THE PARAMETRIZATION OF ALL STABILIZING MULTI-PERIOD REPETITIVE CONTROLLERS WITH THE SPECIFIED FREQUENCY CHARACTERISTICS

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Abstract: In this paper, we examine the parametrization of all stabilizing multi-period repetitive controllers with the specified frequency characteristics. The parametrization of all stabilizing multi-period repetitive controllers, those are used to improve the disturbance attenuation characteristics of the repetitive controller, for non-minimum phase systems was solved by Yamada et al. However, when we design a stabilizing modified repetitive controller using the parametrization by Yamada et al., the frequency characteristics of the control system cannot be settled so easily. From the practical point of view, the frequency characteristics of the control systems are required to be easily settled. This problem is solved by obtaining the parametrization of all stabilizing multi-period repetitive controllers with the specified frequency characteristics. However, no paper has proposed the parametrization of all stabilizing multi-period repetitive controllers with the specified frequency characteristics. In this paper, we expand the result by Yamada et al. and propose the parametrization of all stabilizing multi-period repetitive controllers with the specified frequency characteristics.

Keywords: repetitive control, multi-period repetitive controller, parametrization, low-pass filter

1. INTRODUCTION

In this paper, we investigate the parametrization of all stabilizing multi-period repetitive controllers with the specified frequency characteristics. The parametrization problem is to find all stabilizing controllers (Youla *et al.*, 1976; Kucera, 1979; Dedoer *et al.*, 1980; Glaria and Goodwin, 1994; Vidyasagar, 1985). First, the parametrization of all stabilizing modified repetitive controllers which follows the periodic reference input with small steady state error even if there exists a periodic disturbance or the uncertainty of the plant was studied by (Hara and Yamamoto, 1986). In (Hara and Yamamoto, 1986), since the stability sufficient condition of repetitive control system is decided as H_∞ norm problem, the parametrization for repetitive control system is given by resolving into the interpolation problem of Nevanlinna-Pick. Katoh and Funahashi gave the parametrization of all stabilizing repetitive controllers for minimum phase systems by solving exactly Bezout equation (Katoh and Funahashi, 1996). However, Katoh and Funahashi (Katoh and Funahashi, 1996) assumed the plant is asymptotically stable. This implies that they gave the parametrization of all causal repetitive controllers for an asymptotically stable and minimum phase plant. That is, they do not give the explicit parametrization for minimum phase systems (Katoh and Funahashi, 1996). In addition, in (Katoh and Funahashi, 1996) it is assumed that the relative degree of low-pass filter in the repetitive compensator is equal to that of the plant. Extending the results in (Katoh and Funahashi, 1996), Yamada and Okuyama gave the parametrization of all stabilizing repetitive controllers for minimum phase systems (Yamada and Okuyama, 2000). Yamada et al. gave the parametrization of all stabilizing repetitive controllers for the certain class of non-minimum phase systems (Yamada *et al.*, 2002*a*). They obtained the parametrization of all repetitive controllers using fusion of the parallel compensation technique and the solution of Bezout equation. However, they gave the parametrization of all repetitive controllers for limited class of non-minimum phase systems. Yamada et al. gave the complete parametrization of all stabilizing modified repetitive controllers for non-minimum phase single-input/single-output systems (Yamada *et al.*, 2002*b*). In addition, the parametrization of all stabilizing repetitive controllers for non-minimum phase multivariable systems was considered in (Yamada *et al.*, 2004*a*). The parametrization of all stabilizing multi-period repetitive controllers for non-minimum phase systems which is used to improve the disturbance attenuation characteristics of the repetitive controller was solved in (Yamada *et al.*, 2004*b*). However, when we design a stabilizing modified repetitive controllers using the parametrization in (Yamada *et al.*, 2004*b*), the frequency characteristics of the control system cannot be settled so easily. From the practical point of view, the frequency characteristics of the control systems are required to be easily settled. This problem is solved by obtaining the parametrization of all stabilizing multi-period repetitive controllers with the specified frequency characteristics. However, no paper has proposed the parametrization of all stabilizing multi-period repetitive controllers with the specified frequency characteristics.

