A HYBRID MECHATRONIC TILTING ROBOT: MODELING, TRAJECTORIES, AND CONTROL

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Abstract: Tilting around the foot edges is rarely considered in the modeling of walking robots. This paper uses a hybrid modeling framework to account for different possible ground contact situations of a robot foot. Heuristic trajectory planning is discussed for a robot with five joints and two feet and a method to compensate deficiencies of the planned trajectories by controlling the ZMP is introduced.

Keywords: Trajectory Planning, Hybrid Systems

1. INTRODUCTION

Most trajectory planning methods for biped robots neglect tilting around foot edges. Control strategies that rely on precomputed trajectories, avoid tilting by an appropriate higher-level control law, see Hirai et al. (1998). However, no strategies are known, that apply if accidental tilting occurs.

Incorporating controlled tilting as a regular state into the trajectory planning process could enhance the flexibility of biped robots. Nishiwaki et al. (2002) added toe joints to their humanoid robot to take advantage from this degree of freedom.

Considering configurations with non-flat ground contact of a foot enhances the number of possible ground contact situations of the total system. Then, conditions for transition between different contact situations must be formulated. These transition conditions include collision of the foot with the environment, which causes discontinuous behavior. We therefore choose a hybrid modeling framework, see e.g. Buss et al. (2002), to allow for discontinuities in the model of a mechatronic system, that still acts continuous almost everywhere else.

As an experimental platform, we consider a simple-structured biped robot with five actuated joints and two feet, thus having a variety of possible ground contact configurations. For this robot, trajectories with planned and well defined tilting motions are examined. For a similar robot Albro and Bobrow (2004) use a purely continuous modeling framework, that models ground contact by introducing spring-mass-damper-systems. Based on the model, optimal motions are determined. Hardt and von Stryk (2002) present an optimal control algorithm for general systems with hybrid structure and apply it e.g. to multiped walking machines, but without considering tilted contact states.

Our approach to determine hybrid trajectories is heuristic and does not provide optimal trajectories. The desired trajectories are designed in repeated simulation experiments by variation of parameters of predefined trajectories for the

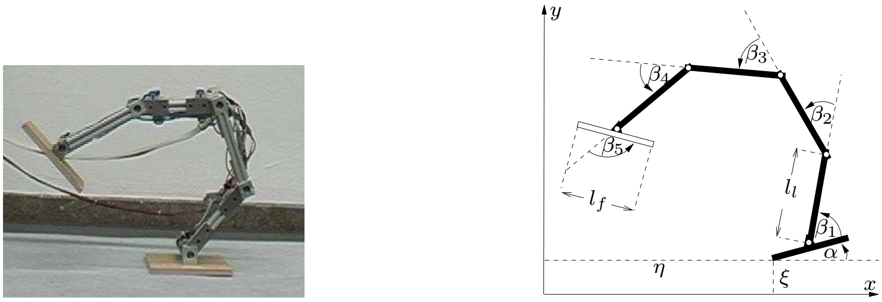

Fig. 1. Left: Robot with four links, five actuated joints and two feet. Right: Schematic illustration as basis for modeling. The generalized coordinate vector $\boldsymbol{q} = (\xi,\, \eta,\, \alpha,\, \beta_1,\, \dots,\, \beta_5)^T$ is defined as pictured.

actuated joints. To compensate for deficiencies, a correcting controller is introduced that acts in the motion phases with stable ground contact and keeps the stable contact phase invariant by ZMP manipulation. The correcting method is related to the Dynamics Filter Method that was proposed by Yamane and Nakamura (2003) and the trajectory planning method for biped robots of Kondak and Hommel (2003).

For the organization of this paper: In Sec. 2 we introduce the robot and establish the hybrid model. The heuristic trajectory planning method is presented in Sec. 3 where we compute stepover and walking trajectories.

2. MODELING

This paper considers a robot that moves only in a two-dimensional plane. The robot consists of five actuated (motor-driven) joints, four links, and two feet arranged as sketched in Fig. 1. The foot with ground contact at the initial time is termed reference foot in the following.

