LIFE EXTENDING CONTROL BY A VARIANCE CONSTRAINED MPC APPROACH

Donglin Li, Tongwen Chen, Horacio J. Marquez [*]
R. Kent Gooden [**]

[*] *Dept. of Electrical and Computer Engineering, University of Alberta*
[**] *Syncrude Canada Ltd.*
Email: marquez@ece.ualberta.ca

Abstract: The objective of life extending control (LEC), also known as damage mitigating control, is to design a controller to achieve a good tradeoff between structural durability and dynamic performance in a system. In this paper, continuum fatigue damage theory for a boiler system is discussed. To reduce the accumulated damage, a variance constrained model predictive control (VCMPC) problem is developed and an algorithm via linear matrix inequalities (LMIs) is derived. Moreover, the controller obtained by this algorithm can assign the resultant closed-loop poles in a prescribed region. Finally, we apply the algorithm in a LEC design for a boiler system.

Keywords: Damage, boilers, linear control systems, variance matrices, multivariable control systems.

1. INTRODUCTION

Traditional control system design focuses on stability and performance only. In other words, the control design process ignores the effects of aging, fatigue, and damage in the materials involved. The material involved, however, inevitably suffers from fatigue and even damage, especially for the systems operating in specific surroundings, e.g., high temperature and high pressure. Motivated by this, Noll *et al.* (1991) pointed out the need to address the trade-off between system performance and durability (of critical components). This formed the basis of the so called *life extending control* (LEC). Since then, LEC has attracted a great deal of attention from industry and academia. An important step in LEC design is to construct damage models for the critical plant components. Because fatigue damage is cycle-dependent, fatigue damage models are usually based on stress-strain hysteresis loops. In contrast, most control theories are formulated in the time domain. To fit the framework of control theories, the damage dynamics should be expressed as a vector differential equation with respect to time. Ray *et al.* (1994) produced the first breakthrough in this area and proposed a continuous time damage modeling theory. But their method leads to a complex nonlinear model. It is difficult to see which factor contributes the most to the damage from their models. In this paper, a further research is carried out to find the most important factor for the thermal fatigue damage of a critical component in a system.

A theory of covariance control was suggested (Hotz and Skelton, 1987). In this theory a hard constraint on the variance of outputs in addition to the cost function is considered. If there exists

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a controller such that the closed-loop system is stable and the variance constraints are satisfied, then we say the problem is feasible. If the variance constrained problem is feasible, one can optimize LQ performance over the feasible controller set and this kind of problem is referred to as variance constrained LQ problem (VCLQ) (Huang, *et al.*, 2003).

Linear matrix inequality (LMI) has emerged as a powerful formulation and design technique for a variety of linear control problems. Since solving LMI is a convex optimization problem, such formulations offer a numerically tractable means of attacking problems that lack an analytical solution. Thus much effort has been made to solve such multi-objective feedback control problems via LMI optimization (Scherer, *et al.*, 1997). However, due to the limitation of the LQ theory, all the earlier research did not deal with the constraints on outputs of controllers. But due to physical limits of actuators, safety margins, limited manufacturing tolerances and other possible reasons, such constraints are also necessary to be considered in engineering design. Therefore, in this paper, a variance constrained MPC (VCMPC) problem is studied via LMI.

For a discrete time system, the resultant closed-loop poles should locate within the unit circle. Furthermore, for satisfactory performance, the closed-loop poles are often assigned in a prescribed region within the unit circle. Pole assignment problem has been studied for years (Haddad and Bernstein, 1992; Li, 2000). In all these references, a scaler $\epsilon > 0$ should be searched first, this increases the complexity of the algorithms. In this paper, an algorithm is developed, which does not involve this particular scalar search.

This paper is organized as follows. In section 2, the fatigue damage modeling of a boiler system is discussed. In section 3, a VCMPC problem is studied and an algorithm via LMIs to solve the VCMPC problem is presented. In section 4, we apply the algorithm to a boiler system to design an LEC.