In this paper, we expand the result in (Yamada *et al.*, 2004*b*) and propose the parametrization of all stabilizing multi-period repetitive controllers with the specified frequency characteristics. The basic idea to obtain the parametrization of all stabilizing multi-period repetitive controllers with the specified frequency characteristics is very simple. If the multi-period repetitive controller stabilizes the plant, then the multi-period repetitive controller must be included in the class of all stabilizing controllers for the plant. The parametrization of all stabilizing controllers for the plant are obtained using the method in (Youla *et al.*, 1976; Vidyasagar, 1985). The parametrization of all stabilizing controllers include the free parameter, which is the set of stable causal function. That is, the parametrization of all stabilizing multi-period repetitive controllers with the specified frequency characteristics can be designed using the free parameter in the parametrization. Using this idea, we obtain the parametrization of all stabilizing multi-period repetitive controllers with the specified frequency characteristics.

NOTATIONS

R	the set of real numbers.
$R(s)$	the set of real rational functions with s.
RH_∞	the set of stable proper real coefficient rational functions.
H_∞	the set of stable causal function.
$\mathcal{U}$	the unimodular procession in H_∞. That is, $U(s) \in \mathcal{U}$ means that $U(s) \in H_\infty$ and $1/U(s) \in H_\infty$.

2. PROBLEM FORMULATION

Consider the unity feedback control system in

$$\begin{cases} y = G(s)u \\ u = C(s)(r-y) \end{cases}, \quad (1)$$

where $G(s) \in R(s)$ is the single-input/single-output plant, $G(s)$ is assumed to be coprime. $C(s)$ is the multi-period repetitive controller defined later, $y \in R$ is the output and $r \in R$ is the periodic reference input with period T satisfying

$$r(t+T) = r(t) \quad \forall t \geq 0. \quad (2)$$

According to (Gotou *et al.*, 1987), the multi-period repetitive controller $C(s)$ in (1) is written by the form in

$$C(s) = C_0(s) + \frac{\displaystyle\sum_{i=1}^{N} C_i(s) q_i(s) e^{-sT_i}}{1 - \displaystyle\sum_{i=1}^{N} q_i(s) e^{-sT_i}}, \quad (3)$$

where N is arbitrary positive integer, $C_0(s) \in R(s)$, $C_i(s) \neq 0 \in R(s)(i = 1, \ldots, N)$, $q_i(s) \in RH_\infty(i = 1, \ldots, N)$ are low-pass filter satisfying $\sum_{i=1}^{N} q_i(0) = 1$ and $T_i \in R(i = 1, \ldots, N)$.

From (Gotou *et al.*, 1987), it is note that if low-pass filter $q_i(s)(i = 1, \ldots, N)$ satisfy

$$1 - \sum_{i=1}^{N} q_i(s) = 0 \quad \forall s = j\omega_k (k = 0, \ldots, N) \quad (4)$$

$$\omega_k = \frac{2\pi k}{T}(k = 0, \ldots, N), \quad (5)$$

then the output y in (1) follows reference input r with small steady state error. In order for $q_i(s)(i = 1, \ldots, N)$ to satisfy (4) in wide frequency range, $q_i(s)(i = 1, \ldots, N)$ must be stable and of minimum phase. Using result in (Yamada *et al.*, 2004*b*), it is difficult to settle $q_i(s)(i = 1, \ldots, N)$ to be stable and of minimum phase. If we obtain the parametrization of all stabilizing multi-period repetitive controllers such that $q_i(s)(i = 1, \ldots, N)$ in (3) is settled beforehand, we can easily design the multi-period repetitive controller in (3) satisfying (4). Since $q_i(s)(i = 1, \ldots, N)$ works to specify the frequency characteristics of the control system in (1), we call the parametrization of all stabilizing multi-period repetitive controllers such that $q_i(s)(i = 1, \ldots, N)$ in (3) is settled beforehand the parametrization with specify the frequency characteristics.

The problem considered in this paper is that when the low-pass filter $q_i(s)(i = 1, \ldots, N)$ in (3) is settled beforehand, we find the parametrization of all stabilizing multi-period repetitive controllers with the specified frequency characteristics written in (3) such that the system in (1) is internally stable.

3. THE PARAMETRIZATION OF ALL STABILIZING MULTI-PERIOD REPETITIVE CONTROLLERS WITH THE SPECIFIED FREQUENCY CHARACTERISTICS

In this section, we give the parametrization of all stabilizing multi-period repetitive controllers with the specified frequency characteristics.

