AN INTRODUCTION TO GEOPLEX: PORT BASED MODELING AND CONTROL

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Abstract: The paper presents a survey of the concepts which are handled in the the FP5 IST-2001-34166 project called GEOPLEX which was launched in March 2002 by the European Commission. The paper is purely intented as an introduction to the special session of which it makes part of and as such does not mean to be a scientific contribution *Copyright© 2005 IFAC.*

Keywords: Port Hamiltonian Systems, Energy, Physical Modeling and Control

1. INTRODUCTION

A fundamental concept in engineering sciences is the notion of an *open system*, that is a system having a direct interface with its environment. The concept of an open system is directly linked to the notion of a *network*, where open systems are coupled to each other through their interfaces. Complementary to the network modeling of complex systems is the *design* and *control* of systems with a required functionality by coupling open system components.

GEOPLEX is aimed at the network modeling and control of complex *physical* systems, using an integrated system approach allowing to deal with physical components stemming from different physical domains (electrical, mechanical, thermodynamic, ..), both in the lumped-parameter and in the distributed-parameter case. In order to describe and to manipulate these dynamical models in a systematic way it is mandatory to develop a coordinate-free, *geometric* framework for their mathematical formulation, especially because of the intrinsic and strong nonlinearities in their system behavior. The methodology used is based on the framework of port-Hamiltonian systems, where the physical components are formulated as generalized Hamiltonian systems, coupled to each other through power ports. The resulting complex physical system is then geometrically described as a Hamiltonian system with respect to the geometric object of a Dirac structure. Apart from the great advantages for simulation and analysis, this Hamiltonian framework immediately provides a powerful starting point for design and control of multi-domain technological systems.

The leading idea of the project can thus be summarized as:

> *"to develop new techniques for modeling, simulation and control of complex physical systems using recent concepts in the geometric formulation of network dynamics as port-Hamiltonian systems"*

The project consortium is composed of seven universities (University of Twente (NL), Universite Claude Bernard Lyon 1 (F), Universitat Politecnica de Catalunya (ES), Ecole Superieure d'electricite (F), Johannes Kepler Universitaet Linz (A), K.U. Leuven Research and Development (B), University of Bologna (I) and the Centre National de la Recherche Scientifique (F)) and one small enterprise, Control Lab Product BV (NL),

[1] This work has been done in the context of the European sponsored project GeoPlex with reference code IST-2001-34166. Further information is available at http://www.geoplex.cc

whose purpose is to implement the results of the project in their modeling and simulation software and make it available for engineering users.

The paper will start with an introduction to the basic features which are needed in order to be able to tackle problems related to multidomain complex systems.

2. PORTS, DIRAC STRUCTURES AND HAMILTONIAN SYSTEMS

As already briefly explained in the previous section, the back-bone of the project is the use of port-based concepts for modeling and control, but what is a power-port?. A power port is the means by which interaction can take place between parts of a physical system or, in certain situations, between a physical system and a controller which has been designed following this philosophy.

A network structure defines then a certain relation on the "to be connected ports" which should be power continuous (does not destroy or generate energy) and which will describe the power flows in the system. Beside the network structure, there will be elements which will store energy like a spring, a mass, a capacitor or an inductor or which will transform energy irreversibly to heat like a resistor or friction.

With these components we will be able to describe a big variety of physical (sub)-systems stemming out from different domains, and this will give rise to a real systematic analysis, control and design of physical systems.

2.1 Power Ports

Mathematically a power port is the pair of two physical variables which, if properly combined, will express power flowing between the subsystems the port connects. Consider for example the interconnection of a resistor with a capacitor in parallel. This interconnection can be described by considering the pair (v, i) of voltage and current common to the two elements. Instantaneously, the power flowing from the capacitor to the resistor will be equal to $P = vi$. The product of these two variable should be always power and for this reason they are called *power conjugate variables.* Other power conjugate variables are force and velocities, pressure and flow-rate, temperature and entropy flow. These variables are called efforts and flows and this nomenclature is the usual one used also in bond-graphs (Paynter, 1960).

