RELIABLE GUARANTEED COST CONTROL FOR PARAMETERIZED INTERCONNECTED SYSTEMS WITH LMI CHARACTERIZATION

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Abstract: This paper presents the design of a reliable decentralized state feedback control for a class of uncertain interconnected polytopic continous systems. A model of failures in actuators is adopted which considers outages or partial degradation in independent actuators. The control is developed using the concept of guaranteed cost control and a new LMI characterization using polytopic Lyapunov functions.

Keywords: Reliability, Interconnected Systems, LMI

1. INTRODUCTION

The decentralized control of interconnected systems in the presence of uncertainties has attracted considerable attention in the last years. With the aim of stabilizing the overall system while ensuring a satisfactory performance, the guaranteed cost control approach (GCC) has been recently considered (see (Gong *et al.*, 1996), (Mukaidini *et al.*, 2002), (Xie *et al.*, 1999) and (Yang *et al.*, 2000)). The guaranteed cost control is concerned with the design of a state feedback controller such that the closed-loop system is stable and an upper bound of a quadratic cost function is minimized.

Another important issue when dealing with interconnected systems is the design of fault-tolerant control systems. Reliable control is concerned with the design of a closed-loop system to maintain key properties, in spite of sensor or actuator outage or partial degradation. Two main approaches have been proposed in the literature. One uses multiple controllers in a redundant control scheme ((Siljak, 1980), (Yang *et al.*, 1998)). The other approach seeks a reliable control design without redundancy by ensuring stability and some performance bounds for specified class of admissible failures of particular control components (Veillette, 1995). Within this approach, the class of failures have been usually modelled as outages. This model considers that the control set can be partitioned into two subsets: one subset includes the actuators whose failures are admissible in the control design; and the complementary subset with the actuators that are assumed to keep a normal operation (see (Liao *et al.*, 2002)). A more general failure scheme is considered in (Yang *et al.*, 2000), where the authors present a model of actuator failures which takes into account the outage case and also the possibility of partial failures. In (Yang *et al.*, 2000) a centralized reliable control is designed for a class of uncertain nonlinear systems without subsystem interconnections. In the present paper, we adopt this failure model extended within a reliable decentralized control scheme for uncertain interconnected linear systems. Linear matrix inequalities (LMI) tech-

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niques have been used in the context of guaranteed cost control problems and reliable control design ((Yang *et al.*, 2001), (Liao *et al.*, 2002)). The currently known LMI characterizations are potentially conservative in the sense that they use a common Lyapunov function regardless of the parameter values. In order to reduce conservatism in the case of constant parameters, the notion of parameter-dependent function was introduced (Barmish, 1989). The Lyapunov conditions leads to nonconvex optimization problem which does not seem tractable in general. In (Apkarian *et al.*, 2000) and (Tuan *et al.*, 2003), this weakness is overcome using auxiliary variables, as well as replacing the functions of singular Lyapunov for multiple functions to obtain more robust tools and to reduce the conservatism in the control problems, and provides additional flexibility in a wide range of problems. In (Tuan *et al.*, 2003), a way to "convexifying" the general affine Lyapunov problem has been proposed, leading to a parametric linear matrix inequality.

This paper has two objectives. One is related to the decentralized reliable guaranteed cost control problem for interconnected continous systems. To our knowledge, there are not available results dealing with reliable control for uncertain interconnected systems with guaranteed cost. We present a control design that allows to consider reliability and guaranteed cost. This reliable control shows that the admission of control failures imposes some restriction in the control weighting matrices in the performance criterion. Thus the designer can take some trade-off between control performance and admitted reliability. The second objective is to give a new LMI characterization and its relation with polytopic systems and polytopic Lyapunov functions. In the centralized control design of uncertain systems with guaranteed cost and reability, matrix inequalities are obtained which lead to LMI's through a linealization process. When dealing with decentralized control of interconnected systems this process needs to be extended. In this paper, we obtain a general result that can be applied to different classes of problems, such as guaranteed cost, H_2 or H_∞ control design. We apply this result in the design of decentralized reliable control of uncertain interconnected systems with guaranteed performance.