In order to obtain the parametrization of all multi-period repetitive controllers with the specified frequency characteristics, $q_i(s) \in RH_\infty(i = 1, \ldots, N)$ is assumed to be settled beforehand. The parametrization of all multi-period repetitive controllers with the specified frequency characteristics written by the form in (3) such that the system in (1) is internally stable is given by following theorem.

Theorem 1. The parametrization of all repetitive controllers written by the form in (3) such that the control system in (1) is internally stable if and only if $C(s)$ is written by

$$C(s) = \frac{\tilde{X}(s) + D(s)Q(s)}{\tilde{Y}(s) - N(s)Q(s)}, \tag{6}$$

where $N(s) \in RH_\infty$, $D(s) \in RH_\infty$, $\tilde{N}(s) \in RH_\infty$ and $\tilde{D}(s) \in RH_\infty$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = N(s)D^{-1}(s) = \tilde{D}^{-1}(s)\tilde{N}(s). \tag{7}$$

$\tilde{X}(s)$ and $\tilde{Y}(s)$ are RH_∞ function satisfying

$$\begin{bmatrix} Y(s) & X(s) \\ -\tilde{N}(s) & \tilde{D}(s) \end{bmatrix} \begin{bmatrix} D(s) & -\tilde{X}(s) \\ N(s) & \tilde{Y}(s) \end{bmatrix} = I, \tag{8}$$

where $X(s)$ and $Y(s)$ are RH_∞ function. $Q(s) \in H_\infty$ is written by

$$Q(s) = \frac{Q_{n0}(s) + \sum_{i=1}^{N} Q_{ni}(s) q_i(s) e^{-sT_i}}{Q_{d0}(s) + \sum_{i=1}^{N} Q_{di}(s) q_i(s) e^{-sT_i}}, \tag{9}$$

where $Q_{n0}(s)$, $Q_{d0}(s) \neq 0$, $Q_{ni}(s)(i = 0, \cdots, N)$, $Q_{di}(s)(i = 0, \cdots, N)$ are any RH_∞ functions satisfying

$$\begin{aligned} &\tilde{Y}(s)(Q_{d0}(s) + Q_{di}(s)) \\ &\quad -N(s)(Q_{n0}(s) + Q_{ni}(s)) = 0 (i = 1, \ldots, N) \end{aligned} \tag{10}$$

and

$$\begin{aligned} &\tilde{X}(s)(Q_{d0}(s) + Q_{di}(s)) \\ &\quad +D(s)(Q_{n0}(s) + Q_{ni}(s)) \neq 0 (i = 1, \ldots, N). \end{aligned} \tag{11}$$

Proof of this theorem requires following lemma.

Lemma 1. Unity feedback control system in

$$\begin{cases} y = G(s)u \\ u = -C(s)y \end{cases} \tag{12}$$

is internally stable if and only if $C(s)$ is written by

$$C(s) = \frac{\tilde{X}(s) + D(s)Q(s)}{\tilde{Y}(s) - N(s)Q(s)}, \tag{13}$$

where $N(s) \in RH_\infty$, $D(s) \in RH_\infty$, $\tilde{N}(s) \in RH_\infty$ and $\tilde{D}(s) \in RH_\infty$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = N(s)D^{-1}(s) = \tilde{D}^{-1}(s)\tilde{N}(s). \tag{14}$$

$\tilde{X}(s)$ and $\tilde{Y}(s)$ are RH_∞ function satisfying (8) and $Q(s) \in H_\infty$ is free parameter (Vidyasagar, 1985).

Using above Lemma 1, we shall show the proof of Theorem 1.

(Proof) First, necessity is shown. That is, if the controller written by (3) stabilize the control system in (1), then $C(s)$ and $Q(s)$ are written by (6) and (9), respectively. From Lemma 1, the parametrization of all stabilizing controllers $C(s)$ for $G(s)$ is written by (6). In addition, $Q(s) \in H_\infty$ is any function. In order to prove necessity, we will show that if $C(s)$ written by (3) stabilizes

the control system in (1), then the free parameter $Q(s) \in H_\infty$ is written by (9). Substituting $C(s)$ in (3) into (6), we have

$$Q(s) = \frac{Q_{n0}(s) + \sum_{i=1}^{N} Q_{ni}(s) q_i(s) e^{-sT_i}}{Q_{d0}(s) + \sum_{i=1}^{N} Q_{di}(s) q_i(s) e^{-sT_i}}, \quad (15)$$