2. FATIGUE DAMAGE MODELING

According to Ray's method (Ray *et al.*, 1994), the elastic damage rate can be calculated by

$$\frac{d\delta_e}{dt} = 2 \times \frac{d}{d\sigma}\left(\left(\frac{\sigma - \sigma_r}{2(\sigma_f^{'} - \sigma_m)}\right)^{-\frac{1}{b}}\right) \times \frac{d\sigma}{dt}, \quad (1)$$

where δ_e is the elastic damage, σ the stress, σ_r the stress of the starting point of one cycle determined from the rainflow cycle counting method (Bannantine, *et al.*, 1990), and σ_m the mean stress of one cycle. The parameters $\sigma_f^{'}$ and b are material constants: $\sigma_f^{'}$ is the fatigue strength coefficient and b the fatigue strength exponent. For many materials, we have $\sigma_m = 0$. To determine the thermal stress, the following equation can be used (Lu and Wilson, 1998):

$$\sigma = \frac{E\alpha}{1 - \upsilon}\left(T_m(t) - T(t)\right) + \beta(T_\infty).$$

Here E is the modulus of elasticity, υ the Poisson's ratio, α the coefficient of linear expansion, β the temperature coefficient, $T(t)$ the temperature of the component at the critical point, $T_m(t)$ the mean temperature up to current time and T_∞ the steady temperature. Now we define

$$K_1 = 2 \times \left(\frac{1}{2\sigma_f^{'}}\right)^{-\frac{1}{b}} \times \left(-\frac{1}{b}\right),$$
$$K_2 = \frac{E\alpha}{1 - \upsilon},$$
$$K_3 = -\frac{1}{b} - 1,$$

and

$$K_2 \times \left(T_m(\tau) - T(\tau)\right) + \beta(T_\infty) - \sigma_r = m$$

yields

$$K_2 \times \left(\dot{T}_m(\tau) - \dot{T}(\tau)\right) = \dot{m}.$$

From Eq. (1) and the above definitions, it is easy to show

$$\frac{d\delta_e}{dt} = K_1 \times (m)^{K_3} \times \dot{m}.$$

Therefore

$$\begin{aligned}\delta_e &= \int_0^t K_1 \times m^{K_3} \times \dot{m} \times d\tau = K_1 \times \left.\frac{m^{K_3+1}}{K_3+1}\right|_0^t \\ &= \frac{K_1}{K_3+1}((K_2 \times (T_m(t) - T(t)) + \beta(T_\infty) \\ &\quad - \sigma_r)^{K_3+1} - (\beta(T_\infty) - \sigma_r)^{K_3+1}). \quad 0 \leqslant t \leqslant t_{fi}\end{aligned} \quad (2)$$

where t_{fi} is the final time of the ith cycle.

For the plastic damage δ_p, we can get the similar result.

The accumulate damage is the weighted sum of δ_e and δ_p. From Eq. (2), the variance and the steady value of the temperature affect the accumulated damage. Because the steady temperature is a required parameter for proper operation of a boiler system, we can only reduce the variate of the temperature to reduce the accumulated damage. Therefore, a VCMPC problem is developed in the following section to achieve the objective of LEC for a boiler system.

3. VARIANCE CONSTRAINED MODEL PREDICTIVE CONTROL

In this section, in view of designing an LEC for a boiler system, a VCMPC problem is formulated and all conditions to make the closed-loop system satisfy the constraints are derived in terms of LMIs.

3.1 Problem formulation

Let us consider the following linear time-invariant discrete system

$$\begin{cases} x(k+1) &= Ax(k) + B_1u(k) + B_2w(k), \\ y(k) &= Cx(k) + v(k), \\ Z_i(k) &= C_ix(k) + D_iu(k), \qquad i = 1, \cdots, m \end{cases} \tag{3}$$

where $x \in R^n$ is the state, $y \in R^m$ is the measured output, $u \in R^l$ is the input and $Z \in R^m$ is a vector of output signals related to the performance of the control system. The vectors $w \in R^n$ and $v \in R^m$ are uncorrelated zero-mean white noises with covariance matrices $W > 0$ and $V > 0$, respectively.