The links are constructed equal and have length $l_l = 0.205\,\mathrm{m}$ and mass $m_l = 0.107\,\mathrm{kg}$. The feet have length $l_f = 0.2\,\mathrm{m}$, height $h_f = 0.00706\,\mathrm{m}$ and mass $m_f = 0.299\,\mathrm{kg}$. For simulation, mass, length, mass center and inertia matrix of the links are taken from a CAD-model of the experimental platform. Only the motors are neglected in the presented modeling and trajectory planning. As a consequence the model is symmetric concerning both, left and right tilting, and the exchange of reference foot and opposite foot.

Four different ground contact situations are possible for each foot: One foot has either flat ground contact, is tilted around the left or right foot edge, or does not have contact with the ground at all. Tilting is defined as a free rotational motion around a foot edge. The combination of the contact situations of the single feet result in 16 possible configurations of the whole robot. For simulation purposes the total amount of configurations is reduced to a subset of relevant configurations where at most one foot has contact at the same time. It results in only seven possible contact configurations, as illustrated in Fig. 2.

The system behavior is mainly continuous, characterized by ordinary differential equations, that are switched subject to the actual contact situation. Switching between the contact situations is triggered by the orbit crossing transition surfaces, that model for example the touching of the ground of another foot edge. Discontinuities in the orbit are then allowed to model collisions. Systems, that combine continuous and discrete behavior are defined as hybrid systems. Therefore, the hybrid modeling notation proposed in Buss et al. (2002) is applied as a basis for simulation and trajectory planning.

2.1 Hybrid State Vector

A state vector at a given time constitutes the evolution of the state of a dynamical system for all future times, if the system input is known. The hybrid state vector

$$\boldsymbol{\zeta} = \begin{pmatrix} \boldsymbol{q} \\ \dot{\boldsymbol{q}} \\ z \end{pmatrix} \quad \in \mathbb{R}^{16} \times \mathbb{N}$$

for the considered system is composed of a continuous part $\boldsymbol{q} = (\xi,\, \eta,\, \alpha,\, \beta_1, \dots, \beta_5)^T \in \mathbb{R}^8$ with its derivative $\dot{\boldsymbol{q}}$ and a discrete part $z \in \mathbb{N}$, where the value of the discrete variable z stands for the contact situation, see Fig. 2. In the next subsection the continuous part, i.e. the differential equation for each contact situation, is derived. Then the transition conditions and the discontinuous transition behavior are discussed.

2.2 Continuous Model

A differential description is established for the flying system ($z = 0$), the contact situation with stable contact on the reference foot ($z = 2$) and for the contact state where the system tilts around the left edge of the reference foot ($z = 3$). The descriptions for the remaining contact states use the same equations with appropriate