where

$$Q_{n0}(s) = \left(\tilde{Y}(s) C_{0n}(s) - \tilde{X}(s) C_{0d}(s)\right) \bar{C}_d(s), \quad (16)$$

$$Q_{ni}(s) = -\left(\tilde{Y}(s) C_{0n} - \tilde{X}(s) C_{0d}(s)\right) \bar{C}_d(s) + \tilde{Y}(s) C_{0d}(s) \bar{C}_{in}(s) \quad (i = 1, \cdots, N), \quad (17)$$

$$Q_{d0}(s) = \left(D(s) C_{0d}(s) + N(s) C_{0n}(s)\right) \bar{C}_d(s) \quad (18)$$

and

$$Q_{di}(s) = -\left(D(s) C_{0d}(s) + N(s) C_{0n}(s)\right) \bar{C}_d(s) + N(s) C_{0d}(s) \bar{C}_{in}(s) \quad (i = 1, \cdots, N). \quad (19)$$

Here, $C_{0n}(s)$ and $C_{0d}(s)$ are coprime factors of $C_0(s)$ on RH_∞ satisfying

$$C_0(s) = \frac{C_{0n}(s)}{C_{0d}(s)}. \quad (20)$$

$\bar{C}_{in}(s) \in RH_\infty (i = 1, \ldots, N)$ and $\bar{C}_d(s) \in RH_\infty$ are written by

$$\bar{C}_{in}(s) = C_{in}(s) \prod_{j=1}^{i-1} C_{jd}(s) \prod_{j=i+1}^{N} C_{jd}(s) \quad (i = 1, \ldots, N) \quad (21)$$

and

$$\bar{C}_d(s) = \prod_{i=1}^{N} C_{id}(s), \quad (22)$$

respectively. $C_{in}(s)(i = 1, \cdots, N)$ and $C_{id}(s)(i = 1, \cdots, N)$ are coprime factors of $C_i(s)(i = 1, \cdots, N)$ on RH_∞ satisfying

$$C_i(s) = \frac{C_{in}(s)}{C_{id}(s)} \quad (i = 1, \cdots, N). \quad (23)$$

Since $N(s) \in RH_\infty$, $D(s) \in RH_\infty$, $\tilde{X}(s) \in RH_\infty$, $\tilde{Y}(s) \in RH_\infty$, $C_{0n}(s) \in RH_\infty$, $C_{0d}(s) \in RH_\infty$, $\bar{C}_{in}(s) \in RH_\infty (i = 1, \ldots, N)$ and $\bar{C}_d(s) \in RH_\infty$, we find that $Q_{n0}(s) \in RH_\infty$ in (16), $Q_{ni}(s) \in RH_\infty (i = 1, \ldots, N)$ in (17), $Q_{d0}(s) \in RH_\infty$ in (18) and $Q_{di}(s) \in RH_\infty (i = 1, \ldots, N)$ in (19). From this expression and (15), we have proved that if the controller written by (3) stabilize the control system in (1), then the free parameter $Q(s)$ in (6) is written by (9). From the assumption of $C_i(s) \neq 0 \ (i = 1, \cdots, N)$, (11) holds true. From (16) $\sim$ (19), (10) is satisfied.

Next the sufficiency is shown. That is, if $Q(s)$ in (6) is written by (9), then the controller $C(s)$ is written by (3) under the assumption of (10) and (11). Substituting (9) into (6), we have

$$C(s) = C_0(s) + \frac{\sum_{i=1}^{N} C_i(s) q_i(s) e^{-sTi}}{1 - \sum_{i=1}^{N} q_i(s) e^{-sT_i}}, \quad (24)$$

where $C_0(s)$, $C_i(s)(i = 1, \cdots, N)$ are denoted by

$$C_0(s) = \frac{\tilde{X}(s) Q_{d0}(s) + D(s) Q_{n0}(s)}{\tilde{Y}(s) Q_{d0}(s) - N(s) Q_{n0}(s)}, \quad (25)$$

$$C_i(s) = \frac{\tilde{X}(s)(Q_{d0}(s) + Q_{di}(s))}{\tilde{Y}(s) Q_{d0}(s) - N(s) Q_{n0}(s)} \frac{+D(s)(Q_{n0}(s) + Q_{ni}(s))}{} \quad (i = 1, \cdots, N). \quad (26)$$

We find that if $C(s)$ and $Q(s)$ is settled by (6) and (9), then the controller $C(s)$ is written by the form in (3). From (11), $C_i(s) \neq 0$ hold true.

