ROBUST POLE ASSIGNMENT BY STATE FEEDBACK CONTROL USING INTERVAL ANALYSIS

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Abstract: An interval analysis approach for the design of robust state feedback controllers is proposed. It is shown that when regional pole placement specifications are represented as spectral sets of interval polynomials, the robust state feedback design problem can be entirely formulated and solved in the context of the concepts and methods of interval analysis. Explicit convex polyhedral representations of a class of robust state feedback controllers satisfying an *interval Ackerman's equation* are derived. A design procedure based on nonlinear programming which aims at maximizing the non-fragility of the resulting robust controller is introduced. Numerical examples illustrate the design of robust state feedback controllers through the interval analysis approach proposed.

Keywords: Uncertain linear systems, pole assignment, intervals, polynomials, numerical algorithms.

1. INTRODUCTION

The problem of designing state feedback controllers for linear time-invariant systems has been extensively treated in the control system literature. Stabilizability conditions via constant state feedback, as well as state feedback solutions for pole placement problems under the assumption of a precisely known system have been completely characterized (Chen, 1999). However, linear models of real systems sometimes include parameters whose values are unknown but bounded in compact sets, often described in the form of closed intervals. In this case, stabilization and performance via state feedback must be addressed in a robust sense. The robust control problem consists in finding a state feedback gain so as to place all closed-loop poles in the left-half side of the complex plane (robust stabilization) or in some prescribed region of it (robust performance) for every possible set of system parameters. The robust stabilization problem has been tackled through two distinct approaches (Wei, 1994). In the first one, the uncertain system is viewed as a nominal system subject to perturbations. The problem is decomposed into subproblems of stabilizing the nominal system and then proving that the closed-loop system remains stable in spite of all the admissible perturbations. According to the second approach, the stabilizability of the system is initially determined and then a stabilizing control is designed.

In this paper a robust state feedback approach for linear time-invariant interval systems which combines some of the above characteristics is

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proposed. As in (Keel and Battacharyya, 1999), regional pole placement specifications are formulated as *spectral sets of interval polynomials*, which can be efficiently described through the *Edge Theorem* (Bartlett *et al.*, 1988). However, the design of robust state feedback controllers is based on the application of concepts and methods of *interval analysis* (Alefeld and Herzberger, 1983) to the interval matrix representation of the system. Interval analysis is becoming an important tool in several areas related to control system design, as comprehensively discussed in (Jaulin *et al.*, 2001). As in (Smagina and Brewer, 2002), the core of our approach is the *interval Ackermann's equation*, the equation associated with the Ackerman's formula, whose *inner* (or *tolerable*) solutions are known to represent robust stabilizing controllers. This basic idea is then combined with results about inner solutions of linear interval equations (Rohn, 1986) and the formalism derived in (Lordelo and Ferreira, 2002) for systems described by transfer functions, according to which robust pole placement controllers are viewed as inner solutions of an *interval Diophantine equation.*

The paper is organized as follows. In Section II, the robust pole placement principle used for the design of state feedback controllers is presented. Section III addresses the problem of designing robust controllers assuming complete access to the state vector. The interval Ackermann's equation for robust pole placement is presented; convex polyhedral representations of the set of robust controllers associated with the Ackermann's equation are derived. The design of non-fragile state feedback controllers as a *design centering problem* is addressed in Section IV. In Section V, a sufficient condition for robust controllability (observability) of interval systems based on an interval QR–factorization method is proposed in the context of multivariable systems. Finally, in Section VI some conclusions are presented.

