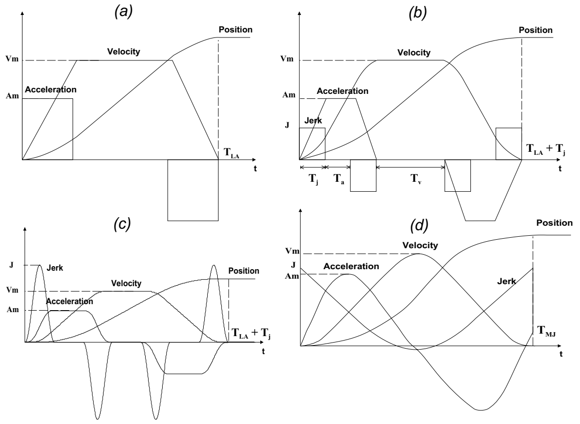


Fig. 4. Classical movement laws: (a) Limited-acceleration,(b) Limited-jerk, (c) Harmonic-jerk and (d) Minimum-jerk.

where $J_i$, $T_i$ denote the $i^{th}$ element of $\mathbf{J}$ and $\mathbf{T}$ vectors respectively, $n$ represents the jerk step considered ($n \leq 8$). The succession of jerk steps leads to linear and constant acceleration stages. These two types of movement stages will have a different influence on vibratory dynamics of the system. In previous works (Barre *et al.*, 2004) it has been demonstrated that the maximum vibratory error is mainly concerned with the constant acceleration stage. This error can be calculated by applying the constant-jerk profile (equation (5)) to the axis drive model described by equation (2). The resulting maximum amplitude of the vibratory error during the first constant acceleration stage ($n = 1$) can be expressed as

$$max|\varepsilon_{vib}(t)| = \frac{A}{\omega_n^2}\sqrt{\frac{1+kf_v^2\omega_n^2/k_v^2}{1+\omega_n^2/k_v^2}}\left|\mathrm{sinc}\left(\frac{T_j\omega_n}{2}\right)\right| \qquad (6)$$

with $\mathrm{sinc}(x) = \sin(x)/x$. Thus, in the particular case where $T_j$ is a multiple integer of the predominant mode period, there will be no residual vibration in the constant acceleration stage. Apart from these particular points, one finds, incidentally, that the higher the position loop gain, the more significant the vibratory error; and the maximum error for a given jerk is obtained in the case of a totally feedforwarded axis. It will also be noted that the maximum vibratory error is obtained while making $T_j$ tend towards nullity, which is the same as considering the case of the limited-acceleration law (figure 4(a)).

At the end of movement, the residual vibrations still follow the previously described evolution; only the error amplitude differs. According to the various phase times and damping factor, the error obtained with equation (6) can be weighted by a factor 8. Figure 5 shows the experimental results obtained on the two test-setups compared with the theoretical evolution. The results confirm the expected parametric evolution of the maximum

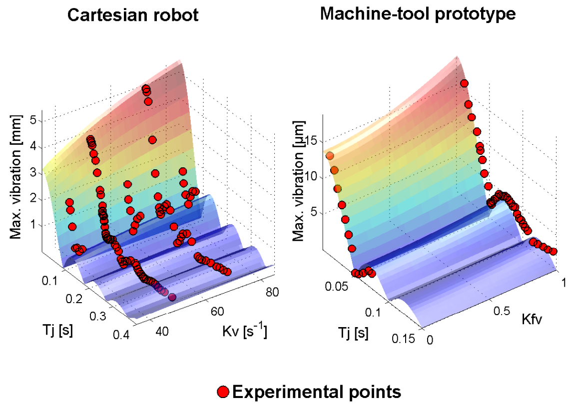


Fig. 5. Maximum residual vibration according to $T_j$, $k_v$ and $kf_v$ as compared to the predicted evolution.

residual vibration. Industrially, the maximum jerk value is tuned on each axis in an iterative manner in order to obtain an acceptable dynamic behaviour during a point-to-point movement type. Equation (6) shows that it is from now on possible to quantify *a priori* the maximum jerk value. The constant jerk stage times leading to an expected residual oscillation criterion can result from one simple measurement of the oscillation level of the concerned axis (see figure 5).

