STABILIZATION OF A PVTOL AIRCRAFT WITH DELAY IN THE INPUT [1]

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Abstract: In this paper, a model of PVTOL aircraft with two delayed inputs is considered. The origin of this system is globally asymptotically stabilized by state feedbacks determined through recent extensions of the forwarding approach to systems with a delay in the input. In a second step, this result is extended to the case where only the variables of position are supposed to be available by measurement. The output feedbacks are obtained through a technique which extensively exploits the presence of delays in the inputs.

Keywords: Stabilization, delay, nonlinear system, vanishing perturbations.

1. INTRODUCTION

The problem of the stabilization of feedforward systems in the absence of delay has been studied by many researchers (Teel, 1996*a*; Mazenc and Praly, 1996; Sepulcre and Kokotovic, 1997; Marconi and Isidori, 2000; Tsinias and Tzamtzi, 2001), during more than ten years. The theoretical results obtained in this field of research have been successfully applied to different physical devices such as, for example, 'the card-pendulum system'(see (Mazenc and Bowong, 2003)), 'the ball and beam' with a friction term (see (Sepulcre and Kokotovic, 1997)), 'the TORA system' (see (Sepulcre and Kokotovic, 1997)) and 'the PVTOL' (*Planar Vertical Takeoff and Landing Aircraft*), (see (Teel, 1996*a*)).

Three recent works (Mazenc *et al.*, 2003; Mazenc *et al.*, 2004*b*; Michiels and Roose, 2002) are devoted to the problem of designing globally asymptotically stabilizing control laws for particular families of feedforward systems with an arbitrarily large delay in the input: this problem is solved for chains of integrators in (Mazenc *et al.*, 2003; Michiels and Roose, 2002) and for nonlinear feedforward systems admitting a chain of integrators as linear approximation at the origin in (Mazenc *et al.*, 2004*b*). The basic idea of these three papers consists in selecting, according to the value of the delay, appropriate stabilizing control laws in a family of control laws whose explicit formulae generalize those of the control laws provided by A. Teel in (Teel, 1992).

In the present work we will use the aforementioned theoretical results, and especially the one of (Mazenc *et al.*, 2004*b*), to stabilize a PVTOL model, when the control inputs are subject to delays. The PVTOL aircraft model is well-known by the control community. Due to the fact that flight control is an essential control problem, this simple model, which retains main features that must be considered when designing control laws for a real aircraft, has been studied extensively by many researchers. Some of the works devoted to this system are the following. In 1992 J. Hauser et al. (Hauser *et al.*, 1992) developed for this model an approximate input-output linearization procedure which results in bounded tracking and asymptotic stability. In 1996 A. Teel (Teel, 1996*b*)

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illustrated his nonlinear small gain theorem based approach for the stabilization of feedforward systems by applying it to the PVTOL aircraft. In 1996 P. Martin et al. (Martin *et al.*, 1996) proposed an extension of (Hauser *et al.*, 1992) relying extensively on the concept of flatness. In 1999, F. Lin et al. (Lin *et al.*, 1999) studied the robust hovering control of the PVTOL and designed a nonlinear state feedback by applying an optimal control approach. The recent publications by L. Marconi et al. (Marconi *et al.*, 2002) within an internal model approach and by K.D. Do et al. (Do *et al.*, 2003), who have solved an output feedback tracking problem, show that this system still captures the attention of researchers.

Observe that, due to the number of papers devoted to the PVTOL system, the list of works on the PVTOL aircraft we give is not exhaustive. However, to the best of our knowledge, all the theoretical results available in the literature on the asymptotic stabilization of the PVTOL assume that there is no delay in the inputs. Nevertheless, such a delay, due to sensors and information processing, is often present in practice. This is in particular the case of the experimental PVTOL setup presented in the work of Palomino et al. (Palomino *et al.*, 2003) where the position and roll angle of the system are measured with the help of a vision system that induces a delay of approximately $40ms$.

The main features of our contribution can be summarized as follows. In the first part of the work, we construct state feedbacks which globally asymptotically and locally exponentially stabilize the origin of the equations modelling the PVTOL when there are known delays in the inputs. These constructions extensively rely on the control design techniques proposed in (Mazenc *et al.*, 2003; Mazenc *et al.*, 2004*b*; Mazenc *et al.*, 2004*a*). The control laws obtained that way are bounded and involve a distributed term. Moreover they depend on the variables of position and velocity. In the second part of the work, we complement this result by showing that using the presence of known non-zero delays in the inputs (or by introducing artificially delays in the inputs), one can determine globally asymptotically and locally exponentially stabilizing control laws depending only on the variables of position and not on the variables of velocity, which in practice cannot be easily measured. This result is proved through ideas borrowed from the recent works (Mazenc *et al.*, 2002) and (Kharitonov *et al.*, 2003) on the output feedback stabilization of linear systems by means of delayed feedbacks. The main feature of the original approach proposed in these works is that it does not rely on the construction of an observer or on the introduction of dynamic extensions but only on the presence of a delay. In the present paper, it is applied for the first time to a nonlinear system. This strategy of output feedback stabilization for the PVTOL has clearly no similarity with the one adopted in (Do *et al.*, 2003), since the latter relies on the construction of an observer.