Notation. The sets of real (complex) numbers and real $m \times n$ matrices are represented as $\mathbb{R}$ ($\mathbb{C}$) and $\mathbb{R}^{m\times n}$, respectively. The symbol $:=$ means equal by definition. The *transpose* of $A \in \mathbb{R}^{m\times n}$ is denoted as A^T, and defining $A = \{\alpha_{ij}\}$, the *absolute value* matrix of A equals $|A| = \{|\alpha_{ij}|\}$. The sets of interval real numbers and interval real $m \times n$ matrices are represented as $\mathbb{IR}$ and $\mathbb{IR}^{m\times n}$, respectively. A *closed interval* $[\alpha] \in \mathbb{IR}$ is denoted as $[\alpha] = [\alpha^-, \alpha^+]$, where the $\alpha^-, \alpha^+ \in \mathbb{R}$, with $\alpha^+ \geq \alpha^-$. An interval matrix $[A]$ is defined by $[A] = \{[a_{ij}]\}$, where $[a_{ij}] := [a_{ij}^-, a_{ij}^+]$ for each i, j. Alternatively,

$$[A] = [A^-, A^+] = \{A \ : \ A^- \leq A \leq A^+\},$$

where $A^- := \{a_{ij}^-\}$, $A^+ := \{a_{ij}^+\}$ and the inequality is meant to be componentwise. The *width*, *center* and *radius* of $[A]$ are defined by $A_w = A^+ - A^-$, $A_c = \frac{1}{2}(A^+ + A^-)$ and $A_\delta = \frac{1}{2}(A^+ - A^-)$, respectively. An inclusion of the form $[A] \subset [B]$ means that $A^- \geq B^-$ and $A^+ \leq B^+$. If the symbol $*$ represents one of the arithmetic operations $+, -$ or $\cdot$, and $[A]$ and $[B]$ are interval matrices of compatible dimensions, then

$$[A] * [B] := \\ \{A * B \ : \ A \in [A^-, A^+], \ B \in [B^-, B^+]\},$$

As usual the symbol $\cdot$ for multiplication of reals or intervals is omitted. A monic n-degree interval polynomial is defined by $p(s) = s^n + p_{n-1}s^{n-1} + \cdots + p_0$, where $p_i \in [p_i] := [p_i^-, p_i^+]$, $i = 0, 1, \ldots, n-1$; another representation is $[p(s)] = s^n + [p_{n-1}]s^{n-1} + \cdots + [p_0]$. The numerical operations on intervals reported in this paper have been performed using INTLAB, an interval arithmetic software package developed by S. Rump (`http://www.ti3.tu-harburg.de`).

2. ROBUST POLE PLACEMENT

Consider the linear time-invariant single input, single output interval system

$$\dot{x} = [A]x + [b]u, \quad (1)$$

$$y = [c]x, \quad (2)$$

where $x = x(t) \in \mathbb{R}^n$, $u = u(t) \in \mathbb{R}$ and $y = y(t) \in \mathbb{R}$ are the state, control and output variables of the system, respectively. The interval state matrix $[A] \in \mathbb{IR}^{n\times n}$ and the interval control and output vectors $[b] \in \mathbb{IR}^{n\times 1}$ and $[c] \in \mathbb{IR}^{1\times n}$ are introduced so as to model structured uncertainties in the form of unknown but bounded system parameters. In this paper the robust pole placement design principle originally introduced in (Soh *et al.*, 1987) is adopted. According to that principle, it is required to robustly assign closed-loop characteristic polynomials in an interval family of characteristic polynomials

$$[p(s)] := s^n + [p_{n-1}]s^{n-1} + \cdots + [p_0],$$

where $[p_i] := [p_i^-, p_i^+]$, $i = 0, 1, \ldots, n-1$ are interval coefficients. Explicit representations for $[p(s)]$ when the closed-loop poles assume simple forms are discussed in (Soh *et al.*, 1987). An alternative robust pole placement design procedure is proposed in (Keel and Battacharyya, 1999) via the concept of *spectral set* of an interval polynomial, defined as

$$\mathcal{S}([p(s)]) := \{s \in \mathbb{C} \ : \ p(s) = 0, \\ p_i \in [p_i^-, p_i^+], \ i = 0, 1, \ldots, n-1\}.$$

The basic idea in (Keel and Battacharyya, 1999) is to create a regional pole placement specification in the form of the spectral set of an interval polynomial, taking advantage of the fact that that spectral sets of interval polynomials can be effectively described through the *Edge Theorem* (Bartlett *et al.*, 1988). Assuming that adequate closed-loop characteristic polynomials have been previously specified, the following robust control system design problems can be formulated.