This can be generalized to much more complicated physical entities which are not scalars but do have a tensorial (geometrical) structure like vector forces and velocities or even more generally like twists and wrenches in multi-body mechanics (Stramigioli, 2001). In this case the power flow will still be expressible as the pairing of two dual [2] variables. After a proper coordinate choice, this product turns out to be the usual scalar product of 2 vectors:

$$P : V \times V^* \to \mathbb{R} \; ; \; (e, f) \mapsto e^T f.$$

where V is the vector space of either efforts or flows which will dependent on the domain to be modeled. The remarkable feature is that this concept can also be generalized to distributed parameters systems like continuous mechanics, electromagnetism and fluid mechanics using the mathematical concept of *differential forms* and *Grassmannian Algebras* (van der Schaft and Maschke, 2002). The dual product in the distributed case would mathematically be expressed by

$$P : \Omega^k \times \Omega^{n-k} \to \mathbb{R} \; ; \; (e, f) \mapsto \int_{V^n} e \wedge f \qquad (1)$$

where the effort $e \in \Omega^k$ and flow $f \in \Omega^{n-k}$ are *differential forms* that can be paired using the *wedge product* '$\wedge$' in order to give what is called a *volume form* which is an object that can be integrated on the all n-dimensional domain V^n. A typical example is the well known Pointing vector representing the power transfer through a boundary of a closed volume, due to electromagnetic waves. The Pointing vector is defined as the "wedge" of E and H, respectively the electric and magnetic field intensity.

2.2 The Dirac structure

Once we have the concept of power ports available, which will be the interface between subcomponents, we can look at a physical system as it would be a collection of parts like springs, dampers, resistors, flying wheels and others, connected through ports to a network structure which represents the energetic interconnections. This network structure is mathematically represented by what is called a Dirac structure.

If for example we consider 2 capacitors, a resistor and an inductor, there are many ways how we can interconnect them, and the resulting behavior, due to the different interconnections, will be completely different; the elements are the same, but the network structure is different.

2.3 The conceptual elements

Once we have a proper definition of the interfaces (power ports) and a description of the network

[2] Duality is a well defined mathematical concept which allows in an intrinsic way to associate to two dual variables a scalar.

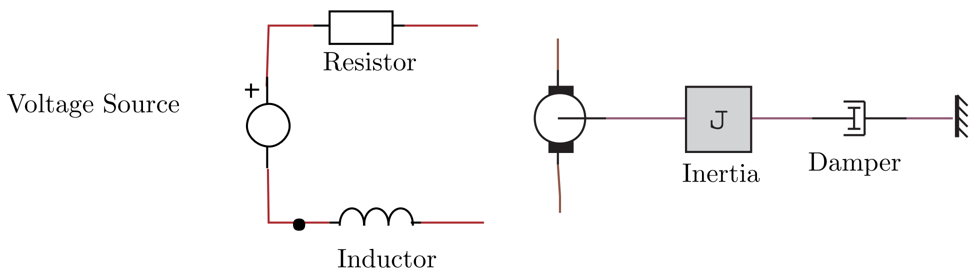

Fig. 1. The electrical (left) and electro-mechanical part (right)

topological structure (Dirac structure), we clearly need the components to be connected like springs, dampers etc. Properly speaking, it is correct to talk about ideal elements instead of components since a "component spring" could be modeled as an interconnection of an "ideal element" spring with some extra parasitic effects like "ideal damping" and "ideal inertial" properties.

For this reasons, we will classify ideal elements as *pure storages* of energy like potential or kinetic energy, electrical or magnetic energy and *irreversible transducers* like resistors.

3. A WORKING EXAMPLE

We will now introduce as an example the modeling of a DC motor in the port-Hamiltonian setting. The example is chosen to be very simple on purpose, in order to make the procedure clear to the reader.

3.1 Power Ports

In the electrical network reported on the left of Fig. 1, there are 4 ports which will be used: a port connected to a voltage source (e_s, f_s), a port connected to a an inductor which is a storage element (e_i, f_i), a port connected to a resistor (e_r, f_r) and an interconnection port which will be used to interconnect the system to something else (e_c, f_c).

3.2 The Dirac structure

We can represent the Dirac structure in many different ways, but the most straight forward in this case will be by means of a skew-symmetric matrix:

$$\begin{pmatrix} f_s \\ e_i \\ f_r \\ f_c \end{pmatrix} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & -1 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} e_s \\ f_i \\ e_r \\ f_c \end{pmatrix} \qquad (2)$$

The skew symmetry ensures power continuity following from Tellegen's theorem:

$$e_s^T f_s + e_i^T f_i + e_r^T f_r + e_c^T f_c = 0$$

3.3 The Conceptual Elements

Consider the electrical network on the left of Fig. 1. It is a series or a source, an inductor, a resistor and an open port which can be used to interconnect the system to the rest of the world.