We have thus proved Theorem 1. ∎

4. A DESIGN METHOD OF $Q(S)$

In this section, we present a design method of the free parameter $Q(s)$ satisfying Theorem 1. From Theorem 1, $Q(s)$ in (6) must be included in H_∞. Since $Q_{n0}(s) \in RH_\infty$ and $Q_{ni}(s) \in RH_\infty$ in (9), if $1/\left(Q_{d0}(s) + \sum_{i=1}^{N} Q_{di}(s) q_i(s) e^{-sT_i}\right) \in H_\infty$, then $Q(s) \in H_\infty$. $Q_{d0}(s) + \sum_{i=1}^{N} Q_{di}(s) q_i(s) e^{-sT_i}$ is rewritten by

$$Q_{d0}(s) + \sum_{i=1}^{N} Q_{di}(s) q_i(s) e^{-sT_i} = Q_{d0}(s) \left(1 + \sum_{i=1}^{N} \frac{Q_{di}(s) q_i(s)}{Q_{d0}(s)} e^{-sT_i}\right). \quad (27)$$

From above equation and Rouche's Theorem (Levine, 1996), if $Q_{d0}(s)$ is settled to be included in $\mathcal{U}$ and $Q_{di}(s) \in RH_\infty$ is settled satisfying

$$\left\| \sum_{i=1}^{N} \frac{Q_{di}(s)q_i(s)}{Q_{d0}(s)} \right\|_\infty < 1, \tag{28}$$

then $Q(s) \in H_\infty$.

5. NUMERICAL EXAMPLE

In this section, a numerical example is shown to illustrate the effectiveness of the proposed parametrization.

Let us consider to obtain the parametrization of all stabilizing multi-period repetitive controllers with the specified frequency characteristics for the plant $G(s)$ written by

$$G(s) = \frac{-s + 400}{s^2 + 4s - 21}. \tag{29}$$

Here, the period T of the reference input is $T = \pi/2$[sec], N in (3) is $N = 3$. Using the method of (Okuyama *et al.*, 2002), $T_i(i = 1, 2, 3)$ and $q_i(s)(i = 1, 2, 3)$ in (3) are settled by

$$T_1 = 1.5608, \tag{30}$$

$$T_2 = 3.2285, \tag{31}$$

$$T_3 = 4.6166, \tag{32}$$

$$q_1(s) = \frac{1}{0.01s + 1}, \tag{33}$$

$$q_2(s) = \frac{1}{0.01s + 1} \cdot \frac{0.047s(s^2 + 4^2)}{(s + 2)^3} \tag{34}$$

$$q_3(s) = \frac{1}{0.01s + 1} \cdot \frac{0.047s(s^2 + 4^2)}{(s + 2)^3} \cdot \frac{25.3772(s^2 + 8^2)}{(s + 2)^3}, \tag{35}$$

respectively. $D(s)$, $N(s)$, $\tilde{X}(s)$ and $\tilde{Y}(s)$ in (6) are given by

$$D(s) = \tilde{D}(s) = \frac{s^2 + 4s - 21}{s^2 + 30s + 200}, \tag{36}$$

$$N(s) = \tilde{N}(s) = \frac{-s + 400}{s^2 + 30s + 200}, \tag{37}$$

$$\tilde{X}(s) = X(s) = \frac{14.7349s + 96.0783}{s^2 + 30s + 200} \tag{38}$$

and

$$\tilde{Y}(s) = Y(s) = \frac{s^2 + 56s + 998.9967}{s^2 + 30s + 200}, \tag{39}$$

respectively. According to Theorem 1, the parametrization of all stabilizing multi-period repetitive controllers with the specified frequency characteristics is written by (6) and (9).

