EXACT PARAMETER ESTIMATION USING RELAY FEEDBACK CONTROL

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Abstract: Obtaining the parameters for PID controllers based on limit cycle information from the process in a relay controlled feedback loop is, in many cases, an acceptable practical procedure. If the form of the plant transfer function is known, exact expressions for the limit cycle frequency and amplitude can be derived in terms of the plant parameters, so that their measurements, assumed error free, can be used to calculate two unknown plant parameter values. In the literature to date the solutions have been considered for stable or unstable first order plus dead time (FOPDT) or second order plus dead time (SOPDT) plant transfer functions. This paper reports on exact parameter estimation for an SOPDT plant transfer function with the further addition of a , stable or unstable, zero by a single relay feedback test.

Keywords: Identification, Relay control, Transfer functions, Time delay

1. INTRODUCTION

Current commercial controllers, such as PID controllers, have used microprocessors for several years and this provides an opportunity to do more than perform simple control algorithms. Auto-tuning may be defined as an experiment performed by a controller in order to find suitable parameters for it to control the system. One of the most well known of these procedures is relay auto-tuning where the controller acts as a relay and measures the amplitude and frequency of the resulting loop limit cycle to set its parameters. The most common procedure is to use the expressions from describing function, DF, analysis of the closed loop to estimate the frequency, ω_c, at which the plant has 180^o phase shift and also the corresponding plant gain, $1/K_c$. These estimates for ω_c and K_c because of the approximations of the DF analysis will have some error. Another approach is to assume a simple plant transfer function, such as an FOPDT, $Ke^{-T_d s}/(1+Ts)$, and estimate two of its unknown parameters, usually 'T_d' and 'T' with 'K' assumed known, again using the DF approximation. Results can be improved by doing two experiments, the second with a filter in the loop to make the limit cycle sinusoidal. Also, if the form of the plant transfer function is known it is possible to determine exactly the limit cycle waveform and this can in theory be used to obtain accurate values of the plant parameters. In practice, of course, due to system noise and measurement errors the exact frequency and amplitude of the limit cycle will not be found from any measurements.

Luyben (1987) was one of the first to consider estimating the plant transfer function from limit cycle measurements and used the approximate DF method. Several authors (Li *et al.*, 1991; Shen *et al.*, 1996) have presented further approaches, which make use of the approximate DF method.

The fact that exact expressions can be found for a limit cycle in a relay feedback system has been known for many years (Bohn, 1961; Chung and Atherton, 1966; Atherton, 1966). Atherton (1997) showed how knowledge of the exact solution for limit cycles in relay controlled FOPDT plants could be used to give more accurate results using the DF method. Recently, several papers (Chang *et al.*, 1992; Wang *et al.*, 1997; Kaya and Atherton, 1999) have been written on using exact analysis for parameter estimation in a relay feedback system, assuming a specific plant transfer function and an odd symmetrical limit cycle. There are also some publications (Kaya and Atherton, 1998; Kaya and Atherton, 2001) which use asymmetrical limit cycle data. All these papers consider the identification problem for a stable or an unstable FOPDT and/or SOPDT process. Atherton and Majhi (1998) used relay feedback control for parameter estimation of processes with a zero using a state space approach.

In this paper exact expressions have been derived, using the Tsypkin approach, for the simple features of asymmetrical limit cycles in relay controlled loops with both stable and unstable FOPDT and SOPDT plant transfer functions with a left-hand side (l.h.s) or right-hand side (r.h.s) s-plane zero. This generalizes the identification problem for a process. That is, with the expressions provided one can estimate the parameters for stable and/or unstable FOPDT process with and/or without a zero. Similarly, the expressions provided can also be used for parameter identification of a stable or unstable SOPDT process with or without a zero.

2. GENERAL SOPDT PROCESS MODEL

In process control problems, it is generally assumed that the process transfer function is a stable or an unstable FOPDT or SOPDT. Some complex chemical processes, however, have a non-minimum phase characteristic and the identification problem for this type of process is considered here.

Consider the following general SOPDT plant transfer function

$$G(s) = \frac{K(\mp T_0 s + 1)e^{-\theta s}}{(T_1 s \mp 1)(T_2 s + 1)} \qquad (1)$$

When $T_0=0$ and $T_2=0$, *G(s)* becomes a stable or an unstable FOPDT plant transfer function model. If only $T_0=0$, then *G(s)* is a stable or an unstable SOPDT plant transfer function. *G(s)*, given by eqn. (1), therefore, represents a general SOPDT process model.