The inductor is a storage of magnetic energy. The energy stored in the inductor is a function of the flux which is the proper energy variable [3]. If we consider a linear inductor, the energy function would be:

$$H(\lambda) = \frac{1}{2L}\lambda^2 \qquad (3)$$

The generalized effort corresponding to each storage element is the partial derivative of the energy function to the energy variable which in this case is the flux:

$$e = i = \frac{\partial H}{\partial \lambda} = \frac{\lambda}{L} \qquad (4)$$

$$f = v = \dot{\lambda} \qquad (5)$$

The resistor will satisfy Ohms law: $e_r = Rf_r$. It could be shown that the system's equations could be easily written in the following form:

$$\dot{x} = (J(x) - R(x))\frac{\partial H}{\partial x} + g(x)u$$
$$y = g^T(x)\frac{\partial H}{\partial x} \qquad (6)$$

where $x = \lambda$ is the physical state of the subsystem, $H(x) = \frac{1}{2L}\lambda^2$ is the energy stored in the sub-system, $J(x)$ is in general a skew symmetric energy representing the network structure, $R(x)$ is a semi-positive matrix representing dissipation

[3] If we consider the energy function, the inductor is a function of the flux linkage and not of the current. Properly speaking, the energy expressed as function of current is called co-energy.

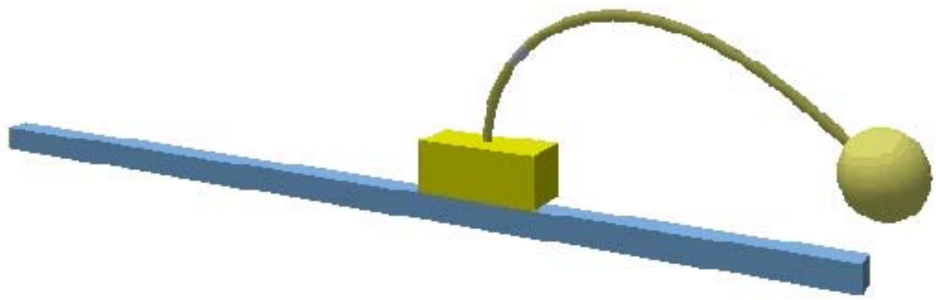

Fig. 2. Infinite Dimensional System with varying boundary conditions.

and $g(x)$ is an input matrix representing the interconnection with external ports or power supply. In details we have for our case:

$$J(x) = 0 \qquad R(x) = R$$

$$g(x) = \begin{pmatrix} 1 & 1 \end{pmatrix} \qquad u = \begin{pmatrix} u_s \\ u_i \end{pmatrix}$$

where u_i indicates the voltage source value and (u_i, i_i) the interconnection port which will be connected to the mechanical part.

3.4 The Electro-mechanical Part

It would be possible to follow the same procedure for the mechanical part represented in Fig. 1. and get to an equation of exactly the same form as Eq. (6). In this case would be

$$x = p \qquad J(x) = 0 \qquad R(x) = b$$

$$g(x) = K \qquad H(p) = \frac{1}{2I}p^2$$

where b is the damping coefficient and K the motor constant relating current to torque and angular velocity to the e.m.f.

It could be also seen that the interconnection of the electrical part and the electro-mechanical one would result once again in equations of the same form as Eq. (6) with $H(\lambda, p) = \frac{1}{2L}\lambda^2 + \frac{1}{2m}p^2$ and:

$$x = \begin{pmatrix} \lambda \\ p \end{pmatrix} \qquad J(x) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$R(x) = \begin{pmatrix} R & 0 \\ 0 & b \end{pmatrix} \qquad g(x) = 0$$

3.5 Conclusion on the example

Using this trivial example, it has been shown that each physical subsystem is characterized by a network structure $J(x)$, an energy storage function $H(x)$, some extra terms representing dissipation $R(x)$ and external interconnection $g(x)$. Furthermore, it has also been shown that the interconnection of two subsystems still results in a system of the same form whose energy is the sum of the energy of the subsystems.

Something which is very important to note is that an interconnection of systems with ports does result in the real physical behavior of the interconnected parts. This is NOT the case in general if physical parts are represented by signal transfer functions, like it can be seen by a cascade interconnection of electrical filters models.

4. HOW FAR CAN WE GO WITH PORT BASED MODELING?

The example only shows a trivial system which could have been easily modeled by other means. The real interesting feature of the GEOPLEX methodology is that exactly in the same way it is possible to interconnect lumped parameters systems with distributed parameter systems. This would result in models of distributed parameter systems with time varying boundary conditions.