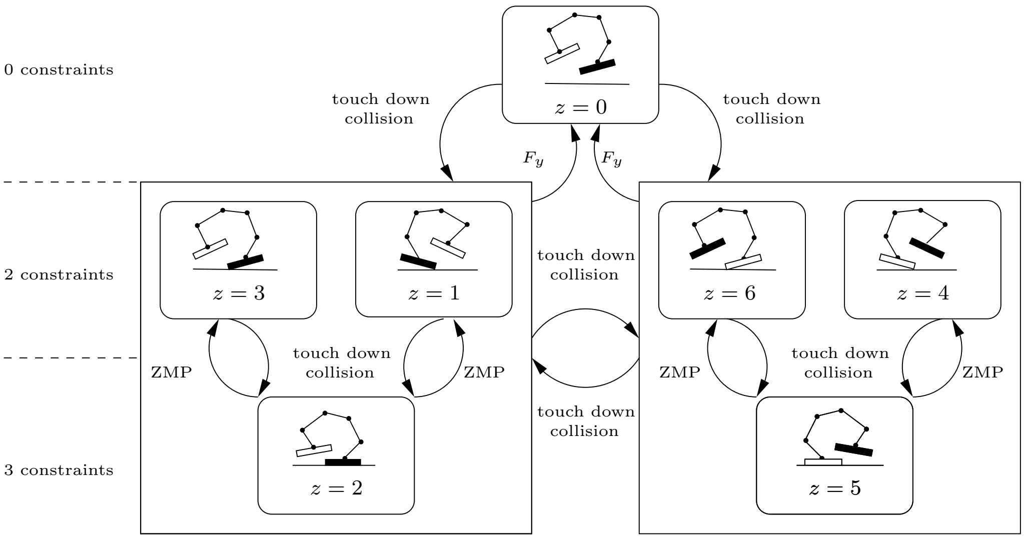

Fig. 2. Transition structure of the contact situations. On the one hand a grouping into situations where the "black" reference foot (left box), where the "white" foot is (right box) and where no foot has contact (top) is done. On the other hand contacts with no constraints (top), two constraints (middle) and three constraints (bottom) are distinguished. Conditions for contact situation changes are either of dynamic nature (ZMP, force in y-direction) or of kinematic nature (foot touches ground). For contact state transitions, where the number of active constraints increases, collisions occur.

state transformations. This is possible due to the symmetry of the construction.

At first, we have to obtain a set of equations of motion for the unconstrained robot (flight phase), e.g. by applying the Euler-Lagrange Method. A Lagrange function $L = U - V$ is composed of the total kinetic energy U and the total potential energy V of the robot and Euler-Lagrange Equations are applied:

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \tau_i, \;\; i = 1, \ldots, 8$$

This results in equations commonly written as

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + N(\boldsymbol{q}, \dot{\boldsymbol{q}}) + G(\boldsymbol{q}) = \boldsymbol{\tau}, \tag{1}$$

where M is the mass matrix, in N we collect the terms of coriolis and centrifugal forces and G describes gravity. The input torque is $\boldsymbol{\tau} = (0,\,0,\,0,\,\tau_1, \ldots, \tau_5)$, since only the joints are actuated. The torque τ_i is the control input for the i-th joint.

The equations of motion (1) for a flying system are the basis for the derivation of the dynamical equations for any kind of ground contact. Ground contact is then introduced by algebraic constraints to the equations of motion for the unconstrained system. For example for stable contact with the ground, the reference foot has zero velocity relative to the ground. Therefore the constraints are

$$\dot{\xi} = 0, \;\; \dot{\eta} = 0, \;\; \dot{\alpha} = 0,$$

or expressed with the appropriate Jacobian J as

$$J\dot{\boldsymbol{q}} = \begin{pmatrix} 1\,0\,0\,0\,0\,0\,0\,0 \\ 0\,1\,0\,0\,0\,0\,0\,0 \\ 0\,0\,1\,0\,0\,0\,0\,0 \end{pmatrix} \dot{\boldsymbol{q}} = \boldsymbol{0}. \tag{2}$$

Accordingly for tilted contact the constraints are

$$\dot{\xi} = 0, \;\; \dot{\eta} = 0. \tag{3}$$

The transposed Jacobian matrix J for the constraint condition scaled by the multiplier $\boldsymbol{\lambda}$ is added to the equation of motion (1), resulting in

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + N(\boldsymbol{q}, \dot{\boldsymbol{q}}) + G(\boldsymbol{q}) = \boldsymbol{\tau} + J^T(\boldsymbol{q})\boldsymbol{\lambda}. \tag{4}$$

For stable ground contact $\boldsymbol{\lambda} = (f_x,\, f_y,\, \tau_z)^T$ is the vector of ground contact forces and moments. The combination of (4) and the derivative of (2),

$$J\ddot{\boldsymbol{q}} + \dot{J}\dot{\boldsymbol{q}} = 0,$$

results in the differential equation for the system with stable ground contact and algebraic equations for the constraint forces $\boldsymbol{\lambda}$. See Murray et al. (1996) for a discussion of the derivation of equations of motion.

The set of continuous systems is linked by the discrete aspect of the system.

2.3 Discrete Model

The contact state $z = i$ transforms into the contact state $z = j$ at a time t, whenever a transition surface function $s_{ij}(t, \boldsymbol{\zeta}, \boldsymbol{\tau}) = 0$ equals zero. Then the hybrid state $\boldsymbol{\zeta}^+$ immediately after the transition is calculated from $\boldsymbol{\zeta}^-$, which is the hybrid state immediately before the transition.

Two classes of transition conditions arise for the modeled robot.