3. A–FUNCTION METHOD

When the nonlinearity in Fig. 1 is a relay, then exact solutions for the limit cycle frequency and amplitude are possible and Tyspkin's approach is one procedure for doing this. The method was developed many years ago by Tyspkin and further developed by Atherton (1966) who introduced the A-Function.

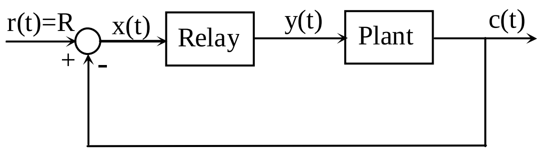

Fig. 1: Relay Feedback System

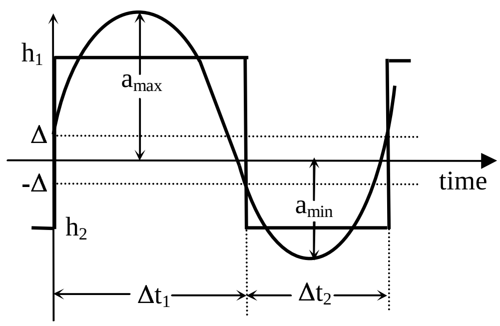

Fig. 2: Relay input and output

The A-Function of a linear transfer function is a complex function of both time and frequency and for a transfer function, *G(s)*, for a specific time, *t*, or phase, $\theta = \omega t$, and frequency, ω, is given by

$$A_G(\theta,\omega) = \operatorname{Re} A_G(\theta,\omega) + j \operatorname{Im} A_G(\theta,\omega) \qquad (2)$$

where,

$$\operatorname{Re} A_G(\theta,\omega) = \sum_{n=1}^{\infty} \{V_G(n\omega)\sin n\theta + U_G(n\omega)\cos n\theta\} \qquad (3)$$

$$\operatorname{Im} A_G(\theta,\omega) = \sum_{n=1}^{\infty} \frac{1}{n}\{V_G(n\omega)\cos n\theta - U_G(n\omega)\sin n\theta\} \qquad (4)$$

Here "Re" and "Im" stand for the real and imaginary parts of the A-locus and U_G and V_G are the real and imaginary parts of the transfer function $G(jn\omega)$.

The summation can be found analytically from known summations for simple transfer functions or computed from a suitable number of terms in the series, when the plant parameters are known.

The plant output and its derivative in a relay feedback system, assuming either a constant input or a biased relay, can easily be found (Atherton, 1981) as

$$c(t)=\frac{G(0)(h_1\Delta t_1+h_2\Delta t_2)}{P}+\frac{(h_1-h_2)}{\pi}\{\operatorname{Im}A_G(-\omega t,\omega)-\operatorname{Im}A_G(-\omega t+\omega\Delta t_1,\omega)\} \quad (5)$$

and

$$\dot{c}(t)=\frac{(h_1-h_2)\omega}{\pi}\{\operatorname{Re}A_G(-\omega t,\omega)-\operatorname{Re}A_G(-\omega t+\omega\Delta t_1,\omega)\} \quad (6)$$

where *G(0)* is the steady state gain, Δt_1 and Δt_2 are the negative and positive pulse durations of the relay output, h_1 and h_2 are the relay heights and P is the period so that $P = \Delta t_1+\Delta t_2$ as shown in Fig. 2.

When the relay has hysteresis, Δ, the limit cycle conditions can easily be obtained, by imposing the switching requirements at time $t=0$ and $t=\Delta t_1$, and using eqns. (5) and (6) to give

$$\operatorname{Im}A_G(0,\omega)-\operatorname{Im}A_G(\omega\Delta t_1,\omega)=\frac{\pi}{(h_1-h_2)}\{R-\Delta-\frac{G(0)(h_1\Delta t_1+h_2\Delta t_2)}{P}\} \quad (7)$$

$$\operatorname{Re}A_G(0,\omega)-\operatorname{Re}A_G(\omega\Delta t_1,\omega)<0 \quad (8)$$