4.1 Distributed Parameters Systems

With a new GEOPLEX methodology (Golo *et al.*, 2003), distributed parameter systems can be discretized using novel methods which seem to give more accurate results than usual Finite Element Methods. Using this discretization method, the “lumps” still retains a physical structure expressible using equations like Eq. (6) and therefore do have a clear physical interpretation. An example of a model composed of a discretized distributed parameter system and a lumped one is reported in Fig. 2: the 3D flexible beam is connected on one side to a mass and on the other side to a sliding link.

In the future, by means of the methodologies explored in this direction, new analysis methods could become available for the analysis of vibration and vibration propagation in structures which is a vary valuable mechatronic problem.

4.2 Multi-domain, Object oriented Modeling

Another attractable feature of the GEOPLEX techniques is the intrinsical multi-domain approach: any physical domain like mechanical and electrical, but also hydraulic, pneumatic and even thermal can be analyzed in a systematic and uniform way. Furthermore, the modeling procedure is really object oriented since sub-models are developed which can be interconnected through compatible interfaces. This is very advantageous from a modeling point of view since reusability, openness and ease of use can be enforced. A good example was given through the interconnection of the flexible beam to a mass and a slider which are just interconnected through the common power port structure.

5. CONTROL

These techniques do not only offer new modeling concepts, but also new methodologies and paradigms for control design. There are applications in which stabilization or tracking is not the goal but rather controlling of a certain impedance. Typical examples would be “impedance control” in robotics (Stramigioli, 2001) or impedance adaptation and matching to avoid over-voltage phenomena (Ortega *et al.*, n.d.). Two controlling methods based on the GEOPLEX techniques can be classified in *state feedback* or *control by interconnection*.

5.1 State Feedback and IDA-PBC

Using state feedback it is possible to change the behavior of the original system by trying to let it behave like another physical system. The most general technique to do this is called Interconnection and Damping Assignment, Passivity Based Control (IDA-PBC) (Ortega *et al.*, 2002). Giving a system a desirable different physical behavior, can improve the analysis and stability of the system since it automatically provides a Lyapunouv function beside giveing the possibility to shape its internal energy “distribution channels”.

5.2 Control by Interconnection and IPC

A different approach is instead to use collocated control by developing a controller which can be interconnected to the system to be controlled via a power port. This can be done only for collocated situations but has extremely good robustness properties. An example of these techniques for robotics applications is called Intrinsically Passive Control and can be found in (Stramigioli, 2001). It is remarkable, that using these approach, control of complex systems with non-holonomic constraints like the snakeboard can be tackled (Duindam and S.Stramigioli, 2004).

6. CONCLUSIONS AND FUTURE WORK

In this paper a quick survey of the basic ideas and methodology used in the European Sponsored project GEOPLEX have been presented. Major features of the methodology are multi-domain, object oriented, open, systematic and general methods to model, analyze and control physical systems. The idea is to use the common entity of a power port to talk about interconnection and composition of subsystems in order to yield a general, multidomain methodology for modeling and control.

REFERENCES

Batlle, C., A. Dòria and R. Ortega (n.d.). Power flow control of a doubly–fed induction machine coupled to a flywheel. IEEE Conference on Control Applications (CCA04), September 2-4, 2004, Taipei (Taiwan). Contributed paper.

Duindam, V. and S.Stramigioli (2004). Port-based asymptotic curve tracking for mechanical systems. *European Journal of Control*. Accepted for Pubblication (EJC 105-03).

Golo, G., V. Talasila, A. J. van der Schaft and B.M. Maschke (2003). Hamiltonian discretization of boundary control systems. Internal Report.

M. Pérez, R. Ortega and J. Espinoza (2004). Passivity–based pi control of switched power converters. *IEEE Transactions on Control Systems Technology*.

Ortega, R., A. van der Schaft, B. Maschke and G. Escobar (2002). Interconnection and damping assignment passivity–based control of port–controlled hamiltonian systems. *Automatica*.

Ortega, R., M. Spong and G. Nam S. Lee (n.d.). On compensation of wave reflections in transmission lines and applications to the overvoltage problem in ac. *IEEE TAC*.

Paynter, Henry M. (1960). *Analysis and Design of Engineering Systems*. M.I.T. Press. Cambridge, Massachusetts. Course 2.751.

Stramigioli, Stefano (2001). *Modeling and IPC Control of Interactive Mechanical Systems: a coordinate free approach*. LNCIS. Springer. London.

van der Schaft, Arjan and Bernhard M.J. Maschke (2002). Hamiltonian formulation of distributed-parameter systems with boundary energy flow. *Journal of Geometry and Physics* **42**, 166–194.