The notation throughout the paper is fairly standard. In symmetric block matrices or long matrix expressions, we use $*$ as an ellipsis for terms that are induced by symmetry, e.g.

$$\begin{pmatrix} S+(*) & * \\ M & Q \end{pmatrix} \equiv \begin{pmatrix} S+S^T & M^T \\ M & Q \end{pmatrix} .$$

We use bold to denote dependence on the parameter α, e.g. $\mathbf{A} = A(\alpha)$; I denotes the identity matrix.

2. PROBLEM STATEMENT

Consider a class of large-scale interconnected system composed of N polytopic subsystems described by the following state equations:

$$\begin{cases} \dot{x}_i = A_i(\alpha)x_i + B_i(\alpha)u_i + \sum_{j\neq i} G_{ij}g_{ij}(t,x_j) \\ x_i(0) = x_{i0},\ i = 1,\ldots,N\ . \end{cases} \tag{1}$$

We define:

- $x_i \in \mathbb{R}^{n_i}$ state of the ith subsystem
- $u_i \in \mathbb{R}^{s_i}$ control of the ith subsystem
- $A_i(\alpha) \in \mathbb{R}^{n_i\times n_i}$ parametric state matrix
- $B_i(\alpha) \in \mathbb{R}^{n_i\times s_i}$ parametric control matrix
- $g_{ij}(t,x_j) \in \mathbb{R}^{l_i}$ unknown interconnection vector function
- $G_{ij} \in \mathbb{R}^{n_i\times l_i}$ constant interconnection matrix

It is assumed that the unknown vectors $g_{ij}(t,x_j)$ are continuous and sufficiently smooth in x_j and piecewise continuous in t.

Uncertain model
The parameter uncertainties considered here are assumed to be of the following form:

$$A_i(\alpha) = \sum_{k=1}^{L} \alpha_k A_{i_k}\ ,\ B_i(\alpha) = \sum_{k=1}^{L} \alpha_k B_{i_k}\ . \tag{2}$$

The parameter vector α is in the simplex Π defined by

$$\Pi = \{\alpha \in \mathbb{R}^L,\ \sum_{k=1}^{L} \alpha_k = 1,\ \alpha_k \geq 0,\ k = 1,\ldots,L\}\ . \tag{3}$$

That is, $A_i(\alpha) \in \mathrm{co}\{A_{i_1},\ldots,A_{i_L}\}$ and $B_i(\alpha) \in \mathrm{co}\{B_{i_1},\ldots,B_{i_L}\}$ are convex combinations of the matrix A_k and B_k, respectively.

Interconnection assumptions

Assumption 1. There exist known constant matrices W_{ij} such that, for all $x_j \in R^{n_j}$,

$$||g_{ij}(t,x_j)|| \leq ||W_{ij}\ x_j|| \tag{4}$$

for all i, j and for all $t \geq 0$, where $||\ ||$ denotes the Euclidean norm.

Assumption 2. For all i, $W_i := \sum_{j=1,j\neq i}^{N} W_{ji}^T\, W_{ji} > 0$.

The Assumption 1 allows some linear structure to the interconnection vector g_{ij} in terms of the state vector x_j. The Assumption 2 ensure that almost two subsystems must be interconnected, because at least one W_{ij} must be nonzero, for each i.

Cost function
Consider the following cost function associated with system (1):

$$J(x,u) = \sum_{i=1}^{N} \int_0^\infty \left(x_i^T Q_i x_i + u_i^T R_i u_i\right)\, dt \tag{5}$$

where $Q_i \in \mathbb{R}^{n_i \times n_i}$ and $R_i \in \mathbb{R}^{s_i \times s_i}$ are given constant symmetric positive definite matrices, for all i.

Failure model
Let u_i^F denote the vectors with the signals from the s_i actuators which control the ith subsystem. Here we consider the following failure model:

$$u_i^F = \Lambda_i u_i + \phi_i(u_i), \qquad i = 1, \ldots, N \qquad (6)$$

where $\Lambda_i = \mathrm{diag}(\lambda_{i1}, \ldots, \lambda_{is_i}) \in \mathbb{R}^{s_i \times s_i}$ is a diagonal positive definite matrix. The uncertain function $\phi_i(u_i) = (\phi_{i1}(u_{i1}), \ldots, \phi_{is_i}(u_{i\,s_i}))$ satisfies, for each i,

$$\phi_{ij}^2(u_{ij}) \leq \gamma_{ij}^2 \, u_{ij}^2 \; , \; j = 1, \ldots, s_i \qquad (7)$$

where $\gamma_{ij} \geq 0$. If (7) holds, then

$$||\phi_i(u_i)||^2 \leq ||\Gamma_i u_i||^2, \qquad i = 1, \ldots, N \qquad (8)$$

where $\Gamma_i = \mathrm{diag}(\gamma_{i1}, \ldots, \gamma_{is_i}) \in \mathbb{R}^{s_i \times s_i}$ is a diagonal positive semidefinite matrix.