and

$$\operatorname{Im}A_G(0,\omega)-\operatorname{Im}A_G(-\omega\Delta t_1,\omega)=\frac{-\pi}{(h_1-h_2)}\{R+\Delta-\frac{G(0)(h_1\Delta t_1+h_2\Delta t_2)}{P}\} \quad (9)$$

$$\operatorname{Re}A_G(0,\omega)-\operatorname{Re}A_G(-\omega\Delta t_1,\omega)<0 \quad (10)$$

provided that $\lim_{s\to\infty} sG(s)=0$, otherwise some corrections should be made to the right hand side of eqns. (7)-(10). The reader may refer to reference (Atherton, 1981) for the corrections when this condition does not hold. Eqns. (7) and (9) give the value of the limit cycle frequency ω and pulse duration Δt_1 and satisfaction can be checked by eqns. (8) and (10).

4. PARAMETER ESTIMATION FOR THE SOPDT

Two cases are considered to obtain expressions for estimating unknown parameters of the SOPDT plant transfer function given by eqn. (1): parameter estimation for the stable SOPDT with zero, either stable or unstable, and parameter estimation for the unstable SOPDT with zero, either stable or unstable. The expressions obtained will be valid when the SOPDT does not possess a zero or for a stable or an unstable FOPDT process as well.

4.1 Parameter estimation for stable SOPDT with or without zero

It can be shown using eqn. (4) that the imaginary part of the A-Function for the stable SOPDT transfer function with zero is given by

$$\begin{aligned}\operatorname{Im}A_G(\theta,\omega)=&\frac{A(\omega t+\omega T_d+\lambda_1-\pi)}{2}-\frac{A\pi e^{(\omega t+\omega T_d)/\lambda_1}}{(e^{2\pi/\lambda_1}-1)}+..\\ &..+\frac{B(\omega t+\omega T_d+\lambda_2-\pi)}{2}-\frac{B\pi e^{(\omega t+\omega T_d)/\lambda_2}}{(e^{2\pi/\lambda_2}-1)}\end{aligned} \quad (11)$$

where, $\lambda_1=\omega T_1$ and $\lambda_2=\omega T_2$. The 'A' and 'B' coefficients are

$$A=\frac{K(T_1-T_0)}{(T_1-T_2)},\ B=\frac{K(T_0-T_2)}{(T_1-T_2)} \quad (12)$$

and

$$A=\frac{K(T_1+T_0)}{(T_1-T_2)},\ B=\frac{-K(T_0+T_2)}{(T_1-T_2)} \quad (13)$$

for the stable SOPDT process with l.h.s and r.h.s s-plane zero, respectively.

Using eqn. (11) in eqns. (7) and (9), respectively, the following two equations for obtaining the limit cycle frequency, ω, and pulse duration, Δt_1, are found:

$$\begin{aligned}&\frac{-\omega\Delta t_1(A+B)}{2}+\frac{A\pi e^{T_d/T_1}(e^{\Delta t_1/T_1}-1)}{(e^{2\pi/\lambda_1}-1)}+...\\ &...+\frac{B\pi e^{T_d/T_2}(e^{\Delta t_1/T_2}-1)}{(e^{2\pi/\lambda_2}-1)}=RHS_1\end{aligned} \quad (14)$$

$$\begin{aligned}&\frac{(\omega\Delta t_1-2\pi)(A+B)}{2}+...\\ &...+\frac{A\pi e^{T_d/T_1}(e^{(2\pi-\omega\Delta t_1)/\lambda_1}-1)}{(e^{2\pi/\lambda_1}-1)}+...\\ &...+\frac{B\pi e^{T_d/T_2}(e^{(2\pi-\omega\Delta t_1)/\lambda_2}-1)}{(e^{2\pi/\lambda_2}-1)}=RHS_2\end{aligned} \quad (15)$$

where,

$$RHS_1=\frac{\pi}{(h_1-h_2)}(R-\Delta-\frac{(h_1\Delta t_1+h_2\Delta t_2)G(0)}{P})$$

$$RHS_2=\frac{-\pi}{(h_1-h_2)}(R+\Delta-\frac{(h_1\Delta t_1+h_2\Delta t_2)G(0)}{P})$$

