STABILIZATION SCHEME FOR FORCE REFLECTING TELEOPERATION WITH TIME-VARYING COMMUNICATION DELAY BASED ON IOS SMALL GAIN THEOREM

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Abstract: The problem of stabilization of force-reflecting teleoperators with time-varying delay in the communication channel is addressed. A control scheme is proposed which guarantees stability of the closed-loop telerobotic system in the presence of an arbitrary time-varying possibly unbounded transmission delay satisfying a set of technical assumptions. The proposed scheme also guarantees (in the "semiglobal" sense with respect to initial conditions and external forces) that the slave manipulator tracks the delayed trajectory of the master with error which is ultimately bounded by an arbitrarily small bound. The proof of this result is based on a special version of the IOS (input-to-output stability) small gain theorem.

Keywords: Teleoperation, Time Delay, Stabilization

1. INTRODUCTION

Force-reflecting (or bilateral) teleoperation is a challenging area of modern technology which inspires researchers from both the control theory and robotics. A teleoperator system consists of two manipulators connected by a communication channel. The first manipulator, called master, is moved by a human operator, and the information about the master's trajectory is sent through the communication channel to the remotely located second manipulator called slave. The slave is controlled to follow the trajectory of the master. In force-reflecting teleoperators, a contact force due to environment is measured on the slave side and sent back to the motors of the master. In the presence of such a force feedback, the ability of the operator to perform complex tasks that include interaction with the environment may be essentially increased. However, as first shown by Ferrell (1966), the force feedback has a strong destabilizing effect if transmission delays are present in the communication channels. This problem was widely addressed in the literature, and several approaches were proposed (see, for example (Anderson and Spong, 1989; Niemeyer and Slotine, 1991), among other papers). Recently, an idea of using a version of the ISS (input-to-state stability) small gain theorem (Jiang *et al.*, 1994) for the proof of stability of bilaterally controlled teleoperators with communication delay was presented in (Polushin and Marquez, 2003b). In this paper, we present further contributions to the IOS (ISS) small gain approach to the stabilization of force-reflecting teleoperators with delay in the communication channel. In particular, we address the situation when the transmission delay is a possibly unbounded function of time rather than a constant. The main motivation for the development of a control scheme that can handle a time-varying communication delay is based on the recent tendency to use the Internet as a communication medium. When teleoperation is performed over the Internet, the transmission delays may vary with such factors as congestion and bandwidth, which leads to decreasing performance and arising of the instability issues. This is a relatively new area of research, and not so many results are obtained (see (Chopra *et al.*, 2003)). In this paper, we propose a control scheme which guarantees stability of the closed-loop telerobotic system in the presence of an arbitrary time-varying possibly unbounded transmission delay satisfying a set of technical assumptions. Our stability analysis is based on a special version of the IOS small gain theorem. Also, the proposed scheme guarantees (in the "semiglobal" sense with respect to initial conditions and external forces) that the slave manipulator tracks the delayed trajectory of the master with error which is ultimately bounded by an arbitrarily small bound.

The structure of the paper is as follows. A statement of the problem is formulated in section 2. In section 3, a special version of the IOS small gain theorem is given which is a main tool for our proof of stability of the telerobotic system with communication delay. The main result is presented in section 4. Some computer simulations of the proposed algorithm can be found in section 5. Conclusions are given in section 6. All the proofs are omitted due to space reasons.

2. PROBLEM STATEMENT

Mathematical model of the master and the slave manipulators. We consider a force-reflecting telerobotic system where the master and the slave manipulators are described by Euler-Lagrange equations of the following standard form

$$\begin{aligned} &H_m(q_m)\,\ddot{q}_m + C_m(q_m,\dot{q}_m)\,\dot{q}_m + G_m(q_m) \\ &= u_m + f_h + \hat{f}_e, \end{aligned} \tag{1}$$

$$H_s(q_s)\,\ddot{q}_s + C_s(q_s,\dot{q}_s)\,\dot{q}_s + G_s(q_s) = u_s + f_e. \tag{2}$$