The paper is organized as follows. In Section 2, we recall a theoretical result which is used to construct the control laws. The PVTOL aircraft model is presented in Section 3. The control laws and the state reconstructor for the PVTOL aircraft model are designed respectively in Sections 4 and 5. Simulation results are presented in Section 6. Concluding remarks in Section 7 end the paper.

Technical preliminaries.

1. A function $\gamma(X)$ is of order one (resp. two) at the origin if for some $c > 0$, the inequality $|\gamma(X)| \leq c|X|$ (resp. $|\gamma(X)| \leq c|X|^2$) is satisfied on a neighborhood of the origin.
2. The argument of the functions will be omitted or simplified whenever no confusion can arise from the context. For example, we may denote $f(x(t))$ by simply $f(t)$ or $f(\cdot)$.
3. By $\sigma : \mathbb{R} \to \mathbb{R}$ we denote a saturation function with the following properties: $\sigma(\cdot)$ is odd, of class C^1 and such that $0 \leq \sigma'(s) \leq 1$, $\forall s \in \mathbb{R}$, $\sigma(s) = 1$, $\forall s \geq \frac{21}{20}$ and $\sigma(s) = s$, $\forall s \in \left[0, \frac{19}{20}\right]$.
4. By $\sigma_i : \mathbb{R} \to \mathbb{R}$ we denote the functions

$$\sigma_i(s) := \varepsilon_i \sigma\left(\frac{1}{\varepsilon_i} s\right),\ \varepsilon_i = \frac{1}{20^{n-i+1}},\ i = 1, \ldots, n. \quad (1)$$

2. THEORETICAL RESULTS

In this section, we recall the main stabilization result of (Mazenc *et al.*, 2004*a*) for nonlinear feedforward systems with a delay in the input and subject to vanishing perturbations. It is a generalization of the result presented in (Mazenc *et al.*, 2004*b*) for nonlinear feedforward systems in absence of vanishing perturbations, which in turn is a generalization of the recursive methodology developed in (Mazenc *et al.*, 2002) to solve the problem of stabilizing chains of integrators.

Theorem 1. Consider the following feedforward system

$$\begin{cases} \dot{x}_1(t) &= x_2(t) + h_1(x_2(t), \ldots, x_n(t)) + r_1(t), \\ \dot{x}_2(t) &= x_3(t) + h_2(x_3(t), \ldots, x_n(t)) + r_2(t), \\ \vdots & \\ \dot{x}_{n-1}(t) &= x_n(t) + h_{n-1}(x_n(t)) + r_{n-1}(t), \\ \dot{x}_n(t) &= u(t - \Theta), \end{cases} \quad (2)$$

where $x_i \in \mathbb{R}$, $u \in \mathbb{R}$ is the input, $\Theta \geq 0$ is the delay and where each function $h_i(\cdot)$ is a function of a class C^2 and of order 2 at the origin, that satisfies the inequality

$$|h_i(x_{i+1}, x_{i+2}, \ldots, x_n)| \leq M(x_{i+1}^2 + x_{i+2}^2 + \cdots + x_n^2) \quad (3)$$

where M is a strictly positive constant when $|x_j| \leq 1, \quad j = i+1, \ldots, n$, and where each function $r_i(\cdot)$ is a function continuously differentiable and such that, for some real-valued nonnegative and nonincreasing function $R \in L^2\,[0, +\infty)$ the inequalities

$$|r_i(t)| \leq R(t) \quad (4)$$

are satisfied for all $t \geq 0$. Consider the control law bounded in norm

$$
\begin{aligned}
u(x_1,\dots,x_n) \ =\ & -\frac{L}{Mk^n}\sigma_n(p_n(k^{n-1}\frac{M}{L}x_n)+\dots \\
& +\sigma_{n-1}(p_{n-1}(k^{n-2}\frac{M}{L}x_{n-1},k^{n-1}\frac{M}{L}x_n)+\dots \\
& +\sigma_1(p_1(\frac{M}{L}x_1,\dots,k^{n-2}\frac{M}{L}x_{n-1},k^{n-1}\frac{M}{L}x_n)))\dots)
\end{aligned} \qquad (5)
$$

where $p_i(x_i,...,x_n) = \sum_{j=i}^{n} \frac{(n-i)!}{(n-j)!(j-i)!}x_j$, where the functions $\sigma_i(\cdot)$ are those defined in the preliminaries (see (1)) and

$$
k \geq \frac{\Theta}{\min\left\{\frac{1}{16n^3\left[4n\sqrt{n}(1+n^2)^{n-1}+1\right]^2}, \frac{1}{4.20^{n+1}n(n+2)}\right\}}, \qquad (6)
$$

$$
0 < L \leq \min\left\{\frac{\eta k}{n^3(n!)^3}, \frac{Mk}{(n+1)!}, M\right\}, \qquad (7)
$$

$$
0 < \eta \leq \min\left\{\frac{1}{8(1+n^2)^{n-1}}, \frac{1}{10.20^n n}\right\}.
$$