## 4. HARMONIC-JERK MOVEMENT LAW

Because of their easiness of implementation, the movement laws containing trigonometric functions are relatively widespread in the numerical control units; in particular the square sine harmonic laws, which make it possible to manage a smooth velocity profile. Within the framework of this article, based on the influence of the jerk profile, we want to compare the influence of the preceding constant-jerk profile with a jerk law of square sine type (figure 4(c)).

Noting $\mathbf{J} = [J, -J, -J, J]$ the vector of maximum jerk amplitude and $\mathbf{T} = [0, T_j+T_a, T_j+T_v, T_j+T_a]$ the vector of each jerk stage time, the resulting profile can be defined in the laplace domain as follows

$$x_{ref}(s) = \frac{1}{s^3}\left[\frac{2\pi^2}{4\pi^2 s + T_j s^3}\right]\sum_{i=1}^{n}\frac{J_i}{s}e^{-\sum_{k=1}^{i}T_k s} \tag{7}$$

with $n$ representing the jerk stage considered ($n \leq 4$). Following the same methodology as described in the previous section, the maximum vibratory error during the first jerk stage ($n = 1$) noted $[max|\varepsilon_{vib}(t)|]_{\mathrm{HJ}}$ is expressed as

$$[max|\varepsilon_{vib}(t)|]_{\mathrm{HJ}} = \frac{4\pi}{|4\pi^2 - T_j^2\omega_n^2|}[max|\varepsilon_{vib}(t)|]_{\mathrm{LJ}} \tag{8}$$

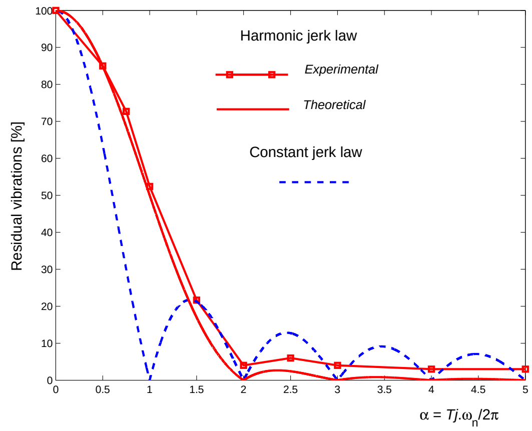


Fig. 6. Residual vibrations evolution for harmonic law according to $\alpha$.

where $[max|\varepsilon_{vib}(t)|]_{\mathrm{LJ}}$ represented the maximum error for the Limited-Jerk profile described in equation (6).

As in the limited-jerk case, if we calculate the residual vibrations ($n = 4$ in equation (7)), we find that the remaining oscillations still follow the evolution rule described by equation (8), but obviously the effective amplitudes are pondered by the neglected damping and by the different stage times. Figure 6 presents the evolution of the residual vibration, according to the ratio $\alpha$ of the jerk stage time and the natural pulsation of the system ($\alpha = 2\pi\omega_n/T_j$). The reference amplitude of 100% corresponds to the constant-acceleration law case. As confirmed by the experimental points obtained on the Cartesian robot, it is particularly notable that for the harmonic-jerk law, the jerk time cancelling the oscillations is at least twice the natural period of the dominating mode (see figure 7). If the frequency associated with the natural mode does not evolve during the movement, this kind of movement law unnecessarily lengthens the execution time, cancelling the residual oscillations as compared to the constant-jerk law. The main interest of this harmonic law lies in the strong attenuation of the amplitudes of residual oscillations obtained for the jerk times higher than twice the natural period (see figure 6). Thus, the trajectory planning will be less sensitive to the parametric variations, to the bias in the estimated dominating frequency or to the neglected modes.