Here $q_m \in \mathbb{Q}_m$, $q_s \in \mathbb{Q}_s$, where $\mathbb{Q}_m$, $\mathbb{Q}_s$ are the configuration spaces of the master and the slave manipulators respectively. For simplicity, assume that each manipulator has l rotational joints and $n-l$ prismatic joints with a finite range of motion, $l \in \{0,\ldots,n\}$. Thus $\mathbb{Q}_m = \mathbb{Q}_s = \mathbb{D} \times \mathbb{T}^l$, where $\mathbb{T}^l$ is an l-dimensional torus, and $\mathbb{D}$ is a compact connected subset of $\mathbb{R}^{n-l}$. Further, $H_m(q_m)$, $H_s(q_s)$ are inertia matrices, $C_m(q_m,\dot{q}_m)$, $C_s(q_s,\dot{q}_s)$ are matrices of centrifugal and Coriolis forces, and $G_m(q_m)$, $G_s(q_s)$ are vectors of potential forces of the master and the slave manipulators respectively. Also, $f_h \in \mathbb{R}^n$ is a force applied by the human operator to move the master manipulator, $f_e \in \mathbb{R}^n$ is the contact force due to the environment applied to the slave, and $\hat{f}_e \in \mathbb{R}^n$ is the force applied to the motors of the master that reflects the contact force due to environment on the slave side. Finally, $u_m, u_s \in \mathbb{R}^n$ are the control inputs of the master and the slave respectively. It is assumed that the dynamics of the master and the slave manipulators (1), (2) possess several well known properties (see, for example, Section 2.1 of (Spong, 1996)).

Communication delay. The joint positions and velocities of the master and the contact force due to the environment applied to the slave are sent to the opposite manipulator over a communication channel with a communication delay. Let $\tau_i : \mathbb{R} \to \mathbb{R}^+$, $i \in \{f,b\}$ be time-dependent time delays in the forward ($i=f$) and backward ($i=b$) communication channel respectively. The joint positions and velocities of the master are transmitted to the slave side with communication delay $\tau_f(\cdot)$, so that the following signals

$$\begin{aligned} \hat{q}_m(t) &= q_m(t-\tau_f(t)), \\ \hat{\dot{q}}_m(t) &= \dot{q}_m(t-\tau_f(t)), \end{aligned} \tag{3}$$

are available for the controller on the slave side. On the other hand, a contact force due to the environment f_e is measured on the slave side and transmitted back to the master with a communication delay $\tau_b(\cdot)$, *i.e.*

$$\hat{f}_e(t) = f_e(t-\tau_b(t)). \tag{4}$$

Both $\tau_f(t)$, $\tau_b(t)$ are assumed to be time-varying and possibly unbounded. More precisely, the assumption imposed on $\tau_f(\cdot)$, $\tau_b(\cdot)$ is given below.

Assumption 1. Both τ_i, $i \in \{f,b\}$, satisfy the following set of properties:

i) $\tau_i(t_2) - \tau_i(t_1) \le t_2 - t_1$ for any $t_2 \ge t_1$;

ii) $-\Upsilon(t_2-t_1) \le \tau_i(t_2) - \tau_i(t_1)$ for some $\Upsilon \ge 0$ and for any $t_2 \ge t_1$.

iii) $t - \tau_i(t) \to +\infty$ as $t \to +\infty$. •

Note that Assumption 1 can always be satisfied for any lossless communication channel.

Dynamical model of the environment. It is assumed that the environment can be described as an unknown dynamical system whose dynamics satisfy a form of input-to-output stability property (Sontag and Wang, 1999). More precisely, the following is valid:

Assumption 2. Suppose $f_e(\cdot)$ is a measurable locally essentially bounded function satisfying the following property: there exist $\gamma_f \geq 0$, $\gamma_e \geq 0$ such that the contact force due to the environment, $f_e(t)$, satisfies the following two properties:

i) *uniform boundedness:* there exists $C \geq 0$ such that for any $t_0 \in \mathbb{R}$

$$\sup_{t\geq t_0} |f_e(t)| \leq \max \left\{ \begin{array}{c} C, \gamma_e \left(\sup_{t\geq t_0} \left| \begin{array}{c} q_s(t) \\ \dot{q}_s(t) \end{array} \right| \right), \\ \gamma_f \left(\sup_{t\geq t_0} |f_{ext}(t)| \right) \end{array} \right\};$$

ii) *convergence:*

$$\limsup_{t\to+\infty} |f_e(t)| \leq \max \left\{ \begin{array}{c} \gamma_e \left(\limsup_{t\to+\infty} \left| \begin{array}{c} q_s(t) \\ \dot{q}_s(t) \end{array} \right| \right), \\ \gamma_f \left(\limsup_{t\to+\infty} |f_{ext}(t)| \right) \end{array} \right\}.$$

Here $q_s(t)$, $\dot{q}_s(t)$ are the state variables (position and velocity) of the slave manipulator, and f_{ext} is an arbitrary measurement essentially bounded function that represents an equivalent of all external forces imposed on the environment.