Then all the trajectories of the system (2) in closed-loop with the control law (5) converge to the origin. Moreover, the origin of the system (2) in closed-loop with the control law (5) is globally uniformly asymptotically and locally exponentially stable when each function $r_i(\cdot)$ is identically equal to zero.

3. PROBLEM STATEMENT

Given the complexity of the systems describing the behavior of aircraft, it is convenient to study simplified models of them that contemplate a specific number of state variables and controls which capture the essential features of the systems for control purposes. The simplified model of PVTOL we consider in this work is the following

$$
\left.\begin{aligned}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= u_1(t-\tau_1)\sin\theta,
\end{aligned}\right\} \qquad (8)
$$

$$
\left.\begin{aligned}
\dot{y}_1 &= y_2, \\
\dot{y}_2 &= u_1(t-\tau_1)\cos\theta - 1,
\end{aligned}\right\} \qquad (9)
$$

$$
\left.\begin{aligned}
\dot{\theta} &= \omega, \\
\dot{\omega} &= u_2(t-\tau_2).
\end{aligned}\right\} \qquad (10)
$$

The variables x_1, y_1 denote the horizontal and the vertical positions, θ is the roll angle that the aircraft makes with the horizon and $\tau_2 > 0, \tau_1 > 0$ are the delays. The input u_1 is the thrust (directed out of the bottom of the aircraft) and the input u_2 is the angular acceleration (rolling moment). In the next two sections, we will address the following problems:

Problem 1. *Construct state feedbacks which globally uniformly asymptotically and locally exponentially stabilize the system (8), (9), (10) when there are delays in the inputs.*

Problem 2. *Construct output feedbacks which globally uniformly asymptotically and locally exponentially stabilize the system (8), (9), (10) with θ, y_1, x_1 as output variables when there are delays in the inputs.*

4. STABILIZATION RESULT

This section is devoted to Problem 1. Using Theorem 1, we will establish the following result.

Theorem 2. Consider the system (8), (9), (10) with the delays $\tau_1 = 0.2, \tau_2 = 0.3$. The origin of this system in closed-loop with the control laws

$$
u_1 = u_{1s}(y_2(t-\tau_1), y_1(t-\tau_1), \hat{\theta}(t-\tau_2)), \qquad (11)
$$

$$
u_2 = u_{2s}(\omega(t-\tau_2), \theta(t-\tau_2), x_2(t-\tau_2), x_1(t-\tau_2)) \qquad (12)
$$

with

$$
\begin{aligned}
u_{2s}(\omega,\theta,x_2,x_1) \ =\ & -\frac{L}{Mk^4}\sigma_4(k^3\frac{M}{L}\omega + \sigma_3(k^3\frac{M}{L}\omega \\
& +k^2\frac{M}{L}\theta + \sigma_2(k^3\frac{M}{L}\omega + 2k^2\frac{M}{L}\theta + k\frac{M}{L}x_2 \\
& +\sigma_1(k^3\frac{M}{L}\omega + 3k^2\frac{M}{L}\theta + 3k\frac{M}{L}x_2 + \frac{M}{L}x_1)))),
\end{aligned} \qquad (13)
$$

with $M = 0.6$ and $L = 6.45 \times 10^{-10}$, $k = 7.5931 \times 10^{12}$ and

$$
u_{1s}(y_2, y_1, \hat{\theta}) = \frac{1 + v_{1s}(y_2, y_1)}{\cos(\sigma(\hat{\theta}))}, \qquad (14)
$$

with

$$
v_{1s}(y_2, y_1) = -\sigma_2(y_2 + \sigma_1(y_2 + y_1)) \qquad (15)
$$

and

$$
\begin{aligned}
\hat{\theta}(t-\tau_2) \ =\ & \theta(t-\tau_2) + \tau_2\omega(t-\tau_2) \\
& -\int_{t-\tau_2}^{t}(s-t)u_{2s}(s-\tau_2)ds
\end{aligned} \qquad (16)
$$

is globally uniformly asymptotically and locally exponentially stable.