## 5. MINIMUM-JERK MOVEMENT LAW

In the robotic field, the optimisation of the jerk profile is a widespread solution to reduce the effect of the natural modes. The minimum-jerk movement is the most used because it is known to provide a trajectory that is very similar to the motion of the human articulations. The minimum-jerk problem is then formulated as: find the func-

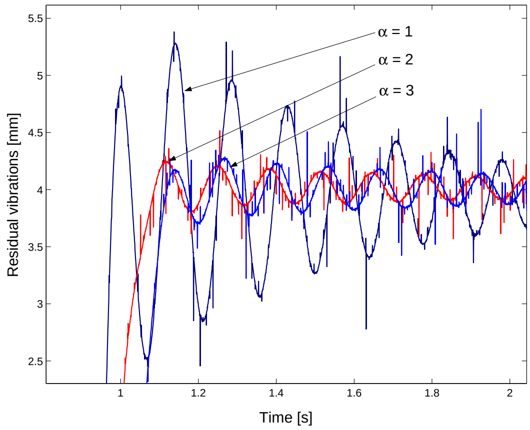


Fig. 7. Residual oscillations of the cartesian robot load according to $\alpha$.

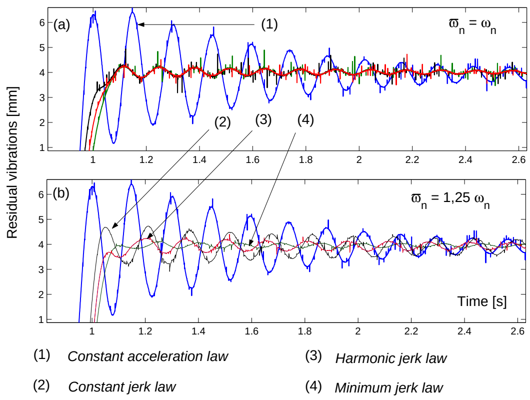


Fig. 8. Vibration reduction ability of the jerk-controlled laws studied.

tion $x_{ref}(t)$ that minimises the performance index which is a square time integral of the jerk

$$J(x_{ref}(t)) = \frac{1}{2}\int_0^T \dddot{x}_{ref}^2(t) \tag{9}$$

where $T$ is the specified execution time.

This optimisation procedure leads to a quintic polynomial, which is depicted in figure 4(d). The optimal movement law is generally associated with a pre-filter, which leads to better tracking performances. The pre-filter design is a problem addressed by several researchers. In concrete terms, it corresponds to the inverse of the system dynamic (for minimal phase systems). But, in our study based on movement law influence, the minimum-jerk profile is directly applied without pre-filtering.

Figure 8(a) shows the residual oscillations of the Cartesian robot load for the previous analysed laws. The limited-jerk and harmonic-jerk profile are tuned to cancel vibration, i.e. the estimated natural pulsation verifies $\widetilde{\omega_n} = \omega_n$ ($\alpha = 1$ and $\alpha = 2$ respectively). It is particularly notable that the three jerk-controlled laws lead to the same level of residual vibration. Table 1 summarises the results obtained for the different laws; they confirm that these laws are equally able to reduce the vibrations.

Table 1 Performances according to the movement law ($P_{ref} = 1$ meter).

| | Movement time [s] | Residual vibration [mm] |
|---|---|---|
| Limited-acceleration | 1,1 | 5,3 (100%) |
| Limited-jerk | 1,215 (+10,45%) | 0,46 (8,6%) |
| Harmonic-jerk | 1,226 (+11,45%) | 0,45 (8,5%) |
| Minimum-jerk | 1,236 (+12,36%) | 0,45 (8,5%) |