Problem statement. Our control design problem can be formulated as follows: design local controllers for both the master and the slave manipulators that

i) guarantee the tracking of the (delayed) master trajectory by the slave manipulator;

ii) guarantee the stability of the overall force-reflecting telerobotic system.

3. TOOLS: INPUT-TO-OUTPUT STABILITY FOR FDE AND IOS SMALL GAIN THEOREM

For the purpose of stability analysis of the telerobotic system with communication delay, we need to establish a specific IOS small gain result that guarantees stability of a feedback systems with components connected through time-varying delay blocks. An appropriate mathematical object that describes such an interconnection is a system of functional-differential equations (FDE). We follow standard notation (Hale, 1977). Given a function $x\colon\ \mathbb{R} \to \mathbb{R}^n$, denote $x_t(s) := x(t-s)$, where $s \geq 0$. Consider a system of functional differential equations with l inputs and r outputs of the following form

$$\begin{aligned} \dot{x}(t) &= F\left(x_t, u_t^{\{1\}}, \ldots, u_t^{\{l\}}, d_t\right), \\ y^{\{1\}}(t) &= H^{\{1\}}\left(x_t, u_t^{\{1\}}, \ldots, u_t^{\{l\}}, d_t\right), \\ &\cdots \\ y^{\{r\}}(t) &= H^{\{r\}}\left(x_t, u_t^{\{1\}}, \ldots, u_t^{\{l\}}, d_t\right), \end{aligned} \tag{5}$$

Here x is the state, $u^{\{i\}}$, $i \in \{1,\ldots,l\}$ are the inputs, $y^{\{j\}}$, $j \in \{1,\ldots,r\}$ are the outputs, and $d(\cdot)$ are the perturbations that are elements of the set of admissible perturbations $\mathcal{D}$. It is assumed that both F and H are continuous operators in x_t, u_t, and d_t. In particular, this guarantees the existence and uniqueness of solutions as well as continuous dependence of the solutions in x_t, u_t .

The following definition presents a version of the notion of input-to-output stability specified for multi-input multi-output systems of FDE. A close definition of input-to-state stability (ISS) for FDE was introduced in (Teel, 1998).

Definition 1. System of the form (5) is said to be input-to-output stable (IOS) at the moment $t = 0$ with $t_d \geq 0$, IOS gains $\gamma^{\{ij\}} \in \mathcal{K}$, $i \in \{1,\ldots,l\}$, $j \in \{1,\ldots,r\}$, and restriction $\left(\Delta_x, \Delta_u^{\{1\}}, \ldots, \Delta_u^{\{l\}}\right) \in \mathbb{R}_{>0}^{l+1}$, if the conditions $\sup_{t\in[-t_d,0]} |x(t)| \leq \Delta_x$, and $\sup_{t\geq -t_d} \left|u^{\{i\}}(t)\right| \leq \Delta_u^{\{i\}}$, $i \in \{1,\ldots,l\}$ imply that the solutions of (5) are well-defined for all $t \in [0,+\infty)$, and the following two properties hold:

i) *uniform boundedness*: there exists a function $\beta \in \mathcal{K}_\infty$ and $C \geq 0$ such that

$$\sup_{t\geq 0} \left|y^{\{j\}}(t)\right| \leq \max \left\{ \begin{array}{c} \beta\left(\sup_{s\in[-t_d,0]} |x(s)|\right), \\ \gamma^{\{1j\}}\left(\sup_{s\geq -t_d} \left|u^{\{1\}}(s)\right|\right), \\ \ldots, \\ \gamma^{\{lj\}}\left(\sup_{s\geq -t_d} \left|u^{\{l\}}(s)\right|\right), C \end{array} \right\}$$

for all $j \in \{1,\ldots,r\}$;

ii) *convergence*:

$$\limsup_{t\to\infty} \left|y^{\{j\}}(t)\right| \leq \max \left\{ \begin{array}{c} \gamma^{\{1j\}}\left(\limsup_{t\to\infty} \left|u^{\{1\}}(t)\right|\right), \\ \ldots, \\ \gamma^{\{lj\}}\left(\limsup_{t\to\infty} \left|u^{\{l\}}(t)\right|\right) \end{array} \right\}$$

for all $j \in \{1,\ldots,r\}$.