Remark.
1. For the sake of simplicity, we have restricted our attention to the case where $\tau_1 = 0.2, \tau_2 = 0.3$. However, one can easily deduce from the proof of Theorem 2 that for any values of τ_1, τ_2, Problem 1 can be solved.
2. From a practical point of view, the smallness of the size of the control law $u_{2s}(\cdot)$ is a drawback. It is important to observe that this drawback can be overcome. Indeed, by constructing a control law by means of the key ideas of Theorem 1 but by taking advantage of the specificity of the nonlinearities of the system (21), one can obtain a control law $u_{2s}(\cdot)$ with respectively much larger and much smaller values for the parameters L and k. For the sake of simplicity, we do not have performed this simple but lengthy construction of feedback and have instead directly applied Theorem 1.

Proof. The proof splits up into three steps. In Step 1 and Step 2, we establish that the control law defined in (12) ensures that the solutions of the subsystem (10) enter in finite time a particular neighborhood of the origin. Next, we show that this property implies that the control law defined in (11) stabilizes the subsystem (9). Then the problem considered reduces to the stability analysis of a four dimensional feedforward system. This analysis is carried out in Step 3.

Step 1. In Appendix A, we will establish the following result.

Lemma 3. The control law defined in (11) is well-defined. The trajectories of system (8), (9), (10) in closed-loop with the feedbacks (11), (12) are defined for all $t \geq 0$. Moreover, there exists $T \geq 2\tau_2$ such that

$$
|\theta(t)| \leq \frac{\pi}{4}, \ \forall t \geq T. \qquad (17)
$$

Step 2. One can establish that, for all $t \geq 2\tau_2$,

$$\hat{\theta}(t-\tau_2) = \theta(t), \tag{18}$$

by observing that, when $t \geq 2\tau_2$,

$$\begin{aligned} \theta(t) &= \theta(t-\tau_2) + \int_{t-\tau_2}^{t} \omega(s)ds \\ &= \theta(t-\tau_2) + \tau_2\omega(t-\tau_2) - \int_{t-\tau_2}^{t} (s-t)\dot{\omega}(s)ds \\ &= \theta(t-\tau_2) + \tau_2\omega(t-\tau_2) - \int_{t-\tau_2}^{t} (s-t)u_{2s}(s-\tau_2)ds. \end{aligned}$$

Equality (18), the definition of $\sigma(\cdot)$ and Lemma 3 ensure that for all $t \geq T$,

$$u_{1s}(t-\tau_1) = \frac{1+v_{1s}(t-\tau_1)}{\cos(\theta(t))} \tag{19}$$

which implies that for all $t \geq T$, the system (9) simplifies as

$$\begin{aligned} \dot{y}_1 &= y_2(t), \\ \dot{y}_2 &= v_1(t-\tau_1) \\ &= -\sigma_2(y_2(t-\tau_1) + \sigma_1(y_2(t-\tau_1) + y_1(t-\tau_1))). \end{aligned} \tag{20}$$

Using Theorem 1 (or the main result of (Mazenc *et al.*, 2003)), one can prove that this system is globally uniformly asymptotically and locally exponentially stable.

Step 3. According to (19), the system (8), (10) in closed-loop with (11), for all $t \geq T + 2\tau_2$, is described by the equations

$$\begin{cases} \dot{x}_1 = x_2(t), \\ \dot{x}_2 = (1+v_{1s}(t-\tau_1))\tan\theta(t), \\ \dot{\theta} = \omega(t), \\ \dot{\omega} = u_{2s}(t-\tau_2), \end{cases}$$

or, equivalently, by the equations

$$\begin{cases} \dot{x}_1 = x_2(t), \\ \dot{x}_2 = \theta(t) + (\tan\theta(t) - \theta(t)) + v_{1s}(t-\tau_1)\tan\theta(t), \\ \dot{\theta} = \omega(t), \\ \dot{\omega} = u_{2s}(t-\tau_2). \end{cases} \tag{21}$$

Observe that the inequalities $|\tan\theta - \theta| \leq \int_0^{|\theta|} \tan^2(l)dl \leq 0.6\theta^2$ hold for all $\theta \in [-1, 1]$. Thus the function $\tan\theta - \theta$ is of class C^2 and of order 2 at the origin: it satisfies the requirement (3) imposed on the function $h_2(\cdot)$ in Theorem 1. Notice also that in (21), the functions corresponding to $h_1(\cdot)$ and $h_3(\cdot)$ in Theorem 1 are identically equal to zero. Moreover, we know that the real-valued functions $y_1(t)$, $y_2(t)$ converge exponentially to zero and that for all $t \geq T$, $|\tan\theta(t)| \leq 1$. Therefore $v_{1s}(t-\tau_1)\tan\theta(t)$ converges exponentially to zero: it follows that this function belongs to $L^2[0, +\infty)$ and thereby can be regarded as a bounded vanishing disturbance ($r_2(t)$ in Theorem 1). Then, using Theorem 1, one can check that all the trajectories of the feedforward system (21) converge to the origin and besides that the system (8), (9), (10) in closed-loop with the feedbacks (12), (15) is globally uniformly asymptotically and locally exponentially stable. (The value of the constant M, in the particular case of the system (21), is $M = 0.6$.) This concludes the proof.