The value of λ_{ij} represents the percentage of failure in the jth actuator of the controller of the ith subsystem. Each actuator can fail independently. If $\lambda_{ij} = 1$ and $\gamma_{ij} = 0$, it corresponds to the normal case for the jth actuator of the ith subsystem ($u_{ij}^F = u_{ij}$). When this is true for all j, we have $\Lambda_i = I_{s_i}$ and $\Gamma_i = 0$ and it corresponds to the normal case in the ith canal ($u_i^F = u_i$). When $\lambda_{ij} = \gamma_{ij}$, (6) and (7) cover the outage case ($u_{ij}^F = 0$) because $\phi_{ij} = -\lambda_{ij} \, u_{ij}$ verify (7). The case $\phi_i(u_i) = -\Lambda_i u_i$ corresponds to the outage of the whole controller of the ith system. Other cases correspond to partial failures or partial degradations of the actuators.

Control objective (Reliable guaranteed cost control)
The objective of this paper is to design a set of decentralized feedback control laws $u_i(t) = K_i x_i(t)$ $(i = 1, \ldots, N)$ and obtain a Lyapunov function defined by $\mathbf{X}_i$, for the interconnected systems (1) with uncertainties model (2)-(3) and Assumptions 1 and 2, in such a way that, in the presence of the failures described by (6) and (7), the following property is satisfied:

$$\sum_{i=1}^{N} \Big(\frac{d}{dt} x_i^T \mathbf{X}_i x_i + x_i^T Q_i x_i + (u_i^F)^T R_i u_i^F \Big) < 0 \; . \qquad (9)$$

This inequality leads to a bound for the cost function (5) in the form $J(x, u^F) \leq \bar{J}$, where $\bar{J}$ is some specified constant, and to ensure that the closed-loop system

$$\begin{cases} \dot{x}_i = A_i(\alpha) x_i + B_i(\alpha) u_i^F + \displaystyle\sum_{j=1, j \neq i}^{N} G_{ij} \, g_{ij} \\ u_i^F = \Lambda_i u_i + \phi_i(u_i) \\ u_i = K_i x_i \end{cases} \qquad (10)$$

is asymptotically stable.

Definition 3. The set of above feedback control laws u_i is said to be a *reliable guaranteed cost control.*

3. MAIN RESULTS

3.1 Instrumental tools

Lemma 4. (Projection lemma). For a symmetric matrix Ψ and matrices P, Q with appropriate dimensions, there exists a matrix X such that:

$$\Psi + P^T X^T Q + Q^T X P < 0 \Leftrightarrow \begin{cases} \mathcal{N}_P^T \Psi \mathcal{N}_P < 0 \\ \mathcal{N}_Q^T \Psi \mathcal{N}_Q < 0 \end{cases}$$

with $\mathcal{N}_P$ and $\mathcal{N}_Q$ any matrices whose columns form bases of P and Q respectively.

Lemma 5. (Schur Complement). Consider a symmetric matrix $M = \begin{pmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{pmatrix}$. Then,

$$M < 0 \Leftrightarrow \begin{cases} P_1 - P_2 P_3^{-1} P_2^T < 0 \\ P_3 < 0 \qquad \textit{invertible.} \end{cases}$$

3.2 LMI characterization

Here we give a general theorem that allows a LMI characterization. The subindex i is omitted because the results can be applied to interconnected systems but also to a single system.

Theorem 6. The following statements, involving symmetric positive definite matrix variables $\mathbf{X}$, $\mathbf{Y}$ and general matrix variables V, N, K, are equivalent.