In this case, a function $\gamma^{\{i,j\}} \in \mathcal{K}$, where $i \in \{1,\ldots,l\}$, $j \in \{1,\ldots,r\}$, is called the IOS gain from $u^{\{i\}}$ to $y^{\{j\}}$. •

The following version of the IOS small gain theorem is our main tool in establishing the results presented in the next section.

Theorem 1. Consider two systems of ordinary differential equations of the following form

$$\Sigma_1\colon \begin{cases} \dot{x}_1 = F_1(x_1, u_1, w_1), \\ y_1^{\{1\}} = H_1^{\{1\}}(x_1, u_1, w_1), \\ y_1^{\{2\}} = H_1^{\{2\}}(x_1, u_1, w_1), \end{cases} \tag{6}$$

where $t_d = t_{d1} \geq 0$, and

$$\Sigma_2\colon \begin{cases} \dot{x}_2 = F_2\left(x_2, u_2^{\{1\}}, u_2^{\{2\}}, w_2, d\right), \\ y_2 = H_2\left(x_2, u_2^{\{1\}}, u_2^{\{2\}}, w_2, d\right), \end{cases} \tag{7}$$

where $d(t)$ is a Lebesgue measurable function that represents disturbances. Suppose $\tau_f, \tau_b\colon \mathbb{R} \to \mathbb{R}^+$ satisfy Assumption 1. Further, suppose the inputs and outputs of the systems Σ_1, Σ_2 satisfy the following constraints

$$u_1(t) \equiv u_2(t) \equiv 0 \quad \text{for } t < 0, \tag{8}$$

and

$$\begin{aligned} |u_1(t)| &\leq |y_2(t - \tau_b(t))|, \\ \left|u_2^{\{j\}}(t)\right| &\leq \left|y_1^{\{j\}}(t - \tau_f(t))\right| \end{aligned} \tag{9}$$

for $t \geq 0$, $j \in \{1, 2\}$. Suppose also

i) subsystem Σ_1 is input-to-output stable, *i.e.*, there exist $\beta_1^{\{1\}}, \beta_1^{\{2\}} \in \mathcal{K}_\infty$, $\gamma_{1u}^{\{1,j\}}, \gamma_{1w}^{\{j\}} \in \mathcal{K}$, and $C_1^{\{j\}} \geq 0$, $j = 1, 2$, such that for all $t_0 \in \mathbb{R}$ the following hold:

$$\sup_{t \geq t_0} \left|y_1^{\{1\}}(t)\right| \leq \max \left\{ \begin{array}{c} \beta_1^{\{1\}}(|x_1(t_0)|), \\ \gamma_{1u}^{\{1,1\}}\left(\sup_{t \geq t_0} |u_1(t)|\right), \\ \gamma_{1w}^{\{1\}}\left(\sup_{t \geq t_0} |w_1(t)|\right), C_1^{\{1\}} \end{array} \right\},$$

$$\sup_{t \geq t_0} \left|y_1^{\{2\}}(t)\right| \leq \max \left\{ \begin{array}{c} \beta_1^{\{2\}}(|x_1(t_0)|), \\ \gamma_{1u}^{\{1,2\}}\left(\sup_{t \geq t_0} |u_1(t)|\right), \\ \gamma_{1w}^{\{2\}}\left(\sup_{t \geq t_0} |w_1(t)|\right), C_1^{\{2\}} \end{array} \right\},$$

$$\limsup_{t \to \infty} \left|y_1^{\{1\}}(t)\right| \leq \max \left\{ \begin{array}{c} \gamma_{1u}^{\{1,1\}}\left(\limsup_{t \to \infty} |u_1(t)|\right), \\ \gamma_{1w}^{\{1\}}\left(\limsup_{t \to \infty} |w_1(t)|\right) \end{array} \right\},$$

$$\limsup_{t \to \infty} \left|y_1^{\{2\}}(t)\right| \leq \max \left\{ \begin{array}{c} \gamma_{1u}^{\{1,2\}}\left(\limsup_{t \to \infty} |u_1(t)|\right), \\ \gamma_{1w}^{\{2\}}\left(\limsup_{t \to \infty} |w_1(t)|\right) \end{array} \right\};$$