Now a state feedback control law is given by

$$u(k+i|k) = Kx(k+i|k). \tag{4}$$

So, the closed loop system becomes

$$\begin{cases} x(k+i+1) &= A_cx(k+i|k) + B_2w(k+i), \\ y(k+i|k) &= Cx(k+i|k) + v(k+i), \\ Z_j(k+i|k) &= C_{cj}x(k+i|k), \end{cases} \tag{5}$$

where

$$A_c = (A + B_1K), \quad C_{cj} = (C_j + D_jK).$$

Therefore, the problem can be stated as follows: For the system in (3), find a controller of the form (4) such that the resultant system in (5) is stable and minimizes the following cost function:

$$J = \lim_{K\to\infty} E(\sum_{i=0}^{\infty}[x(k+i|k)^TQx(k+i|k) + u(k+i|k)^TRu(k+i|k)]), \tag{6}$$

under the constraints

$$\Phi_j = \lim_{k\to\infty} E(Z_j(k+i|k)Z_j(k+i|k)^T) < \bar{\Phi}_j, \tag{7}$$

and the following constraints on the inputs:

$$-u_{i,max} \leqslant u_i \leqslant u_{i,max}. \tag{8}$$

To get satisfactory performance, we also want to assign the closed-loop eigenvalues in a prescribed region $D(q,r)$ within the unit circle, that is

$$\sigma(A_c) \subset D(q,r), \tag{9}$$

where $\sigma(A_c)$ represents eigenvalues of A_c, and $D(q,r)$ is a disc in the complex plane centered at q and with radius r.

Furthermore, this problem can be divided into two parts: feasibility and optimization. If there exists a controller such that the closed-loop system given by (5) is stable and the constraints given by (7), (8), and (9) are satisfied, then we say the problem is feasible. Suppose the problem is feasible; the feasible controller set is denoted by $\mathcal{C}_d$. The task in optimization is to find a controller over the set $\mathcal{C}_d$ to minimize the cost function in (6).

3.2 MPC covariance controller design via LMIs

Now we define the state covariance matrix as follows:

$$X = \lim_{k\to\infty} E(x(k|k)x(k|k)^T), \tag{10}$$

where $x(k|k)$ refers to the sate measured at time k. From the above definition, X is a positive definite and symmetric matrix. To find a state feedback controller K, we also define another matrix

$$P = KX. \tag{11}$$

Therefore, if we can find X and P, then the controller can be determined

$$K = PX^{-1}.$$

3.2.1. Feasibility of the problem *Lemma 1.* (Hsieh and Skelton, 1990) The system in (5) is asymptotically stable iff there exist a steady state covariance matrix X such that

$$A_cXA_c^T - X + B_2B_2^T \square 0. \tag{12}$$

Theorem 1 (Feasibility). The problem is feasible via a state feedback controller for the LTI discrete system in (3) iff there exist matrices $X > 0$ and P such that

$$\begin{bmatrix} X - AXA^T - AP^TB_1^T - B_1PA^T - B_2B_2^T & B_1P \\ P^TB_1^T & X \end{bmatrix}, \geq 0 \tag{13}$$

$$\begin{bmatrix} \bar{\Phi}_i - C_iXC_i^T - C_iP^TD_i^T - D_iPC_i^T & D_iP \\ P^TD_i^T & X \end{bmatrix} > 0, \quad i \in [1, \cdots, m] \tag{14}$$

$$\begin{bmatrix} u_{j,max}^2 & P_j \\ P_j^T & X \end{bmatrix} \geq 0, \qquad j \in [1, \cdots, l] \tag{15}$$

and

$$\begin{bmatrix} r^2X - r^2B_2B_2^T - (A-qI)X(A-qI)^T \\ -(A-qI)P^TB_1^T - B_1P(A-qI)^T & B_1P \\ P^TB_1^T & X \end{bmatrix} \geq 0. \tag{16}$$

Here the matrices X and P relate to the controller K through Eq. (11).

Proof: Obviously, if we can prove that Ineq. (13) and Ineq. (12), Ineq. (14) and Ineq. (7), Ineq. (15) and Ineq. (8), and Ineq. (16) and Ineq. (9) are equivalent, Theorem 1 is proved. Here we only show the proof of Ineq. (13) is equivalent to Ineq. (12) and others are omitted for brevity.