One transition type is the onset of tilting from an initially stable ground contact situation or the onset of flying from an initially stable or tilted contact situation. These transitions have in common, that the state transforms smoothly and only the discrete state variable z changes its value. To detect the onset of tilting we employ the Zero Moment Point (ZMP), see Vukobratovic et al. (2001), to derive a condition of the form $s_{ij}(t, \boldsymbol{\zeta}, \boldsymbol{\tau}) = 0$. The ZMP is the point on ground level, where the ground reaction force acts to compensate all horizontal moments and thus keeps the system in balance. If the ZMP leaves the area covered by the foot plate (supporting area), balance is lost and hence the system starts to tilt. The ZMP is calculated from the vertical contact force f_y and from the contact moment μ_z, whereas only the vertical contact force f_y is used to detect the onset of a ballistic phase.

The second kind of transition is the touch down of a foot edge, that had no ground contact before. This occurs e.g. when the robot lands from a ballistic phase, or when the stance foot changes. These transitions have in common that collisions have to be considered and modeled, resulting in discontinuous behavior. Conservation of angular momentum is chosen as collision model for an instantaneous modeled collision, which results in discontinuous behavior of the joint velocities $\dot{\boldsymbol{q}}^+ = \Phi(\boldsymbol{q}, \dot{\boldsymbol{q}}^-)$, see Grizzle et al. (2001). The equations for the transition surfaces $s_{ij} = 0$, that model the touch down conditions are obtained from the kinematic equations of the positions of the foot edges.

The hybrid model presented in this section is used for simulative evaluation of the control algorithms in the following.

3. TRAJECTORY PLANNING

Optimal trajectory planning for hybrid walking machines is difficult, especially in the presence of nonactuated motion phases and variable, state-dependant transition times. Therefore we choose as a first step a heuristic approach to perform experiments in hardware.

In this work we focus on trajectories, where tilting motion phases and stable motion phases alternate. We have chosen a stepover motion and a walking motion for the following presentation.

The trajectories for the actuated joints are chosen periodic, depending on parameters. To achieve a hybrid periodic motion the parameters are varied and the effect is observed in simulation experiments. This intuitive trajectory planning method requires a good guess of the parameter-dependant trajectories and often unwanted behavior cannot be removed, since the only degree of freedom are parameters and not the overall shape of the trajectories. Therefore insufficiencies in this heuristically planned trajectories are compensated by an additional higher level corrective control, that acts only in the stable contact phase.

3.1 Stepover Motion

The planar robot is to perform a walking motion, where the swing foot is moved over the support foot passing through the singular configuration, see Fig. 3. This motion is in the following referred to as “stepover motion”. For a stepover motion we define the actuated joint angles to follow cosine shaped trajectories. The amplitudes and the frequencies of the cosines are tuned in simulations.

In Fig. 4 details for the resulting stepover motion are shown. The motion of the total robot, and in particular the trajectory of the non-actuated degree of freedom α, is a consequence of controlling the individually actuated joints on the predefined trajectories.

We use a computed torque control law for the tracking of desired trajectories of the actuated joints. For the five actuated joints the desired trajectories are denoted $(\boldsymbol{\beta}^d, \dot{\boldsymbol{\beta}}^d, \ddot{\boldsymbol{\beta}}^d)$. If the equations of motion in the stable contact phase are denoted

$$M_s \ddot{\boldsymbol{\beta}} + N_s + G_s = \boldsymbol{\tau} \qquad (5)$$

the choice of

$$\boldsymbol{\tau} = M_s \boldsymbol{v} + N_s + G_s$$

linearizes (5) resulting in decoupled linear equations

$$\ddot{\boldsymbol{\beta}} = \boldsymbol{v},$$

where $\boldsymbol{v}$ is the control input. Choosing

$$\boldsymbol{v} = \ddot{\boldsymbol{\beta}}^n = \ddot{\boldsymbol{\beta}}^d + K_D(\dot{\boldsymbol{\beta}}^d - \dot{\boldsymbol{\beta}}) + K_P(\boldsymbol{\beta}^d - \boldsymbol{\beta}) \quad (6)$$

leads to tracking of the desired trajectory, where K_D and K_P characterize the dynamic properties. The controller (6) is termed nominal controller in the following.