ii) subsystem Σ_2 is input-to-output stable, *i.e.*, there exist $\beta_2 \in \mathcal{K}_\infty$, $\gamma_{2u}^{\{j,1\}}, \gamma_{2w} \in \mathcal{K}$, $j = 1, 2$, and $C_2 \geq 0$, such that for all $t_0 \in \mathbb{R}$:

$$\sup_{t \geq t_0} |y_2(t)| \leq \max \left\{ \begin{array}{c} \beta_2(|x_2(t_0)|), \\ \gamma_{2u}^{\{1,1\}}\left(\sup_{t \geq t_0} \left|u_2^{\{1\}}(t)\right|\right), \\ \gamma_{2u}^{\{2,1\}}\left(\sup_{t \geq t_0} \left|u_2^{\{2\}}(t)\right|\right), \\ \gamma_{2w}\left(\sup_{t \geq t_0} |w_2(t)|\right), C_2 \end{array} \right\},$$

$$\limsup_{t \to \infty} |y_2(t)| \leq \max \left\{ \begin{array}{c} \gamma_{2u}^{\{1,1\}}\left(\limsup_{t \to \infty} |u_2^{\{1\}}(t)|\right), \\ \gamma_{2u}^{\{2,1\}}\left(\limsup_{t \to \infty} |u_2^{\{2\}}(t)|\right), \\ \gamma_{2w}\left(\limsup_{t \to \infty} |w_2(t)|\right) \end{array} \right\};$$

iii) there exists $\Delta_s > 0$ such that the following small gain condition holds

$$\gamma_{2u}^{\{j,1\}} \circ \gamma_{1u}^{\{1,j\}}(s) < s$$

for all $s \in (0, \Delta_s)$, $j \in \{1, 2\}$.

Suppose $\Delta_{x1}, \Delta_{x2}, \Delta_{w1}, \Delta_{w2} > 0$ satisfy

$$\Delta_s > \Delta_s^* := \max \left\{ \begin{array}{c} \gamma_{2u}^{\{1,1\}} \circ \beta_1^{\{1\}}(\Delta_{x1}), \\ \gamma_{2u}^{\{2,1\}} \circ \beta_1^{\{2\}}(\Delta_{x1}), \\ \beta_2(\Delta_{x2}), \gamma_{2u}^{\{1,1\}}\left(C_1^{\{1\}}\right) \\ \gamma_{2u}^{\{1,1\}} \circ \gamma_{1w}^{\{1\}}(\Delta_{w1}), \\ \gamma_{2u}^{\{2,1\}} \circ \gamma_{1w}^{\{2\}}(\Delta_{w1}), \\ \gamma_{2w}(\Delta_{w2}), \\ \gamma_{2u}^{\{2,1\}}\left(C_1^{\{2\}}\right), C_2 \end{array} \right\}.$$

Then the system (6), (7) subject to constraints (8), (9), with inputs w_1, w_2, and outputs $y_1^{\{1\}}$, $y_1^{\{2\}}$, y_2, being considered as a system of FDE, is input-to-output stable at the moment $t = 0$ in the sense of definition 1 with

$$t_d = \max \left\{ \begin{array}{c} \tau_f(0) + \tau_b(-\tau_f(0)), \\ \tau_b(0) + \tau_f(-\tau_b(0)) \end{array} \right\},$$

and restriction $(\Delta_{x1}, \Delta_{x2}, \Delta_{w1}, \Delta_{w2})$.

4. MAIN RESULT

To stabilize the force reflecting telerobotic system with time-varying communication delay, we propose the following local controllers

$$\begin{aligned} u_m = &-H_m(q_m)\Lambda_m\dot{q}_m \\ &-C_m(q_m, \dot{q}_m)\Lambda_m q_m \\ &+G_m(q_m) - K_m(\dot{q}_m + \Lambda_m q_m), \end{aligned} \tag{10}$$

$$\begin{aligned} u_s = &H_s(q_s)\left(\Lambda\left(\dot{\hat{q}}_m - \dot{q}_s\right)\right) \\ &+C_s(q_s, \dot{q}_s)\left(\dot{\hat{q}}_m - \Lambda(q_s - \hat{q}_m)\right) \\ &+G_s(q_s) - K_s\left(\dot{q}_s - \dot{\hat{q}}_m + \Lambda(q_s - \hat{q}_m)\right), \end{aligned} \tag{11}$$

where $\Lambda_m = \Lambda_m^T > 0$, $\Lambda_s = \Lambda_s^T > 0$, $K_m = K_m^T > 0$, $K_s = K_s^T > 0$ are symmetric positive definite matrices that can be chosen by the designer.