By Lemma 1, we have

$$(A + B_1K)X(A + B_1K)^T - X + B_2B_2^T \,\Box\, 0.$$

Substituting Eq. (11) into the above inequality

$$(AX + B_1P)(A^T + K^TB_1^T) - X + B_2B_2^T \,\Box\, 0,$$

we have

$$X - AXA^T - AP^TB_1^T - B_1PA^T - B_1PX^{-1}P^TB_1^T - B_2B_2^T \geq 0,$$

which implies Eq. (13), by Schur complement. □

3.2.2. Optimization via MPC Constrained MPC using LMIs has been studied by many researchers, most of them are based on the work of Kothare *et al.* (1996). In their research, to guarantee stability, an infinite horizon MPC (IHMPC) was adopted. However, for an IHMPC problem, it is hard to minimize the cost function J directly, instead the upper bound of J is minimized. Kothare *et al.* (1996) pointed out that this approach does not lead to much conservatism and a robust controller can be obtained. In this paper, we extend their results into the stochastic VCMPC problem. On the other hand, since the controller should satisfy some conditions, a solution may not exist, we also consider the situation of finite horizon MPC (FHMPC).

Lemma 2. (Kothare *et al.*, 1996) For the cost function

$$J(k) = \sum_{i=0}^{\infty}[x(k+i|k)^TQx(k+i|k) + u(k+i|k)^TRu(k+i|k)],$$

if there exist a matrix $P_1 > 0$ and $V(x)$ defined as $V(x) = x^TP_1x$ satisfy

$$V(x(k+i+1|k)) - V(x(k+i|k)) \leqslant -[x(k+i|k)^T Qx(k+i|k) + u(k+i|k)^TRu(k+i|k)], \tag{17}$$

then

$$J(k) \leqslant V(x(k|k)).$$

Theorem 2 (IHMPC). Let $x(k) = x(k|k)$ be the state of the system (3) measured at sampling time k, then the state feedback matrix K in the control law $u(k+i|k) = Kx(k+i|k)$, $i \geq 0$ that minimizes the upper bound $V(x(k|k))$ of the cost function (6) is given by:

$$K = PX^{-1}.$$

X and P, which are defined in (10) and (11) respectively, can be obtained from the solution (if they exist) of the following linear objective minimization problem:

$$\min \gamma, \tag{18}$$

subject to

$$\begin{bmatrix} \gamma & x(k|k)^T \\ x(k|k) & X \end{bmatrix} > 0, \tag{19}$$

and

$$\begin{bmatrix} X & XA^T + P^TB_1^T & XQ^{1/2} & P^TR^{1/2} \\ AX + B_1P & X & 0 & 0 \\ Q^{1/2}X & 0 & I & 0 \\ R^{1/2}P & 0 & 0 & I \end{bmatrix} \geq 0. \tag{20}$$

Proof: Now we let

$$V(x) = x^TX^{-1}x < \gamma,$$

from Lemma 2, it is immediate to know that $V(x)$ is one upper bound of the cost function J when Eq. (17) in Lemma 2 holds:

$$x(k+i+1|k)^TX^{-1}x(k+i+1|k) - x(k+i|k)^TX^{-1}x(k+i|k) \leqslant -[x(k+i|k)^T Qx(k+i|k) + u(k+i|k)^TRu(k+i|k)].$$

Therefore,

$$x(k+i|k)^T\{(A + B_1K)^TX^{-1}(A + B_1K) - X^{-1} + Q + K^TRK\}x(k+i|k) \leqslant 0.$$

Substituting

$$P = KX,$$

pre- and post-multiplying by X, we have

$$X(A + B_1K)^TX^{-1}(A + B_1K)X - XX^{-1}X + XQX + XK^TRKX \leqslant 0,$$

thus

$$(XA^T + P^TB_1^T)X^{-1}(AX + B_1P) - X + XQ^{1/2}Q^{1/2}X + P^TR^{1/2}R^{1/2}P \leqslant 0.$$