5. A STATE RECONSTRUCTOR

This section is devoted to Problem 2 (see the end of Section 3). We show that one can solve the problem of stabilizing the PVTOL system when only the variables of position are available by measurement. The approach consists in evaluating the exact values of the variables of velocity through a state reconstructor for each subsystem (8), (9), (10). We show that, when the delays are known, the knowledge of the positions and roll angle of the aircraft which correspond to the states $x_1(t)$, $y_1(t), \theta(t)$ along with the control inputs $u_1(t)$ and $u_2(t)$ at present and past time instants is sufficient to determine the derivatives $x_2(t), y_2(t), \omega(t)$. The approach draws inspiration from the ideas on output feedback stabilization used in (Mazenc *et al.*, 2002) for the case of a bounded input delayed simple oscillator and in (Kharitonov *et al.*, 2003) for the case of multiple oscillators and chains of integrators.

Theorem 4. Consider the system (8), (9), (10) with delays $\tau_1 = 0.2, \tau_2 = 0.3$. The origin of this system in closed-loop with the control laws

$$u_2(t-\tau_2) = u_{2s}(\overline{\omega}(t-\tau_2), \theta(t-\tau_2), \overline{x}_2(t-\tau_2), x_1(t-\tau_2)), \tag{22}$$

$$u_1(t-\tau_1) = u_{1s}(\overline{y}_2(t-\tau_1), y_1(t-\tau_1), \overline{\hat{\theta}}(t-\tau_2)), \tag{23}$$

with

$$\begin{aligned} \overline{x}_2(t) &= \frac{1}{\tau_1}\Bigg[x_1(t) - x_1(t-\tau_1) , \\ &\quad - \int_{t-\tau_1}^{t}\left(\int_t^s u_{1s}(l-\tau_1)\sin\theta(l)dl\right)ds\Bigg] \\ \overline{y}_2(t) &= \frac{1}{\tau_1}\Bigg[y_1(t) - y_1(t-\tau_1) - \\ &\quad \int_{t-\tau_1}^{t}\left(\int_t^s (u_{1s}(l-\tau_1)\cos\theta(l) - 1)dl\right)ds\Bigg], \\ \overline{\omega}(t) &= \frac{1}{\tau_2}\Bigg[\theta(t) - \theta(t-\tau_2) - \\ &\quad \int_{t-\tau_2}^{t}\left(\int_t^s u_{2s}(l-\tau_2)dl\right)ds\Bigg], \end{aligned} \tag{24}$$

and

$$\overline{\hat{\theta}}(t-\tau_2) = \theta(t-\tau_2) + \tau_2\overline{\omega}(t-\tau_2) - \int_{t-\tau_2}^{t}(s-t)u_{2s}(s-\tau_2)ds,$$

where $u_{1s}(\cdot), u_{2s}(\cdot)$ are the functions defined respectively in (14) and (13), is globally uniformly asymptotically and locally exponentially stable.

Proof. It follows from (8) that, for all $t \geq 2\tau_1$,

$$\begin{aligned} x_1(t) &= x_1(t-\tau_1) + \int_{t-\tau_1}^{t} x_2(s)ds \\ &= x_1(t-\tau_1) + \tau_1 x_2(t) + \int_{t-\tau_1}^{t}\left(\int_t^s \dot{x}_2(l)dl\right)ds \\ &= x_1(t-\tau_1) + \tau_1 x_2(t) \\ &\quad + \int_{t-\tau_1}^{t}\left(\int_t^s u_{1s}(l-\tau_1)\sin\theta(l)dl\right)ds. \end{aligned}$$