In order to satisfy (10) and (11), $Q_{n0}(s)$, $Q_{ni}(s)(i = 1, 2, 3)$, $Q_{d0}(s)$ and $Q_{di}(s)(i = 1, 2, 3)$ in (9) are settled by

$$Q_{n0}(s) = \frac{s^2 + 56.1s + 992}{s^2 + 30s + 200}, \tag{40}$$

$$Q_{ni}(s) = \frac{-0.1s + 6.9967}{s^2 + 30s + 200} \quad (i = 1, 2, 3), \tag{41}$$

$$Q_{d0}(s) = \frac{0.01s^2 + 4s + 400}{s^2 + 30s + 200} \tag{42}$$

and

$$Q_{di}(s) = \frac{-0.01s^2 - 5s}{s^2 + 30s + 200} \quad (i = 1, 2, 3). \tag{43}$$

From the discussion in Section 4., since $Q_{n0}(s) \in RH_\infty$, $Q_{ni}(s) \in RH_\infty$, $Q_{d0}(s) \in RH_\infty$, $Q_{di}(s) \in RH_\infty$ and $q_i(s) \in RH_\infty$ in (9), if (28) is satisfied, then $Q(s)$ in (9) is included in H_∞. The bode plot of $\sum_{i=1}^{3} \frac{Q_{di}(s)q_i(s)}{Q_{d0}(s)}$ is shown in Fig. 1 . Since $\sum_{i=1}^{3} \frac{Q_{di}(s)q_i(s)}{Q_{d0}(s)} \in RH_\infty$, Fig. 1 shows that (28) holds true. Therefore, $Q(s)$ is included in H_∞.

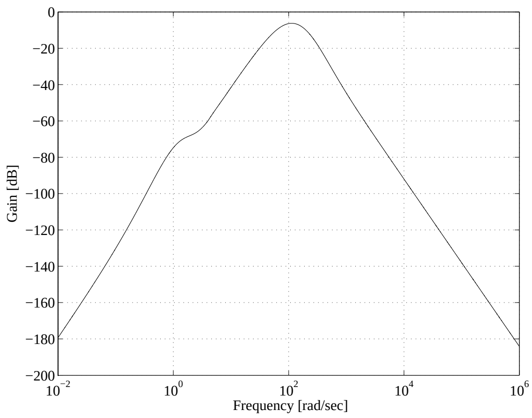

Fig. 1. Bode plot of $\sum_{i=1}^{3} \frac{Q_{di}(s)q_i(s)}{Q_{d0}(s)}$

Using the obtained multi-period repetitive controller $C(s)$, the response of the output $y(t)$ in (1) for the reference input $r(t) = \sin(4t)$ is shown in Fig. 2 . Here, the solid line shows the response of the output $y(t)$ and the dotted line shows that for reference input $r(t)$. Fig. 2 shows that the output $y(t)$ follows the reference input $r(t)$ with very small steady state error.

Next, when disturbance $d(t) = \sin(4t)$ exists, the response of the output $y(t)$ for the disturbance is shown in Fig. 3 . Here, the solid line shows the response of the output $y(t)$ and the dotted line shows the disturbance $d(t)$. Fig. 3 shows that the disturbance is attenuated effectively.

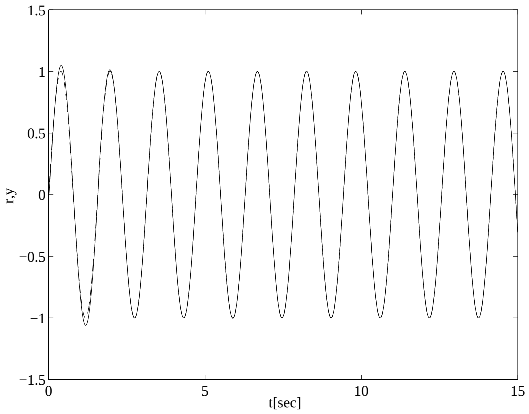

Fig. 2. Response for the reference input $r(t) = \sin(4t)$

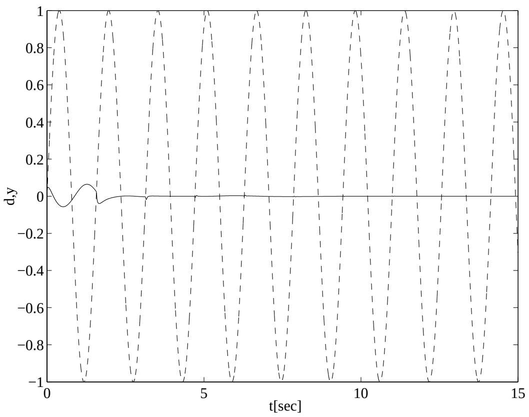

Fig. 3. Response for the disturbance $d(t) = \sin(4t)$

6. CONCLUSION

In this paper, we proposed the parametrization of all stabilizing multi-period repetitive controllers with the specified frequency characteristics.

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