Controller design. Given $[p(s)]$, find a constant state-feedback gain $k \in \mathbb{R}^{1\times n}$ such that

$$\det(sI - A + bk) \in [p(s)]$$

for every $A \in [A]$ and $b \in [b]$.

Observer design. Given $[q(s)]$, find a constant observer gain $l \in \mathbb{R}^{n\times 1}$ such that

$$\det(sI - A + lc) \in [q(s)]$$

for every $A \in [A]$ and $c \in [c]$.

If the poles in $\mathcal{S}([q(s)])$ are sufficiently faster than those in $\mathcal{S}([p(s)])$, and the state x is replaced by its observed value $\hat{x}$, one obtains a robust state-observed feedback controller $u = -k\hat{x}$. The computation of the observer gain l is not explicitly considered in this paper. It can be obtained by applying the interval analysis approach proposed to the *interval dual system*

$$\dot{z} = [A]^T z + [c]^T v,$$
$$w = [b]^T z.$$

It is apparent that the existence of a robust controller assigning closed-loop poles in arbitrary locations of the complex plane requires controllability (observability) in a robust sense. The interval system $([A], [b], [c])$ is said to be controllable if the rank of the $n \times n$ *controllability matrix*

$$M := \left[\, b \;\vdots\; Ab \;\vdots\; \cdots \;\vdots\; A^{n-1}b \,\right]$$

equals n for every pair $(A, b) \in ([A], [b])$. The *pair* $([A], [b])$ is then said to be controllable. Invoking the *duality principle* in control system design (Chen, 1999), one concludes that the interval system $([A], [b], [c])$ is observable if and only if the pair $([A]^T, [c]^T)$ is controllable.

The *interval matrix extension* of the controllability matrix is given by (Alefeld and Herzberger, 1983)

$$[\mathcal{M}] = \left[\, [b] \;\vdots\; [A]\,[b] \;\vdots\; \cdots \;\vdots\; [A]^{n-1}\,[b] \,\right].$$

It should be observed that $[\mathcal{M}] \in \mathbb{IR}^{n\times n}$ contains every possible controllability matrix of the interval system, but not all matrices in $[\mathcal{M}]$ are controllability matrices. Nevertheless, if $\operatorname{rank}(\mathcal{M}) = n$ for every $\mathcal{M} \in [\mathcal{M}]$, then the pair $([A], [b])$ is controllable. Denoting as $[\det([\mathcal{M}])]$ the interval extension of $\det(\mathcal{M})$, $\mathcal{M} \in [\mathcal{M}]$, it follows that $([A], [b])$ is controllable if $0 \notin [\det([\mathcal{M}])]$. To obtain the interval extension $[\det([\mathcal{M}])]$ is, however, computationally expensive. A numerical procedure for testing controllability (observability) based on an interval extension of the *QR*–factorization method (Bentbib, 2002) is proposed in Section V, in the context of multivariable systems.

3. CONTROLLER DESIGN

An extension to interval systems of the classical state feedback design technique is presented in this section. In the non-interval case, given the characteristic polynomial of A,

$$\det(sI - A) = s^n + \alpha_{n-1}s^{n-1} + \cdots + \alpha_1 s + \alpha_0,$$

and the desired closed-loop characteristic polynomial

$$p(s) = s^n + p_{n-1}s^{n-1} + \cdots + p_1 + p_0,$$

a constant feedback gain $k := [k_0 \;\; k_1 \;\; \cdots \;\; k_{n-1}]$ assigning $p(s)$ can be computed by the Ackermann's equation (Ogata, 1997)

$$kMW + \alpha = p, \tag{3}$$

where

$$p := \left[\, p_0 \; p_1 \; \cdots \; p_{n-1} \,\right], \; \alpha := \left[\, \alpha_0 \; \alpha_1 \; \cdots \; \alpha_{n-1} \,\right]$$

and

$$W := \begin{bmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_{n-1} & 1 \\ \alpha_2 & \alpha_3 & \cdots & 1 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \alpha_{n-1} & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix}.$$