(i) There exist $\mathbf{X}$, K such that

$$\begin{pmatrix} (\mathbf{A} + \mathbf{B}K)^T \mathbf{X} + (*) & * & * \\ G^T \mathbf{X} & Q_{11} & * \\ C & Q_{21} & Q_{22} \end{pmatrix} < 0 \qquad (11)$$

(ii) There exist $\mathbf{Y}$, V and N such that

$$\begin{pmatrix} -(V + V^T) & * & * & * & * \\ \mathbf{A}V + \mathbf{Y} + \mathbf{B}N & -\mathbf{Y} & * & * & * \\ 0 & G^T & Q_{11} & * & * \\ CV & 0 & Q_{21} & Q_{22} & * \\ V & 0 & 0 & 0 & -\mathbf{Y} \end{pmatrix} < 0 \qquad (12)$$

$$\begin{pmatrix} -\mathbf{Y} & * & * \\ G^T & Q_{11} & * \\ 0 & Q_{21} & Q_{22} \end{pmatrix} < 0 \qquad (13)$$

(iii) Consider $\mathbf{A} = A(\alpha)$, $\mathbf{B} = B(\alpha)$, $\mathbf{X} = X(\alpha)$ as in (2). There exist $\mathbf{Y} = Y(\alpha)$, V and N such that

$$\begin{pmatrix} -(V + V^T) & * & * & * & * \\ A_k V + Y_k + B_k N & -Y_k & * & * & * \\ 0 & G^T & Q_{11} & * & * \\ CV & 0 & Q_{21} & Q_{22} & * \\ V & 0 & 0 & 0 & -Y_k \end{pmatrix} < 0 \qquad (14)$$

$$\begin{pmatrix} -Y_k & * & * \\ G^T & Q_{11} & * \\ 0 & Q_{21} & Q_{22} \end{pmatrix} < 0 \, . \qquad (15)$$

The matrix $Q = \begin{pmatrix} Q_{11} & * \\ Q_{21} & Q_{22} \end{pmatrix}$ has to be a negative defined matrix.

PROOF. Taking into account that $\sum_{k=1}^{L} \alpha_k = 1$, the inequality (12) is true when it holds in the vertices of the simplex Π, so (ii) and (iii) are equivalent. To see the equivalence between (i) and (ii), consider a new variable $N_i := K_i V_i$ and apply Projection Lemma 4 to (12). Consider $\bar{\mathbf{A}} = \mathbf{A} + \mathbf{B}K$, $P = (Id, 0, 0, 0, 0)$, $Q = \left(-Id,\ \bar{\mathbf{A}}^T,\ 0,\ C^T,\ Id\right)$ and

$$\psi = \begin{pmatrix} 0 & * & * & * & * \\ \mathbf{Y} & -\mathbf{Y} & * & * & * \\ 0 & G^T & Q_{11} & * & * \\ 0 & 0 & Q_{21} & Q_{22} & * \\ 0 & 0 & 0 & 0 & -\mathbf{Y} \end{pmatrix}.$$

The null spaces bases of P and Q are

$$\mathcal{N}_P = \begin{pmatrix} 0 & 0 & 0 & 0 \\ I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{pmatrix},\ \mathcal{N}_Q = \begin{pmatrix} \bar{\mathbf{A}}^T & 0 & C^T & Id \\ I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{pmatrix}.$$

By $\mathcal{N}_P^T \psi \mathcal{N}_P < 0$, we have

$$\begin{pmatrix} -\mathbf{Y} & * & * & * \\ G^T & Q_{11} & * & * \\ 0 & Q_{21} & Q_{22} & * \\ 0 & 0 & 0 & -\mathbf{Y} \end{pmatrix} < 0. \tag{16}$$

By Schur complement, (16) is equivalent to (13).

From $\mathcal{N}_Q^T \psi \mathcal{N}_Q < 0$, we have

$$\begin{pmatrix} \bar{\mathbf{A}}\mathbf{Y} + \mathbf{Y}\bar{\mathbf{A}}^T - \mathbf{Y} & * & * & * \\ G^T & Q_{11} & * & * \\ C\mathbf{Y} & Q_{21} & Q_{22} & * \\ \mathbf{Y} & 0 & 0 & -\mathbf{Y} \end{pmatrix} < 0.$$

By Schur complement,

$$\begin{pmatrix} \bar{\mathbf{A}}\mathbf{Y} + \mathbf{Y}\bar{\mathbf{A}}^T & * & * \\ G^T & Q_{11} & * \\ C\mathbf{Y} & Q_{21} & Q_{22} \end{pmatrix} < 0. \tag{17}$$