3.2 Corrective Control

Due to the non-optimal trajectory planning, the robot shows undesired behavior after landing. As can be seen in Fig. 4, the robot does not reach a stable contact situation and cycles between tilted right and tilted left after landing. Finally the robot falls down, which is seen in Fig. 3. A model based control approach is applied to prevent undesired leaving of the stable ground

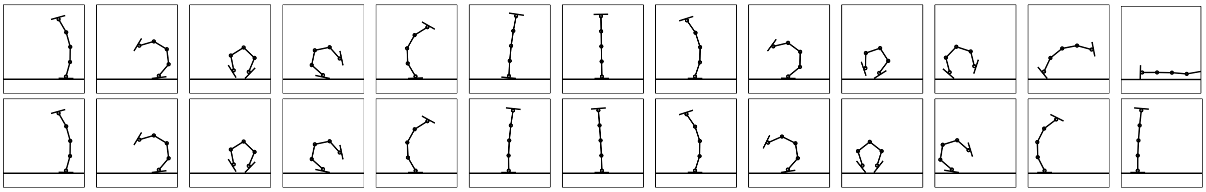

Fig. 3. Snapshot Series of simulation results for stepover trajectory. Top: The robot falls if no corrective control is applied. Bottom: Corrective control is active.

contact state. This is closely related to the Dynamics Filter Method that was introduced by Yamane and Nakamura (2003) to make improperly planned trajectories applicable to locomotion systems.

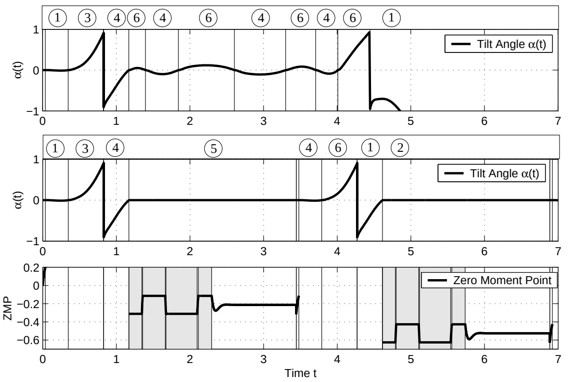

Fig. 4. Details to the snapshot series in Fig. 3. Circled numbers indicate contact situations z. Top: Tilting angle α without corrective control. Although the robot is landed, tilting does not stop ($\alpha > 0$: tilting left, $\alpha < 0$: tilting right). Middle: Tilting angle α with corrective control. Bottom: Zero Moment Point, grey shaded is the activity time of the corrective controller.

The robot leaves the stable contact phase and starts tilting if the ZMP crosses the left or right edge of the supporting area. If tilting must be avoided, then a controller has to supervise the ZMP and has to take corrective action if the ZMP leaves a subset of the supporting area. The equation for the ZMP is written as

$$p_{\text{zmp}} = \frac{\mu_z}{f_y} = \frac{\mu_{z,0}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \sum \mu_{z,i}(\boldsymbol{q}, \dot{\boldsymbol{q}})\ddot{q}_i}{f_{y,0}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \sum f_{y,i}(\boldsymbol{q}, \dot{\boldsymbol{q}})\ddot{q}_i}. \qquad (7)$$

It is used, that generalized forces are linear in the accelerations, where the coefficients $\mu_{z,i}$ and f_y, i depend on the positions $\boldsymbol{q}$ and the velocities $\dot{\boldsymbol{q}}$. The stability condition for the considered planar robot is

$$b_* \leq p_{\text{zmp}} \leq b^*$$

where b_* is the lower bound of a security subset of the supporting area and b^* is the upper bound. Thus if the ZMP with the nominal controller is about to leave the supporting area, a new control moment $\boldsymbol{\tau}$ has to be determined, that either leads to

$$p_{\text{zmp}} = b_* \quad \text{or} \quad p_{\text{zmp}} = b^*. \qquad (8)$$

Eq. (8) are linear equations for the accelerations $\ddot{\beta}_i$ that have an infinite number of solutions for $\ddot{\boldsymbol{\beta}} = (\ddot{\beta}_1, \ldots, \ddot{\beta}_5)^T$. The next step is to determine one solution $\ddot{\boldsymbol{\beta}}^c$ of (8) and to adapt the control law to track the corrective acceleration $\ddot{\boldsymbol{\beta}}^c$.