Following the idea introduced in (Polushin and Marquez, 2003a, b), we will consider the force reflecting telerobotic system with communication delay (1), (2), (3), (4), (10), (11) as a system of functional-differential equations (5). A state of the telerobotic system at time $t \in \mathbb{R}$ can be chosen as follows

$$\mathbf{x}_t := \left(q_m^T, \dot{q}_m^T, \tilde{q}_s^T, \dot{\tilde{q}}_s^T\right)_t^T, \qquad (12)$$

where $\tilde{q}_s = q_s - \hat{q}_m$, and $\dot{\tilde{q}}_s = \dot{q}_s - \dot{\hat{q}}_m$, and, as before, we use the notation $x_t(s) := x(t-s)$, $s \geq 0$. On the other hand, consider the following set of inputs

$$u^{\{1\}} = F_h, \quad u^{\{2\}} = f_{ext}, \qquad (13)$$

and the following set of outputs

$$y^{\{1\}} = \left(q_m^T, \dot{q}_m^T\right)^T, \quad y^{\{2\}} = \ddot{q}_m, \\ y^{\{3\}} = \left(\tilde{q}_s^T, \dot{\tilde{q}}_s^T\right)^T, \quad y^{\{4\}} = f_e, \qquad (14)$$

of the closed-loop telerobotic system. We will show that the closed-loop telerobotic system with the above defined state and outer signals (input and output) can be made input-to-output stable in the sense of definition 1 by an appropriate choice of the matrices Λ_m, Λ_s, K_m, and K_s. Below, given a symmetric matrix A, it's minimal (maximal) eigenvalue is denoted by $\lambda_{min}(A)$ ($\lambda_{max}(A)$). Our main result can be formulated as follows:

Theorem 2. Consider the controlled force-reflecting telerobotic system (1), (2), (3), (4), (10), (11) with state (12), inputs (13), and outputs (14). Suppose $\tau_f(\cdot)$, $\tau_b(\cdot)$ are time-varying communication delays satisfying the Assumption 1. Suppose also that the dynamics of the environment satisfy Assumption 2. Then, given $\Delta_x, \Delta_u^{\{1\}}, \Delta_u^{\{2\}} \in (0, +\infty)$, $\gamma_0 > 0$, there exist $\lambda_m, \lambda_s \geq 0$ such that for any Λ_m, Λ_s satisfying $\lambda_{min}(\Lambda_m) \geq \lambda_m$, $\lambda_{min}(\Lambda_s) \geq \lambda_s$, there exist $\kappa_m, \kappa_s \geq 0$ such that $\lambda_{min}(K_m) \geq \kappa_m$, $\lambda_{min}(K_s) \geq \kappa_s$ implies that the controlled force-reflecting telerobotic system is input-to-output stable at $t = 0$ with

$$t_d = \max \left\{ \begin{array}{c} \tau_f(0) + \tau_b(-\tau_f(0)), \\ \tau_b(0) + \tau_f(-\tau_b(0)) \end{array} \right\},$$

and restriction $\left(\Delta_x, \Delta_u^{\{1\}}, \Delta_u^{\{2\}}\right)$. Moreover, the IOS gains $\gamma^{\{13\}}$, $\gamma^{\{23\}}$ from inputs $u^{\{1\}}$, $u^{\{2\}}$ to output $y^{\{3\}} = \left(\tilde{q}_s^T, \dot{\tilde{q}}_s^T\right)^T$ is less than or equal to γ_0.

5. SIMULATIONS

Here we present an example of computer simulations of the proposed algorithm. Consider a force-reflecting telerobotic system where both the master and the slave are two-degrees-of-freedom manipulators with $H_m(q) = H_s(q) \in \mathbb{R}^{2\times 2}$, where

$$\begin{aligned} h_{11} &= (2l_1 \cos q_2 + l_2) l_2 m_2 + l_1^2 (m_1 + m_2), \\ h_{12} &= h_{21} = l_2^2 m_2 + l_1 l_2 m_2 \cos q_2, \\ h_{22} &= l_2^2 m_2, \end{aligned}$$