Two more equations can be obtained from eqn. (5) for the maximum and minimum amplitudes of the limit cycle at the plant output which are found to be given by

$$\begin{aligned}a_{\max}=&\frac{(h_1\Delta t_1+h_2\Delta t_2)G(0)}{P}+...\\ &...+\frac{h_1-h_2}{\pi}\{\frac{-\omega\Delta t_1(A+B)}{2}+...\\ &...+\frac{A\pi e^{\theta_1/T_1}(e^{\Delta t_1/T_1}-1)}{(e^{2\pi/\lambda_1}-1)}+...\\ &...+\frac{B\pi e^{\theta_1/T_2}(e^{\Delta t_1/T_2}-1)}{(e^{2\pi/\lambda_2}-1)}\}\end{aligned} \quad (16)$$

and

$$a_{\min} = \frac{(h_1\Delta t_1 + h_2\Delta t_2)G(0)}{P} + \ldots$$
$$\ldots + \frac{(h_1 - h_2)}{\pi}\{\frac{(2\pi - \omega\Delta t_1)(A+B)}{2} + \ldots$$
$$\ldots + \frac{A\pi e^{\theta_2/T_1}(e^{\Delta t_1/T_1} - e^{2\pi/\lambda_1})}{(e^{2\pi/\lambda_1} - 1)} + \ldots \quad (17)$$
$$\ldots + \frac{B\pi e^{\theta_2/T_2}(e^{\Delta t_1/T_2} - e^{2\pi/\lambda_2})}{(e^{2\pi/\lambda_2} - 1)}\}$$

where,

$$\theta_1 = \frac{T_1T_2}{(T_1 - T_2)}\ln\{\frac{-BT_1(e^{\Delta t_1/T_2} - 1)(e^{2\pi/\lambda_1} - 1)}{AT_2(e^{\Delta t_1/T_1} - 1)(e^{2\pi/\lambda_2} - 1)}\}$$

$$\theta_2 = \frac{T_1T_2}{(T_1 - T_2)}\ln\{\frac{-BT_1(e^{\Delta t_1/T_2} - e^{2\pi/\lambda_2})(e^{2\pi/\lambda_1} - 1)}{AT_2(e^{\Delta t_1/T_1} - e^{2\pi/\lambda_1})(e^{2\pi/\lambda_2} - 1)}\}$$

One further equation is needed to obtain the five unknowns of the SOPDT plant transfer function. Fourier analysis can be used to find the steady state gain, K;

$$K = G(0) = \frac{\int_t^{t+P} c(t)dt}{\int_t^{t+P} y(t)dt} \quad (18)$$

Therefore, eqns. (14)-(18) can be solved simultaneously to find five unknowns of the stable SOPDT process with a l.h.s or r.h.s s-plane zero.

4.2 Parameter estimation for unstable SOPDT with or without zero

Eqn. (4) can be used to find the imaginary part of the A-Function for the unstable SOPDT transfer function with zero, which is found to be given by

$$\mathrm{Im}\, A_G(\theta,\omega) = \frac{A(-\omega t - \omega T_d + \lambda_1 + \pi)}{2} + \frac{A\pi e^{(-\omega t - \omega T_d)/\lambda_1}}{(e^{-2\pi/\lambda_1} - 1)} + \frac{B(\omega t + \omega T_d + \lambda_2 - \pi)}{2} - \frac{B\pi e^{(\omega t + \omega T_d)/\lambda_2}}{(e^{2\pi/\lambda_2} - 1)} \quad (19)$$

where, $\lambda_1 = \omega T_1$ and $\lambda_2 = \omega T_2$. The 'A' and 'B' coefficients, in this case, are

$$A = \frac{K(T_0 + T_1)}{(T_1 + T_2)},\ B = \frac{(KT_0 - T_2)}{(T_1 + T_2)} \quad (20)$$

for the unstable SOPDT process with l.h.s zero, and

$$A = \frac{K(T_1 - T_0)}{(T_1 + T_2)},\ B = \frac{-K(T_0 + T_2)}{(T_1 + T_2)} \quad (21)$$

for the unstable SOPDT process with r.h.s zero.