It follows that

$$x_2(t) = \overline{x}_2(t), \ \forall t \geq 2\tau_1. \tag{25}$$

Similarly, it follows from (9) that

$$y_2(t) = \overline{y}_2(t), \ \forall t \geq 2\tau_1 \tag{26}$$

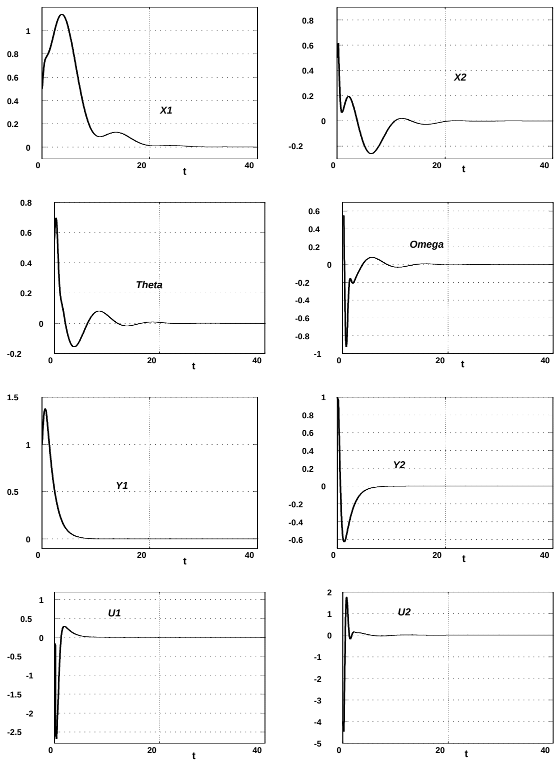
and from (10) that

$$\omega(t) = \overline{\omega}(t), \ \forall t \geq 2\tau_1. \tag{27}$$

We deduce that, for all $t \geq 2(\tau_2 + \tau_1)$, the control laws (23), (22) are equal to the control laws (11), (12) used in Theorem 2. This allows us to conclude the proof.

6. SIMULATION RESULTS

We have performed simulations for the system (8), (9) and (10) in closed-loop with the control laws (11) and (12) where the variables $x_2(t), y_2(t)$ and $\omega(t)$ are substituted by the right hand side of (25), (26) and (27) respectively. The initial conditions we have chosen are: $x_1(0) = x_2(0) = 0.5$, $\theta(0) = \omega(0) = 0.55$, $y_1(0) = y_2(0) = 1$. The behavior of the six state variables and the two control inputs is presented below



7. CONCLUSION

In this work, two problems have been solved. First, we have achieved the global uniform asymptotic and local exponential stabilization of an aircraft PVTOL model with two delays in the inputs, using bounded state feedbacks. In a second step, we have shown how the presence of delays in the inputs can be exploited to achieve the global uniform asymptotic and local exponential stabilization of an aircraft PVTOL model when the variables of velocity are not measured. The main interest of the work is that it illustrates the possibility of applying recent theoretical results for nonlinear systems with delay to a physical system, very relevant from a practical point of view. Much remains to be done. We plan to study the following problems: Investigating whether or not there are possible ways to modify our construction in such a way that the resulting control laws are without distributed terms, determining control laws for the PVTOL with delay using not the forwarding approach but the backstepping approach, extending our results to the case where the exact values of the delay are unknown. At last, we will implement the control laws proposed in the present work in real time on a PVTOL aircraft prototype.

REFERENCES

Do, D. K., Z. P. Jiang and J. Pan (2003). On global tracking control of a aircraft without velocity measurement. *IEEE Trans. Autom. Cotr.* **48(12)**, 2212 – 2217.

Hauser, J., S. Sastry and G. Meyer (1992). Nonlinear control design for slightly nonminimum phase systems: Application to V/STOL aircraft. *Automática* **28(4)**, 665–679.

Kharitonov, V. L., S. I. Niculescu, J. Moreno and W. Michiels (2003). Some remarks on static output feedback stabilization problem: Necesary conditions for multiple delay controllers.. *ECC 2003, Cambridge, U. K.*

Lin, F., W. Zhang and R. D. Brandt (1999). Robust hovering control of a PVTOL aircraft. *IEEE Trans. Contr. Syst. Tech.* **7(3)**, 343 – 351.

Marconi, L., A. Isidori and A. Serrani (2002). Autonomous vertical landing on an oscillating platform: An internal model based approach. *Automatica* **38**, 21 – 32.

Marconi, L. and A. Isidori (2000). Robust global stabilization of a class of uncertain feedforward nonlinear systems. *Sys. Contr. Lett.* **41**, 281–290.

Martin, P., S. Devasia and B. Paden (1996). A different look at output tracking: Control of a VTOL aircraft. *Automatica* **32(1)**, 101 – 107.

Mazenc, F. and L. Praly (1996). Adding an integration and global asymptotic stabilization of feedforward systems. *IEEE. Trens. Autom. Contr.* **41(11)**, 1559–1578.

Mazenc, F. and S. Bowong (2003). Tracking trajectories of the cart-pendulum system. *Automatica* **39**, 677–684.

Mazenc, F., S. Mondié and R. Francisco (2004*a*). Global asymptotic stabilization of feedforward system with delay in the input and vanishing perturbations. *3rd IFAC Symposium on Systems, Structure and Control, Oaxaca, Mexico.*

Mazenc, F., S. Mondié and R. Francisco (2004*b*). Global aysmptotic stabilization of feedforward system with delay in the input. *IEEE. Trans. Autom. Contr.* **49(5)**, 844–850.