$C_m(q, \dot{q}) = C_s(q, \dot{q}) \in \mathbb{R}^{2\times 2}$, where

$$\begin{aligned} c_{11} &= -l_1 l_2 m_2 \sin(q_2) \dot{q}_2, \\ c_{12} &= -l_1 l_2 m_2 \sin(q_2)(\dot{q}_1 + \dot{q}_2), \\ c_{21} &= l_1 l_2 m_2 \sin(q_2), \\ c_{22} &= 0, \end{aligned}$$

and $G_m(q) = G_s(q) \in \mathbb{R}^2$, where

$$\begin{aligned} g_1 &= g\left(m_2 l_2 \sin(q_1 + q_2) + (m_1 + m_2) l_1 \sin(q_1)\right), \\ g_2 &= g m_2 l_2 \sin(q_1 + q_2), \end{aligned}$$

$m_1 = 10$, $m_2 = 5$, $l_1 = 0.7$, $l_2 = 0.5$, $g = 9.81$. The initial conditions are all zeros, *i.e.* $q_{m1}(0) = \dot{q}_{m1}(0) = q_{m2}(0) = \dot{q}_{m2}(0) = q_{s1}(0) = \dot{q}_{s1}(0) = q_{s2}(0) = \dot{q}_{s2}(0) = 0$. The forces applied by the human operator are as follows

$$F_{h1}(t) = 80 \sin(0.5t), \quad F_{h2}(t) = 50 \sin t.$$

The contact with the environment is simulated by the following simple model: for each joint $i = 1, 2$, we have $f_{ei} = 0$, if $q_i \leq \pi/2$, and

$$-f_{ei} = K(q_i - \pi/2) \quad \text{if } q_i > \pi/2,$$

where K is stiffness coefficient of the environment. In the simulations, we put $K = 1000$. The parameters of control law are set as follows: $K_m = 5 \cdot \mathbb{I}$, $K_s = 10 \cdot \mathbb{I}$, $\Lambda_m = 5 \cdot \mathbb{I}$, $\Lambda_s = 10 \cdot \mathbb{I}$, where $\mathbb{I} \in \mathbb{R}^{2\times 2}$ is the identity matrix. The communication channel is modeled by time varying time delay $\tau_f(t) = \tau_b(t)$ shown in figure 1.

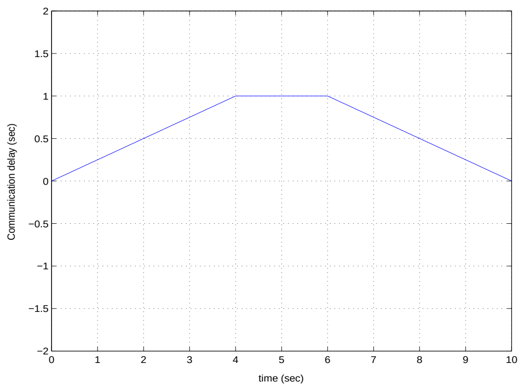

Fig. 1. Communication delay.

Simulation results are presented in figures 2-5. In particular, delayed position of the master vs. position of the slave and the contact force due to environment for 1st joint are shown in figures 2 and 3. Analogous plots for 2nd joint are given in figures 4 and 5. We see that these simulations demonstrate stability as well as good tracking properties of the proposed algorithm which are not destroyed by a contact with the environment.

6. CONCLUSIONS

The problem of stabilization of force reflecting teleoperators with time-varying possibly unbounded communication delay has been addressed.

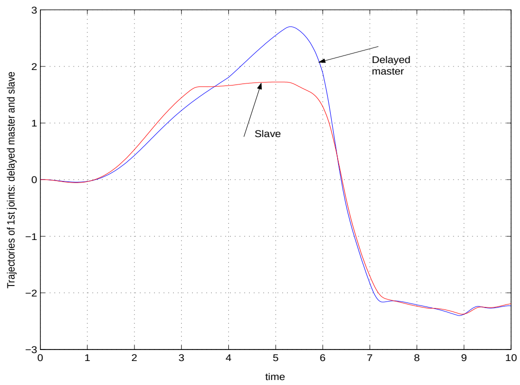

Fig. 2. Trajectories of 1st joints: delayed master vs. slave.