Therefore,

$$X - (XA^T + P^TB_1^T)X^{-1}(AX + B_1P) - XQ^{1/2}Q^{1/2}X - P^TR^{1/2}R^{1/2}P \geq 0.$$

By Schur complement, it is easy to obtain Eq. (20). □

From the above theorem, we see that the solution may not exist. To overcome this problem, the following finite horizon MPC cost function can be used:

$$J = \lim_{K\to\infty} E(\sum_{i=0}^{N_x} x(k+i|k)^TQx(k+i|k) + \sum_{i=0}^{N_u} u(k+i|k)^TRu(k+i|k)), \tag{21}$$

where N_x and N_u are the state prediction and control horizon, respectively.

Theorem 3 (FHMPC). The state feedback matrix K in the control law $u(k+i|k) = Kx(k+i|k)$, $i \geq 0$ that minimizes the cost function J in Eq. (21) is given by:

$$K = PX^{-1}.$$

X and P, which are defined in (10) and (11) respectively, can be obtained from the solution of

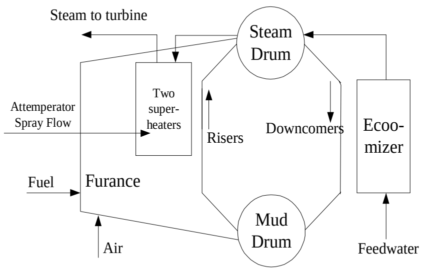

Fig. 1. The operating process schematic diagram of the utility boiler

the following linear objective minimization problem:

$$\min \gamma, \tag{22}$$

subject to

$$\text{trace}(M_x) + \text{trace}(M_u) < \gamma, \tag{23}$$

$$(N_x + 1)QX < M_x, \tag{24}$$

and

$$\begin{bmatrix} \frac{1}{N_u+1} M_u & R^{1/2}P \\ P^T R^{1/2} & X \end{bmatrix} > 0. \tag{25}$$

Here $M_{x(n\times n)}$ and $M_{u(l\times l)}$ are defined serving as intermediate matrices of the optimization process.

The proof is similar to that in *Theorem 2.* and it is omitted here for brevity

Remark: By Theorem 2, because the controller K is based on instant states $x(k)$, thus it is time-dependent; While by Theorem 3, a constant controller will be obtained.

4. LEC DESIGN FOR A BOILER SYSTEM

In this paper, we apply the VCMPC algorithm that we developed to a boiler system in order to reduce the accumulated damage of the system, that is, to extend its life-span. This boiler system is a part of an industrial co-generation plant, owned and operated by Syncrude Canada Ltd in Fort McMurray. The operating process of this boiler can be sketched as shown in Fig 1. Fuel and air are thoroughly mixed and ignited in the furnace, feedwater is preheated by economizer and then is fed into the steam drum. Steam produced by the boiler is fed to power generators through two superheaters: primary and secondary superheaters. By the two superheaters, the steam is further heated. In between the two superheaters is an attemperator which regulates the temperature of the steam exiting the secondary superheater by mixing water at a lower temperature with the steam from the primary superheater. Because the boiler system is responsible for the steam production, the steam *quantity* (measured by its flow rate) and *quality* (measured by its pressure and temperature) are the controlled variables. In brief, in this system there are four inputs: feedwater flow