Since the coefficients of the characteristic polynomial of A are multilinear functions of its elements, the interval extensions of these coefficients, namely $[\alpha_i]$, $i = 0, 1, \ldots, n-1$, can be computed, as well as $[\mathcal{W}] \in \mathbb{IR}^{n\times n}$, the interval matrix extension of W considering that $A \in [A]$. The interval extension of the Ackermann's equation is given by

$$k[\mathcal{M}][\mathcal{W}] + [\alpha] = [p], \tag{4}$$

where $[p] \in \mathbb{IR}^{1\times n}$ represents the interval closed-loop characteristic polynomial. A necessary condition for the solvability of linear interval equations of the form (4) is that $p_w \geq \alpha_w$, meaning that the width of p must be greater than or equal to the width of α. The next step is to remove $[\alpha]$ from the left-hand side of (4), which can not be done by using the standard definition of interval subtraction because, in general, $[\alpha] - [\alpha] \neq [0, 0]$. The *extended subtraction* (Inuiguchi and Kume, 1991)

$$[p] \ominus [\alpha] := [p^- - \alpha^-, p^+ - \alpha^+],$$

is used instead. Noting that $[\alpha] \ominus [\alpha] = [0, 0]$, one obtains the system of linear interval equations

$$k[T] = [f], \tag{5}$$

where $[T] := [\mathcal{M}][\mathcal{W}]$ and $[f] := [p] \ominus [\alpha]$. As happens in the non-interval case, the controller design has a simple solution when the state equation is the *interval controllable form*, in which

$$[A] = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -[\alpha_0] & -[\alpha_1] & -[\alpha_2] & \cdots & -[\alpha_{n-1}] \end{bmatrix}$$

and

$$[b]^T = b^T = \begin{bmatrix} 0 \; 0 \cdots 0 \; 1 \end{bmatrix}.$$

It is readily seen that the above interval pair $([A], [b])$ is always controllable. Interval controllable representations are easily obtained from systems represented by interval transfer functions.

Theorem 1. Let $([A], [b], [c])$ be a linear time-invariant interval system in the interval controllable form. Then

$$k \in [f] = [p] \ominus [\alpha]. \tag{6}$$

solves the robust pole placement problem.

Proof: Given the structures of $[A]$ and $[b]$, it follows that

$$\det(sI - [A] + [b]k) = s^n + ([\alpha_{n-1}] + k_{n-1})s^{n-1} + \cdots + ([\alpha_1] + k_1) + ([\alpha_0] + k_0),$$

and the solution of $\det(sI - [A] + [b]k) = [p(s)]$ by using the extended subtraction leads to (6). □

Clearly, (6) generalizes the solution of the controller design problem based on pole placement, in the sense that if $p^- = p^+ = p$ and $\alpha^- = \alpha^+ = \alpha$, then $k = p - \alpha$. Although any feedback gain satisfying (6) solves the pole placement problem, the *central controller* $k = f_c = \frac{1}{2}(f^+ + f^-)$ seems to be preferable due to its maximal *non-fragility* with respect to gain variations. The maximal variation allowed in the coefficients of f_c, that is, the *radius* of the interval vector centered in f_c, referred as θ, is easily computed.