First we will discuss the choice of a solution of (8) for the lower bound. It is reminded, that we need a solution for $\ddot{\boldsymbol{\beta}}$ of the equation

$$\sum_{i=1}^{5} (\mu_{z,i} - f_{y,i})\ddot{\beta}_i = A\ddot{\boldsymbol{\beta}} = b_* \, f_{y,0} - \mu_{z,0}, \qquad (9)$$

which results from inserting (7) in (8). One possibility is to choose the solution of (9), that is closest in euclidian norm to the solution, that the nominal controller proposes. Formally this is the solution $\ddot{\boldsymbol{\beta}}^c$ of (9), with

$$\ddot{\boldsymbol{\beta}}^c = \underset{\ddot{\boldsymbol{\beta}}}{\text{argmin}} \, \|\ddot{\boldsymbol{\beta}} - \ddot{\boldsymbol{\beta}}^n\|^2. \qquad (10)$$

The minimal norm solution is obtained by Pseudoinverse application. The matrix $A^{\#}$ is the pseudoinverse matrix of A, and the minimal norm solution for the cost function (10) is

$$\ddot{\boldsymbol{\beta}}^c = A^{\#}(b_* f_{y,0} - \mu_{z,0}) + (I - A^{\#}A)\ddot{\boldsymbol{\beta}}^n,$$

where I is the identity matrix.

To achieve exact tracking of desired accelerations the computed torque controller is adapted. In motion phases, where the ZMP must be corrected, the linear control input is chosen $\boldsymbol{v} = \ddot{\boldsymbol{\beta}}^c$ instead of $\boldsymbol{v} = \ddot{\boldsymbol{\beta}}^n$, as given in (6).

For trajectories, where tilting is in general allowed, activity of the tilting avoidance controller must be restricted to time ranges, where tilting is not wanted. This is in particular after collisions, that means, when the control error $\boldsymbol{e} = K_P(\boldsymbol{\beta}^d - \boldsymbol{\beta}) + K_D(\dot{\boldsymbol{\beta}}^d - \dot{\boldsymbol{\beta}})$ is large.

A second possibility to choose a solution of (9) is related to the approaches by Hirai et al. (1998), where the ZMP, if it is too large or too small is corrected by a correcting motion only in the base link. This approach is comprised in our approach, if $\ddot{\beta}_2, \ldots, \ddot{\beta}_5$ are determined by the nominal controller and only $\ddot{\beta}_1$ is adapted to keep the ZMP on the margin of the stability region.

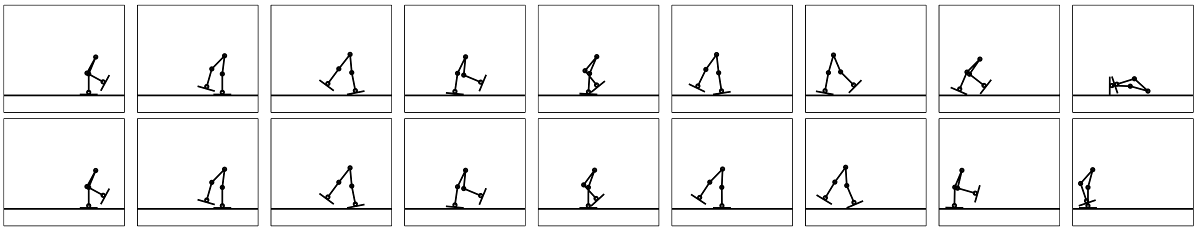

Fig. 5. Snapshot Series of simulation results for walking trajectories. The robot falls, if no corrective control is applied (top).

3.3 Walking

In this example we present heuristically determined trajectories for a walking motion. Like in the previous example periodic parameter dependent trajectories for the actuated joints are chosen and the parameters are tuned by inspection in simulation experiments. There are parameters, where the walking cycle is periodic and stable, but there are also parameters, where the robot falls down. The reason for falling is mostly, that undesired tilting around the foot edges occurs, therefore the corrective controller introduced in the section before is used. In Fig. 5 a snapshots series of such a periodic walking cycle is shown. Details for the simulation experiment are given in Fig. 6.