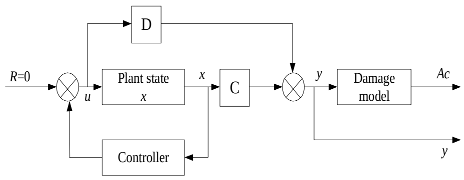

Fig. 2. The structure of the control system

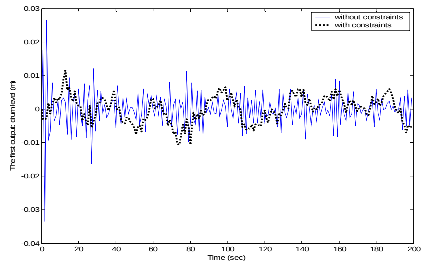

Fig. 3. The first output: drum level

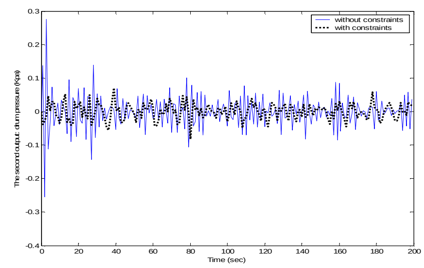

Fig. 4. The second output: drum pressure

(kg/s), fuel flow (kg/s), attemperator spray flow (kg/s) and air flow (kg/s) and three controlled outputs: drum level (m), drum pressure (KPa) and steam temperature (^{0}C). To simplify the control design problem, we assume the ratio of fuel-to-air flow rate is fixed. Therefore, this is a three-input and three-output system. The parameters of this linearized model can be found in (Li, *et al.*, 2003). The continuous time model should be discretized as in the form of Eq. (3), there are 13 states in the discretized model and

$$B_2 = [0.3, \cdots\cdots, 0.3]^T_{(13\times 1)}.$$

The structure of the control system is shown in Fig 2. We use VCMPC algorithm to design the controller, so the controller can limit the variance of the specific output besides minimizing the cost function J, thus the accumulated damage A_c will be reduced.

The cost function is

$$J = \lim_{K\to\infty} E(\sum_{i=0}^{N_x} x(k+i|k)^T Qx(k+i|k)$$
$$+ \sum_{i=0}^{N_u} u(k+i|k)^T Ru(k+i|k)).$$

We set $N_x = 4$, $N_u = 2$, the constraints on the variance of the steam temperature as

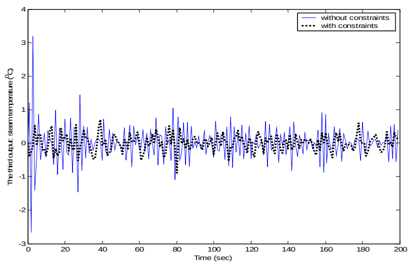

Fig. 5. The third output: steam temperature

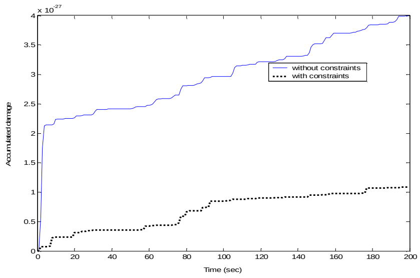

Fig. 6. The accumulated damage

$$\Phi_j = \lim_{k\to\infty} E(Z_3(k+i|k)Z_3(k+i|k)^T) < 0.3,$$

and limit the location of the closed-loop system poles as

$$\sigma(A_c) \subset D(0.5, 0.5).$$

There are constraints on inputs of the plant

$$|u_1| < 120, \qquad |u_2| < 10, \qquad |u_3| < 7.$$

Meanwhile, to show the effectiveness of our algorithm, we also design a controller without the constraints on the variance of outputs and on the location of the closed-loop system poles. Simulation is proceeded under the condition that all initial states are 1 and the reference inputs are zero. Simulation results are shown in Fig. 3 to Fig. 6, in which solid lines represent the results without constraints on the variance and position of poles, dotted lines the results with constraints on the variance and position of poles. In our design, we limit the variance of the third output, the steam temperature, and assign the system poles within a proscribed region to make the system have desired performance. From Fig. 3 to Fig. 5, we see that with the VCMPC algorithm, the response speed is decreased; however, the variance of the steam temperature has been controlled effectively. As a result, the accumulated damage is reduced significantly, as shown in Fig. 6.

5. CONCLUSIONS

The variance of the temperature is the most important factor for LEC to reduce the thermal accumulated damage of some critical components in a boiler system. On the basis of this conclusion, a research on VCMPC problem was proceeded and an algorithm via LMIs was derived. Simulation results showed this controller can significantly reduce the accumulated damage of the critical component while satisfactory performance was obtained. Since the controller is in the form of a state feedback, a condition is assumed: all states of the system are measurable. The dynamic output feedback controller will be the next step of our research.

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