Mazenc, F., S. Mondié and S. Niculescu (2002). Global stabilization of oscillators with bounded delayed input. *41st IEEE. Conference on Decision and Control, Las vegas.*

Mazenc, F., S. Mondié and S. Niculescu (2003). Global asymptotic stabilization for chains of integrators with delay in the input. *IEEE. Trans. Autom. Contr.* **48(1)**, 57–53.

Michiels, W. and D. Roose (2002). Global stabilization of multiple integrators with time - delay and the input constraints. *Santa Fe, New Mexico* pp. 266 – 271.
Palomino, A., P. Castillo, I. Fantoni and R. Lozano (2003). Strategy using vision for the stabilization of an experimental PVTOL aircraft setup. *42th IEEE Conference on Decision and Control, Maui, Hawaii.*
Sepulcre, R. and M. Jankovic Ans P. V. Kokotovic (1997). *Constructive Nonlinear Control.* Springer-Verlag. London.
Teel, A.R. (1992). Global stabilization and restricted tracking for multiple integrators with bounded controls. *Syst. Contr. Lett.* **18**, 165–171.
Teel, A.R. (1996*a*). A nonlinear small gain theorem for the analysis of control system with saturation. *IEEE. Trans. Autom. Contr.* **41(9)**, 1256–1270.
Teel, A.R. (1996*b*). On L_2 performance indueced by feedbacks with multiple saturations. *ESAIM: COCV* pp. 225–240.
Tsinias, J. and M. P. Tzamtzi (2001). An explicit formula of bounded feedback stabilizers for feedforrward systems. *Syst. Contr. Lett.* **43**, 247 – 261.

Appendix A. PROOF OF LEMMA 3

The fact that $\tau_2 \geq \tau_1$ ensures that the control law $u_{1s}(\cdot)$ defined in (11) is well-defined. Due to the feedforward structure of the system (8), (9), (10), it is clear that the trajectories of this system in closed-loop with the bounded feedbacks (15), (12) are defined for all $t \geq 0$ (observe in particular that the finite escape time phenomenon does not occur). The next step of the proof consists in showing that $u_{2s}(\cdot)$ defined in (12) ensures that $|\theta(t)| \leq \frac{\pi}{4}$ when t is large enough. This proof is lengthy but simple.

First observe that

$$\omega(t-\tau_2) - \omega(t) = \int_t^{t-\tau_2} \dot{\omega}(s)ds = \int_{t-\tau_2}^{t} \frac{L}{Mk^4}\sigma_4(\cdot)ds. \tag{A.1}$$

It follows that

$$\dot{\omega} = -\frac{L}{Mk^4}\sigma_4(k^3\frac{M}{L}\omega(t) + \mu_1(t)) \tag{A.2}$$

where $\mu_1(t) = k^3\frac{M}{L}(\omega(t-\tau_2) - \omega(t)) + \sigma_3(\cdot)$ is a function such that, for all $t \geq 0$,

$$|\mu_1(t)| \leq \frac{\tau_2}{k}\varepsilon_4 + \varepsilon_3. \tag{A.3}$$

The derivative of the positive definite and radially unbounded function $V_1(\omega) = \frac{1}{2}\omega^2$ along the trajectories of (A.2) satisfies

$$\dot{V}_1 \leq -\frac{L}{Mk^4}|\omega(t)|\varepsilon_4\sigma(\frac{1}{\varepsilon_4}(k^3\frac{M}{L}|\omega(t)| - \frac{\tau_2}{k}\varepsilon_4 - \varepsilon_3)).$$

It follows that, when $|\omega(t)| \geq 2\frac{L}{Mk^3}\left(\frac{\tau_2}{k}\varepsilon_4 + \varepsilon_3\right)$,

$$\dot{V}_1 \leq -2\frac{L^2\left(\frac{\tau_2}{k}\varepsilon_4 + \varepsilon_3\right)\varepsilon_4}{M^2k^7}\sigma(\frac{1}{\varepsilon_4}\left(\frac{\tau_2}{k}\varepsilon_4 + \varepsilon_3\right)) < 0.$$

It follows that there exists $T_1 \geq 0$ such that, for all $t \geq T_1$,

$$|\omega(t)| \leq 2\frac{L}{Mk^3}\left(\frac{\tau_2}{k}\varepsilon_4 + \varepsilon_3\right). \tag{A.4}$$

Combining (A.3) and (A.4), we deduce that, for all $t \geq T_1$,

$$\frac{1}{\varepsilon_4}\left|k^3\frac{M}{L}\omega(t) + \mu_1(t)\right| \leq \left(3\frac{\tau_2}{k} + 3\frac{\varepsilon_3}{\varepsilon_4}\right) \leq \frac{1}{2}.$$