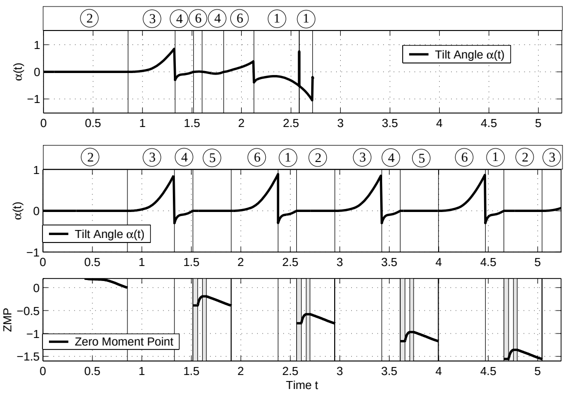

Fig. 6. Details to the snapshot series in Fig. 5. Top: Repetitive tilting destabilizes the periodic motion, the robot falls. Middle/bottom: Regular and stable behavior is achieved by corrective control in stable contact phases.

4. CONCLUSIONS

In this paper we presented an approach to determine periodic trajectories for a hybrid robot with small computational effort, if the basic structure of the desired motion is known. The different ground contact situations are considered in a hybrid model. Since the heuristic planning process often does not deliver feasible trajectories, a corrective controller is presented, that acts in the motion phase with stable ground contact of one foot and manipulates the ZMP. The method was illustrated for a stepover and a walking motion.

For the next time, we plan to automate the manual parameter tuning of the trajectory design by formulation as optimal control problem. In parallel hardware experiments will be performed.

REFERENCES

J. V. Albro and J. E. Bobrow. Motion generation for a tumbling robot using a general contact model. In *Proceedings of the IEEE International Conference on Robotics and Automation (ICRA)*, pages 3270–3275, New Orleans, LA, April 2004.

M. Buss, M. Glocker, M. Hardt, O. von Stryk, R. Bulirsch, and G. Schmidt. Nonlinear hybrid dynamical systems: Modeling, optimal control, and applications. In S.Engell, G.Frehse, and E. Schnieder, editors, *Modelling, Analysis and Design of Hybrid Systems. Lecture Notes in Control and Information Sciences (LNCIS)*. Springer, Berlin, Heidelberg, 2002.

J.W. Grizzle, G. Abba, and F. Plestan. Asymptotically stable walking for biped robots: Analysis via system with impulse effects. *IEEE Transactions on Automatic Control*, 46(1):51–64, January 2001.

M. Hardt and O. von Stryk. The role of motion dynamis in the design, control and stability of bipedal and quadrupedal robots. In *Proceedings of the RoboCup 2002 International Symposium*, Fukuoka, Japan, 2002.

K. Hirai, M. Hirose, Y. Haikawa, and T. Takenaka. The development of honda humanoid robot. In *Proceedings of the IEEE International Conference on Robotics and Automation (ICRA)*, pages 1321–1326, Leuven, Belgium, May 1998.

K. Kondak and G. Hommel. Control algorithm for stable walking of biped robots. In *Proceedings of the International Conference on Climbing and Walking Robots*, Catania, Italy, 2003.

R. M. Murray, Z. Li, and S. S. Sastry. *A Mathematical Introduction to Robotic Manipulation*. CRC Press, 1996.

K. Nishiwaki, S. Kagami, Y. Kuniyoshi, M. Inaba, and H. Inoue. Toe joints that enhance bipedal and fullbody motion of humanoid robots. In *Proceedings of the IEEE International Conference on Robotics and Automation (ICRA)*, pages 3105–3110, Washington, DC, May 2002.

M. Vukobratovic, B. Borovac, and D. Surdilovic. Zero-moment point – proper interpretation and new application. In *Proceedings of the IEEE/RAS International Conference on Humanoid Robots (HUMANOIDS)*, pages 237–244, Tokyo, Japan, 2001.

K. Yamane and Y. Nakamura. Dynamics filter–concept and implementation of online motion generator for human figures. *IEEE Transactions on Robotics and Automation*, 19(3), June 2003.