Using eqn. (19) in eqns. (7) and (9), respectively, the following two equations for the limit cycle frequency, ω, and pulse duration, Δt_1, are found:

$$\frac{\omega\Delta t_1(A-B)}{2} + \frac{A\pi e^{-T_d/T_1}(1 - e^{-\Delta t_1/T_1})}{(e^{-2\pi/\lambda_1} - 1)} + \ldots$$
$$\ldots - \frac{B\pi e^{T_d/T_2}(1 - e^{\Delta t_1/T_2})}{(e^{2\pi/\lambda_2} - 1)} = RHS_1 \quad (22)$$

and

$$\frac{(2\pi - \omega\Delta t_1)(A-B)}{2} + \ldots$$
$$\ldots + \frac{A\pi e^{-T_d/T_1}(1 - e^{(\omega\Delta t_1 - 2\pi)/\lambda_1})}{(e^{-2\pi/\lambda_1} - 1)} + \ldots \quad (23)$$
$$\ldots - \frac{B\pi e^{T_d/T_2}(1 - e^{(2\pi - \omega\Delta t_1)/\lambda_2})}{(e^{2\pi/\lambda_2} - 1)} = RHS_2$$

where,

$$RHS_1 = \frac{\pi}{(h_1 - h_2)}(R - \Delta - \frac{(h_1\Delta t_1 + h_2\Delta t_2)G(0)}{P})$$
$$RHS_2 = \frac{-\pi}{(h_1 - h_2)}(R + \Delta - \frac{(h_1\Delta t_1 + h_2\Delta t_2)G(0)}{P})$$

The maximum and minimum amplitudes of the limit cycle at the plant output can be obtained from eqn. (5) as follows

$$a_{\max} = \frac{(h_1\Delta t_1 + h_2\Delta t_2)G(0)}{P} + \ldots$$
$$\ldots + \frac{h_1 - h_2}{\pi}\{\frac{\omega\Delta t_1(A-B)}{2} + \ldots$$
$$\ldots + \frac{A\pi e^{-\theta_1/T_1}(1 - e^{-\Delta t_1/T_1})}{(e^{-2\pi/\lambda_1} - 1)} + \ldots \quad (24)$$
$$\ldots - \frac{B\pi e^{\theta_1/T_2}(1 - e^{\Delta t_1/T_2})}{(e^{2\pi/\lambda_2} - 1)}\}$$

and

$$a_{\min} = \frac{(h_1\Delta t_1 + h_2\Delta t_2)G(0)}{P} + \ldots$$
$$\ldots + \frac{(h_1 - h_2)}{\pi}\{\frac{(2\pi - \omega\Delta t_1)(A-B)}{2} + \ldots$$
$$\ldots + \frac{A\pi e^{-\theta_2/T_1}(e^{-2\pi/\lambda_1} - e^{\Delta t_1/T_1})}{(e^{-2\pi/\lambda_1} - 1)} + \ldots \quad (25)$$
$$\ldots - \frac{B\pi e^{\theta_2/T_2}(e^{2\pi/\lambda_2} - e^{\Delta t_1/T_2})}{(e^{2\pi/\lambda_2} - 1)}\}$$

where,

$$\theta_1 = \frac{T_1T_2}{(T_1 + T_2)}\ln\{\frac{-AT_2(1 - e^{-\Delta t_1/T_1})(e^{2\pi/\lambda_2} - 1)}{BT_1(1 - e^{\Delta t_1/T_2})(e^{-2\pi/\lambda_1} - 1)}\}$$

$$\theta_2 = \frac{T_1T_2}{(T_1 + T_2)}\ln\{\frac{-AT_2(e^{-2\pi/\lambda_1} - e^{-\Delta t_1/T_1})(e^{2\pi/\lambda_2} - 1)}{BT_1(e^{2\pi/\lambda_2} - e^{\Delta t_1/T_2})(e^{-2\pi/\lambda_1} - 1)}\}$$

Again, one more equation is needed in order to obtain the five unknowns of the unstable SOPDT plant transfer function. Similar to the stable SOPDT case, Fourier analysis can be used to find the steady state gain, K, from eqn. (18).

Therefore, eqns. (22)-(25) together with eqn. (18) can be solved simultaneously to find five unknowns of the stable SOPDT process with a stable or an unstable zero.