We deduce that there exists $T_2 \geq T_1$ such that, for all $t \geq T_2$,

$$\dot{\omega} = -\frac{1}{k}\omega(t-\tau_2) - \frac{L}{Mk^4}\sigma_3(\cdot).$$

It follows that the derivative of the variable

$$\gamma = k\omega + \theta \tag{A.5}$$

satisfies

$$\begin{aligned}\dot{\gamma} &= \omega(t) - \omega(t-\tau_2) - \frac{L}{Mk^3}\sigma_3(k^3\frac{M}{L}\omega(t-\tau_2) \\ &\quad + k^2\frac{M}{L}\theta(t-\tau_2) + \sigma_2(\cdot)) \\ &= -\frac{L}{Mk^3}\sigma_3(\frac{M}{L}k^2\gamma(t) + \mu_2(t)) + \mu_3(t),\end{aligned} \tag{A.6}$$

where $\mu_2(t)$ and $\mu_3(t)$ are continuous functions such that

$$\begin{aligned}|\mu_2(t)| &\leq k^3\frac{M}{L}\,|\omega(t) - \omega(t-\tau_2)| \\ &\quad + k^2\frac{M}{L}\,|\theta(t) - \theta(t-\tau_2)| + \varepsilon_2, \\ |\mu_3(t)| &\leq |\omega(t) - \omega(t-\tau_2)|\,.\end{aligned}$$

From (A.1) and (A.4), we deduce that there exists $T_3 \geq T_2$ such that, for all $t \geq T_3$,

$$\begin{aligned}|\mu_3(t)| &\leq \frac{\tau_2 L\varepsilon_4}{Mk^4}, \\ |\mu_2(t)| &\leq k^3\frac{M}{L}\frac{\tau_2 L\varepsilon_4}{Mk^4} + \varepsilon_2 + k^2\frac{M}{L}\int_{t-\tau_2}^{t}|\omega(s)|\,ds \\ &\leq \varepsilon_2 + \frac{2\tau_2}{k}\varepsilon_3 + \left(\frac{\tau_2}{k} + \frac{2\tau_2^2}{k^2}\right)\varepsilon_4.\end{aligned}$$

It follows that the derivative of the positive definite and radially unbounded function $V_2(\gamma) = \frac{1}{2}\frac{Mk^3}{L}\gamma^2$ along the trajectories of (A.6) satisfies, when $t \geq T_3$

$$\begin{aligned}\dot{V}_2 &\leq -\varepsilon_3|\gamma(t)|\sigma(\frac{1}{\varepsilon_3}(\frac{M}{L}k^2|\gamma(t)| - (\varepsilon_2 + \frac{2\tau_2}{k}\varepsilon_3 \\ &\quad + (\frac{\tau_2}{k} + \frac{2\tau_2^2}{k^2})\varepsilon_4)))) + \frac{\tau_2\varepsilon_4}{k}|\gamma(t)|.\end{aligned}$$

It follows that when $t \geq T_3$ and when $|\gamma(t)| \geq 2\frac{L}{Mk^2}\left(\varepsilon_2 + \frac{2\tau_2}{k}\varepsilon_3 + \left(\frac{\tau_2}{k} + \frac{2\tau_2^2}{k^2}\right)\varepsilon_4\right)$, the inequality

$$\begin{aligned}\dot{V}_2 \leq &-\varepsilon_3|\gamma(t)|\sigma\left(\frac{1}{\varepsilon_3}\left(\varepsilon_2 + \frac{2\tau_2}{k}\varepsilon_3 + \left(\frac{\tau_2}{k} + \frac{2\tau_2^2}{k^2}\right)\varepsilon_4\right)\right) \\ &+ \frac{\tau_2\varepsilon_4}{k}|\gamma(t)|\end{aligned}$$

is satisfied. The values of the parameters present in this inequality and the properties of $\sigma(\cdot)$ imply that when $t \geq T_3$ and when $|\gamma(t)| \geq 2\frac{L}{Mk^2}\left(\varepsilon_2 + \frac{2\tau_2}{k}\varepsilon_3 + \left(\frac{\tau_2}{k} + \frac{2\tau_2^2}{k^2}\right)\varepsilon_4\right)$, the inequality $\dot{V}_2 < 0$ is satisfied. We deduce that there exists $T_4 \geq T_3$ such that, for all $t \geq T_4$, $|\gamma(t)| \leq 2\frac{L}{Mk^2}\left(\varepsilon_2 + \frac{2\tau_2}{k}\varepsilon_3 + \left(\frac{\tau_2}{k} + \frac{2\tau_2^2}{k^2}\right)\varepsilon_4\right)$. From this inequality, the definition of γ (see (A.5)) and (A.4) one can deduce that, when $t \geq T_4$, $|\theta(t)| \leq \frac{\pi}{4}$. This concludes the proof.