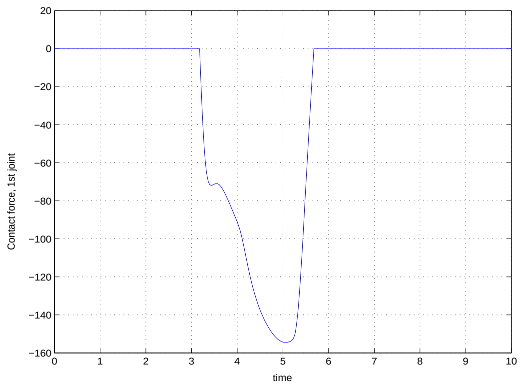

Fig. 3. Environmental forces applied to 1st joint of the slave.

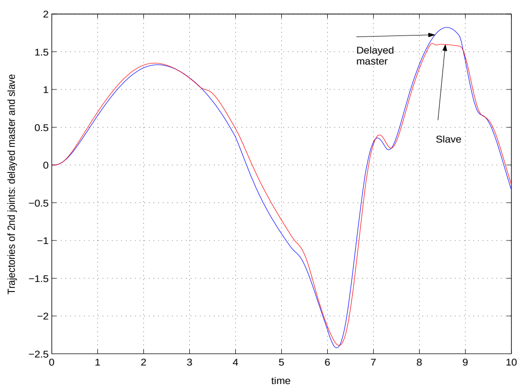

Fig. 4. Trajectories of 2nd joints: delayed master vs. slave.

Our interest to this problem is based on the fact that time-varying communication delays naturally arise when teleoperation is performed over the Internet. We propose a control scheme that guarantees the stability of the overall telerobotic system with time varying possibly unbounded communication delay satisfying a set of technical assumptions. Moreover, the proposed scheme guarantees that the slave tracks the delayed master trajectory with an error ultimately bounded by an arbitrarily small constant. The proof of this result is based on a special version of the IOS small gain theorem.

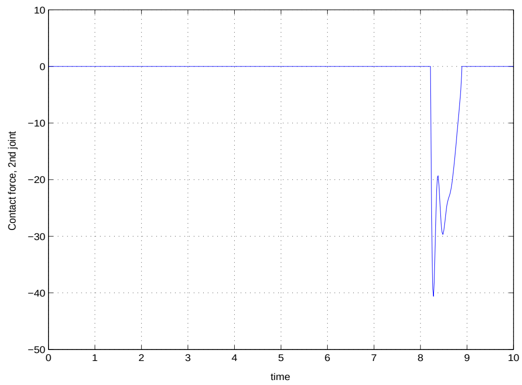

Fig. 5. Environmental forces applied to 2nd joint of the slave.

REFERENCES

Anderson, R. J. and M. W. Spong (1989). Bilateral control of teleoperators with time delay. *IEEE Transactions on Automatic Control*, **34**, 494–501.

Chopra, N., M. W. Spong, S. Hirche and M. Buss (2003). Bilateral teleoperation over the internet: the time varying delay problem. In: *Proceedings of American Control Conference*. Denver, CO.

W. R. Ferrell, W. R. (1966). Delayed force feedback. *Human Factors*, **8**, 449–455.

Hale, J. K. (1977). *Theory of Functional Differential Equations*. Springer-Verlag, New York.

Jiang, Z.-P., A. R. Teel and L. Praly (1994). Small-gain theorem for ISS systems and applications. *Mathematics of Control, Signals, and Systems*, **7**, 95–120.

Niemeyer, G. and J.-J. E. Slotine (1991). Stable adaptive teleoperation. *IEEE Journal of Oceanic Engineering*, **16**, 152–162.

Polushin, I.G. and H. J. Marquez (2003a). Stabilization of bilaterally controlled teleoperators with communication delay: an ISS approach. *International Journal of Control*, **76**, 858–870.

Polushin, I.G. and H. J. Marquez (2003b). The ISS small gain approach to stabilization of bilaterally controlled teleoperators with communication delay. In: *Proceedings of the 7th European Control Conference*. Cambridge, UK.

Sontag, E. D. and Y. Wang (1999). Notions of input-to-output stability. *Systems & Control Letters*, **38**, 235–248.

Spong, M. W. (1996). Motion control of robot manipulators. In: *Handbook of Control* (W. Levine. (Ed.)), pp. 1339–1350. CRC Press, Boca Raton, FL.

Teel, A. R. (1998). Connections between Razumikhin-type theorems and the ISS nonlinear small gain theorem. *IEEE Transactions on Automatic Control*, **43**, 960–964.