The results reveal moderate difference in execution time between the three laws, but for little displacement, the effects of the transitory stage times will be more significant. The limited-jerk and harmonic-jerk profiles require an increase of execution time of one or two natural periods as compared to the reference limited-acceleration profile. For the minimum-jerk law the theoretical movement time is generally limited by the maximum acceleration capability. In this case, the corresponding execution time for a displacement of $P_{ref}$ is

$$T_{\mathrm{MJ}} = \sqrt{\frac{10P_{ref}}{\sqrt{3}A}} \tag{10}$$

The most frequently encountered movements in industrial systems are the *intermediate movements*, i.e. the maximum acceleration is reached, the maximum velocity not. For a limited acceleration profile, the corresponding movement time is

$$T_{\mathrm{LA}} = 2\sqrt{\frac{P_{ref}}{A}} \tag{11}$$

Therefore, equations (10) and (11) lead to the fact that for intermediate displacement the minimum-jerk movement will theoretically be longer by 20% than the limited-acceleration one.

In many cases, the modal frequencies evolve with the various configurations of the axes. Figure 8(b) shows the residual oscillations for an $\widetilde{\omega_n}$ error of 25%. The limited-jerk or harmonic-jerk solutions suffer from a serious limitation, which is the sensitivity to frequency inaccuracy. Confronted to these problem, it is preferable to choose a jerk time for which the specified level of oscillation is respected in spite of parameter variations, i.e. one use the multiples of the period of the cardinal sine function to stay under a maximum oscillation limit. This procedure inevitably increases the rise time. On the other hand, the minimum-jerk law is considerably less sensitive to parameter variations, thanks to the polynomial smoothness. The comparative effects of error in the estimated natural period, for minimum-jerk and limited-jerk law are given in figure 9.

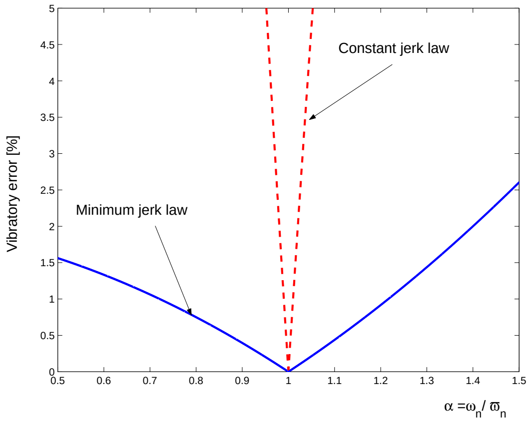


Fig. 9. Residual vibrations due to $\omega_n$ variation.

Finally, the movement law with minimal jerk seems to carry out an interesting compromise between reduction of the residual vibrations, the increase of movement time and the sensitivity to parametric variations of the dominating mode. It must be remarked that the minimum-jerk profile is computationally more expensive, but the resulting quintic polynomial could be efficiently replaced by cubic-splines, which are available in modern numerical control units, as demonstrated in (Brun-Picard, 2004).

## 6. CONCLUDING REMARKS

This paper has presented the influence analysis of some jerk profiles on the vibratory error of industrial machines. The following points underline the pragmatic aspects of the study

(1) The formulation developed in this paper allows for a better understanding of the effects of the jerk profile on the vibration.
(2) Numerical results confirm that for a limited-jerk law, the residual vibrations are cancelled for a $T_j$ being an integral multiple of the predominating period. For an harmonic-jerk law, the cancellation of vibrations require a $T_j$ at least equal to twice the considered modal period. Although the execution time is longer than in the case of limited-jerk profile, the sensitivity to parameters variation and neglected modes is improved.
(3) The minimum-jerk profile leads to the same residual vibrations than the correctly tuned limited-jerk and harmonic-jerk profiles. But, it is considerably less sensitive to the variations of the dominating modal frequency. The only apparent limitation for high-speed systems is that the movement time is inevitably longer (at least +20% in comparison to a limited-acceleration law).

## 7. ACKNOWLEDGMENTS

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