However, it should be noted that, although, a limit cycle always exists for a stable plant transfer function, this is not the case for an unstable one. For example, for an unstable FOPDT plant transfer function, an odd symmetrical limit cycle can only exist when $T_d/T<0.693$ (Kaya and Atherton, 2001). This ratio decreases as the asymmetry in the limit cycle increases and/or the hysteresis in the relay is chosen larger (Kaya and Atherton, 2001). For an unstable SOPDT $T_1<T_2$, where T_1 and T_2 are the time constants for the unstable and stable poles, respectively, must be satisfied for a limit cycle to exist (Kaya and Atherton, 2001). Also, the smaller the value of T_1/T_2 the smaller the value of T_d/T_1 for a limit cycle to exist. Therefore, the relay feedback may not yield a limit cycle if the unstable transfer function parameters do not lie in certain ranges.

5. SIMULATION EXAMPLES

In this section several examples are given to illustrate the use of the proposed identification method. The relay controlled plants were simulated using SIMULINK and measurements of the zero crossing frequency, ω, and the maximum and minimum amplitude values, a_{max} and a_{min}, taken from the limit cycle waveform, $x(t)$. The pulse duration, Δt_1 was obtained from the relay output, $y(t)$.

Example 1

Consider a stable FOPDT

$$G(s)=\frac{e^{-2s}}{5s+1}$$

A relay test with $h_1=1$ and $h_2=-0.7$ was performed to obtain an asymmetrical limit cycle. From simulation, limit cycle frequency $\omega=0.902$, pulse duration $\Delta t_1=3.038$, maximum and minimum of the limit cycle amplitudes $a_{max}=0.330$ and $a_{min}=-0.231$ were measured. From eqn. (18), the steady state gain was found to be $K=0.999$. Using eqn. (16) or (17), the time constant was calculated to be $T=4.999$ and time delay $T_d=1.999$, using eqn. (14) or (15). Note that in this case not all the equations are needed for estimation of the three unknowns.

Example 2

A non-minimum phase plus time delay process

$$G(s)=\frac{(1-s)e^{-s}}{(2s+1)(s+1)}$$

is considered. A limit cycle was obtained with $h_1=1$ and $h_2=-0.8$. From the simulation, the quantities measured were $\omega=0.793$, $\Delta t_1=3.734$, $a_{max}=0.740$ and $a_{min}=-0.642$. $K=0.999$ was found from eqn. (18). Eqns. (14), (15), (16) and (17) were simultaneously used to identify $T_0=-1.002$, $T_1=1.999$, $T_2=1.000$ and $T_d=0.999$.

Example 3

Consider an unstable FOPDT

$$G(s)=\frac{2e^{-s}}{3s-1}$$

A relay test with $h_1=1$ and $h_2=-0.7$ was performed to obtain an asymmetrical limit cycle, which has a limit cycle frequency $\omega=1.148$, pulse duration $\Delta t_1=1.973$, maximum and minimum of the limit cycle amplitudes $a_{max}=0.791$ and $a_{min}=-0.554$. Eqn. (18), was used to find $K=2.001$. Eqn. (24) (or (25)) was used to find the time constant $T=3.003$. The time delay was calculated from eqn. (22) (or (23)) as $T_d=1.000$. As in example 1, not all the equations are needed for estimation of the three unknowns.

Example 4

An unstable SOPDT

$$G(s)=\frac{e^{-0.5s}}{(5s-1)(s+1)}$$

is considered. The relay had heights $h_1=1$ and $h_2=-0.6$ and hysteresis $\Delta=0.1$. Measured quantities were $\omega=0.972$, $\Delta t_1=2.211$, $a_{max}=0.172$ and $a_{min}=-0.0931$. Eqn. (18) gave K=1.000. From eqns. (24) and (25) time constants were calculated to be $T_1=4.996$ and $T_2=1.002$. The time delay was identified from eqn. (26) as $T_d=0.499$.

6. CONCLUSIONS

The relay feedback method has become an accepted practical procedure for obtaining the parameters of PID controllers based on limit cycle information. However, this method may lead to inaccurate results if the approximate DF method is used. Hence, exact expressions have been evaluated for the frequencies and amplitudes of asymmetrical limit cycles for both the stable and unstable SOPDT plant transfer function with or without a zero. The method gives exact results for no measurement errors of the required parameters of the limit cycle waveform. Several examples are given showing the application of the proposed method.

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