Example 1. Consider the interval controllable form associated with a third order interval transfer function discussed in (Jaulin *et al.*, 2001):

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\alpha_0 & -\alpha_1 & -\alpha_2 \end{bmatrix}, \; b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \; c^T = \begin{bmatrix} \gamma \\ 0 \\ 0 \end{bmatrix},$$

where

$$\alpha_0 = \frac{p_3^2}{p_2}, \; \alpha_1 = p_3^2 + \frac{p_3}{p_2}, \; \alpha_2 = p_3 + \frac{1}{p_2}, \; \gamma = \frac{p_1 p_3^2}{p_2}.$$

Assuming that $[p_1] = [p_2] = [p_3] = [0.97, 1.03]$ and using interval arithmetics, one obtains the interval system $([A], [b], [c])$ characterized by

$$[\alpha_0] = [0.9134, 1.0937], \; [\alpha_1] = [1.8826, 2.1227],$$
$$[\alpha_2] = [1.9408, 2.0609], \; [\gamma] = [0.8860, 1.1265].$$

The interval characteristic polynomial

$$[p(s)] = s^3 + [7.469, 8.536]s^2 + [20.89, 27.32]s + [25.98, 38.87],$$

encloses the *nominal* characteristic polynomial $p(s) = s^3 + 8s^2 + 24s + 32$ (poles at -4, $-2 \pm j2$). The spectral set of $[p(s)]$ is illustrated in Figure 1 by using light grey lines. Since the system is in the interval controllable form, a possible solution for the robust pole placement problem is the central controller $k = f_c = [31.42 \quad 22.10 \quad 6.001]$. The spectral set of $\det(sI - [A] + b[k])$ is illustrated in Figure 1 using dark grey lines. The closed-loop poles remain inside the spectral set of $[p(s)]$ for all possible $A \in [A]$ and $k \in [k]$, where $[k]$ is the interval vector with center in f_c and radius $\theta = 0.0793$. □

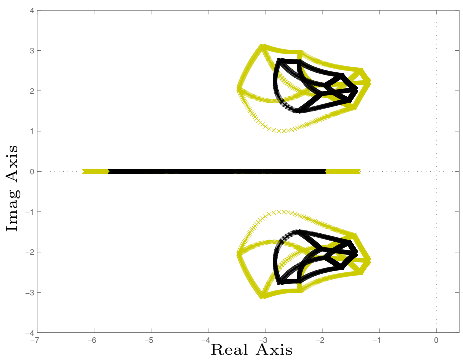

Fig. 1: *Spectra* of $[p(s)]$ and $\det(sI - [A] + b[k])$.

In (Smagina and Brewer, 2002) is shown that the robust stabilization problem may have a solution of the form $k = f_c(T_c)^{-1}$, provided that $f_c(T_c)^{-1}[T] \in [f]$, and that $[p(s)]$ is an interval Hurwitz polynomial satisfying $p_w > \alpha_w$. Here, using concepts and results of interval analysis applied to systems of linear interval equations, the whole set of robust pole placement controllers which can be derived from the interval Ackermann's equation is characterized. The solution set of (5) is defined by (Rohn, 1989)

$$\mathcal{K} := \{k \; : \; kT = f \; \text{ for some } T \in [T], f \in [f]\}.$$

The subset of the *inner solutions* of $\mathcal{K}$, given by (Rohn, 1986)

$$\mathcal{K}_0 := \{k \; : \; kT \in [f], \quad T \in [T]\},$$

characterizes all the robust state feedback controllers associated with (5). The following representations of $\mathcal{K}_0$ are consequences of the application of interval analysis results.

Theorem 2. (Representations of $\mathcal{K}_0$). Let $\mathcal{K}_0$ be the set of all inner solutions of the interval Ackermann's equation $k[T] = [f]$ and define

$$\mathcal{K}_1 := \{k \ : \ |kT_c - f_c| + |k|\,T_\delta \leq f_\delta\};$$

$$\begin{aligned}\mathcal{K}_2 := \{k \ : \ & k = k^1 - k^2,\\ & k^1T^- - k^2T^+ \geq f^-,\\ & k^1T^+ - k^2T^- \leq f^+,\\ & k^1 \geq 0,\ k^2 \geq 0\,\};\end{aligned}$$

$$\begin{aligned}\mathcal{K}_3 := \{(k,\bar{k}) \ : \ & kT_c - \bar{k}T_\delta \geq f^-,\\ & kT_c + \bar{k}T_\delta \leq f^+,\\ & -\bar{k} \leq k \leq \bar{k}\,\}.\end{aligned}$$

Then $\mathcal{K}_0 = \mathcal{K}_1 = \mathcal{K}_2$ and $k \in \mathcal{K}_0$ if and only if there exists $\bar{k}$ such that $(k,\bar{k}) \in \mathcal{K}_3$.

Proof: The proofs involving the equivalences $\mathcal{K}_0 = \mathcal{K}_1 = \mathcal{K}_2$ are derived from (Rohn, 1986). The correspondence between $\mathcal{K}_0$ and $\mathcal{K}_3$ is based on (Kelling, 1994) $\square$

From the equivalence $\mathcal{K}_0 = \mathcal{K}_1$, one concludes that $\mathcal{K}_0$ is a convex set. In addition, if every column of T_δ has at least one nonzero element, then $\mathcal{K}_0$ is bounded.

4. NON-FRAGILE DESIGNS

An important concern while designing feedback controllers is to avoid that small variations in the coefficients of the designed controller, dictated by implementation issues, for example, deteriorate the closed-loop performance significantly (Keel and Battacharyya, 1997). To avoid fragility, a controller design procedure based on the solution of a *design centering problem*, a classical problem in nonlinear programming, is proposed. The idea is to find the *center* k and the largest *radius* $\theta \geq 0$ such that

$$k + \theta\mathcal{C} \in \mathcal{K}_0,$$

where $\mathcal{C}$ is a given set specifying how the controller coefficients can vary and $k+\theta\mathcal{C} := \{k+\theta v,\ v \in \mathcal{C}\}$. The radius θ represents a measure of the fragility of the robust controller k when its coefficients vary as specified by $\mathcal{C}$. Assuming that $\mathcal{C}$ is an hyperretangle (that is, an interval vector), the controller design problem assumes the form

$$(P_\theta)\ \left|\ \begin{array}{ll}\max\ \theta & \\ \text{s.t.} & (I_n \pm \theta V)kT_c - \bar{k}T_\delta \geq f^-,\\ & (I_n \pm \theta V)kT_c + \bar{k}T_\delta \leq f^+,\\ & -\bar{k} \leq (I_n \pm \theta V)k \leq \bar{k},\\ & \theta \geq 0,\end{array}\right.$$

where '±' stands for two inequalities (one for '+', another for '−'), I_n denotes the n-order identity matrix and $V := \mathrm{diag}\,(v_1, v_2, \ldots, v_n)$, where $v_i \geq 0$ represents the relative weight attributed to i-th gain: the larger the value v_i relatively to the other $v_j,\ j \neq i$, the smaller the variation allowed in k_i. The nonlinear problem (P_θ) has been solved by using MATLAB (Optimization Toolbox).

5. MULTIVARIABLE SYSTEMS

The design of state feedback controllers for MIMO systems can be reduced to the design of SISO systems if one assumes that A is *cyclic*, that is, if the characteristic polynomial of A equals its minimal polynomial. If the pair (A,B) is controllable and A is cyclic, then the pair (A,Bq) is controllable for *almost all* $q \in \mathbb{R}^m$, where m is the number of system inputs (colums of B) (Chen, 1999). Adopting the interval extension of this design principle (Smagina and Brewer, 2002), the robust state feedback gain for interval MIMO systems assumes the form $K = qk$, where k can be obtained by letting $[b] := [B]q$ and then applying the design procedure discussed in the previous sections of this paper. Of course, one can simply apply the procedure and try to obtain a robust controller without an *a priori* controllability test. For the sake of completeness, the following controllability (observability) test based on an interval extension of the QR–factorization method (Bentbib, 2002) is proposed. Given any interval matrix $[N] \in \mathbb{IR}^{m\times n}$ with $m \geq n$, one obtains an orthogonal $m \times m$ interval matrix $[Q]$, an upper trapezoidal $m \times n$ interval matrix $[R]$, and then a factorization of the form $[N] = [Q][R]$, meaning that for every $N \in [N]$ there exist matrices $Q \in [Q]$ and $R \in [R]$ such that $N = QR$. Then $\mathrm{rank}([N]) = n$ if $\mathrm{rank}([R]) = n$. Since $[R]$ exhibits the upper trapezoidal interval form

$$[R] = \begin{bmatrix}\left[\widetilde{R}\right]\\ \cdots\\ \left[0\right]\end{bmatrix},$$

where $[\widetilde{R}] \in \mathbb{IR}^{n\times n}$ is an upper triangular interval matrix, it follows that $\mathrm{rank}([R]) = n$ if and only if $0 \notin [\tilde{r}_{ii}]$ for $i = 1, 2, \ldots, n$.

Example 2. Consider the linearized state equation for the longitudinal motion speed of a helicopter, discussed in (Smagina and Brewer, 2002):

$$[A] = \begin{bmatrix}[a_1] & [a_2] & -9.8\\ [a_3] & [a_4] & 0\\ 0 & 1 & 0\end{bmatrix},\ [B] = \begin{bmatrix}[b_1] & 0\\ 0 & [b_2]\\ 0 & 0\end{bmatrix},$$

with $[a_1] = [-0.031, -0.0128]$, $[a_2] = [-3.4, -0.1]$, $[a_3] = [-0.00077, -0.0007]$, $[a_4] = [-0.32, -0.31]$, $[b_1] = [-18, -15]$ and $[b_2] = [-3.3, -3]$. The interval QR–factorization method has been used for checking the controllability of $([A],[B])$. For convenience the algorithm has been applied to $[\mathcal{M}]^T \in \mathbb{IR}^{6\times 3}$. The relevant interval matrix for the analysis is

$$[\widetilde{R}] = \begin{bmatrix} [29.7, 38.7] & [-0.40, 0.24] & [-0.73, 1.21] \\ [0,0] & [3.11, 3.52] & [-1.55, -0.47] \\ [0,0] & [0,0] & [-3.81, -2.43] \end{bmatrix}.$$

Since $0 \notin [\tilde{r}_{ii}]$, $i = 1, 2, 3$, we conclude that $\text{rank}([R]) = n$ and therefore the pair $([A],[B])$ is controllable. In (Smagina and Brewer, 2002), given the interval family of Hurwitz polynomials

$$[p(s)] = s^3 + [3,4]s^2 + [2,8]s + [0.5, 5.5],$$

it is required to find a state feedback gain $K \in \mathbb{R}^{2\times 3}$ such that $\det(sI - A + BK) \in [p(s)]$ for every $A \in [A]$ and $B \in [B]$, thus assuring the robust stability of the closed-loop system. Adopting $q^T = [0.8\ 1.2]$ and solving problem (P_θ) with $V = \text{diag}(1,1,1)$, the robust state feedback gain $k^* = [0.0266 \quad -0.9297 \quad -0.7028]$ is found, which then provides

$$K^* = qk^* = \begin{bmatrix} 0.0213 & -0.7438 & -0.5622 \\ 0.0319 & -1.1157 & -0.8433 \end{bmatrix}.$$

The optimal value of θ has been $\theta^* = 0.0852$, meaning that the state feedback gains may vary up to 8.5% without destabilizing the closed-loop system. □

6. CONCLUSIONS

The design of state feedback controllers for interval systems based on the robust pole placement design principle has been addressed in this paper. Simple conditions for the existence of robust state feedback controllers have been derived; a procedure based on the solution of a design centering problem which aims at maximizing their non-fragility has been proposed and applied, with good simulation results. The authors are currently investigating ways of improving some numerical aspects involved in the application of interval analysis to robust control system design, especially those associated with the concept of controllability (observability) of interval systems.

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