Consider $Q = \begin{pmatrix} Q_{11} & * \\ Q_{21} & Q_{22} \end{pmatrix}$. Applying Schur complement again,

$$\bar{\mathbf{A}}\mathbf{Y} + \mathbf{Y}\bar{\mathbf{A}}^T - (G \quad \mathbf{Y}C^T) Q^{-1} \begin{pmatrix} G^T \\ C\mathbf{Y} \end{pmatrix} < 0.$$

Using now that $\mathbf{Y} = \mathbf{X}^{-1}$, we have

$$\mathbf{X}\bar{\mathbf{A}} + \bar{\mathbf{A}}^T\mathbf{X} - (\mathbf{X}G \quad C^T) Q^{-1} \begin{pmatrix} G^T\mathbf{X} \\ C \end{pmatrix} < 0,$$

which is equivalent to (11). □

3.3 Stability problem

In this subsection, we apply Theorem 6 to stability characterization. We introduce an alternative characterization of the fundamental Lyapunov stability theorem for linear interconnected systems. It introduces a new transformation on the Lyapunov variables which helps to reduce the typical degree of conservatism in some problems. The obtained result will be the base of the development of the reliable guaranteed control in the next section.

Consider the polytopic Lyapunov function:

$$V(x, \alpha) = \sum_{i=1}^{N} x_i^T \mathbf{X}_i x_i, \tag{18}$$

where $\mathbf{X}_i = \sum_{k=1}^{L} \alpha_k X_{i_k}$, $\alpha \in \Pi$ and $X_{i_k} > 0$, for all $k = 1, \ldots, L$ and for all $i = 1, \ldots, N$. This function has to satisfy:

(i) $V(x, \alpha) > 0$,
(ii) $\dot{V}(x, \alpha) < 0$.

Let us evaluate condition (ii). From (18),

$$\dot{V}(x, \alpha) = \sum_{i=1}^{N} \dot{x}_i^T \mathbf{X}_i x_i + x_i^T \mathbf{X}_i \dot{x}_i \ < 0.$$

By using the feedback controls $u_i = K_i x_i$ in (1),

$$\dot{V} = \sum_{i=1}^{N} x_i^T \big((\mathbf{A}_i + \mathbf{B}_i)K_i)^T \mathbf{X}_i + \mathbf{X}_i(\mathbf{A}_i + \mathbf{B}_i K_i)\big) x_i +$$

$$+ \sum_{i=1}^{N} \sum_{j \neq i} g_{ij}^T(t, x_j)\, G_{ij}^T \mathbf{X}_i x_i + x_i^T \mathbf{X}_i G_{ij}\, g_{ij}(t, x_j) < 0.$$

Now, using (4), this inequality leads to

$$\begin{pmatrix} (\mathbf{A}_i + \mathbf{B}_i K_i)^T \mathbf{X}_i + (*) & * & * \\ G_i^T \mathbf{X}_i & Q_{11}^i & * \\ C_i & Q_{21}^i & Q_{22}^i \end{pmatrix} < 0 \tag{19}$$

with $G_i = (G_{i1}, \ldots, G_{iN})$, $C_i = Id$, $Q_{11}^i = diag(-Id, \ \ldots, -Id)$, $Q_{21}^i = 0$, $Q_{22}^i = -W_i^{-1}$. This step involves tedious manipulations. The details are omitted here, but can be found in (Pujol, 2004). So that, if (19) holds for each $i = 1, \ldots, N$, then $\dot{V} < 0$ and the system is stable. In order to obtain an efficient LMI condition, we now use Theorem 6 to characterize the stability.

Theorem 7. Under assumptions 1 and 2, consider the system (1) with uncertain model (2) and (3), and consider the state-feedback controls $u_i = K_i x_i$. Assume that there exist symmetric positive definite matrices $\{Y_{i_k}\}_k$ and matrices V_i and N_i such that the LMI system

$$\begin{pmatrix} -(V_i + V_i^T) & * & * & * & * \\ A_{i_k} V_i + Y_{i_k} + B_{i_k} N_i & -Y_{i_k} & * & * & * \\ 0 & G_i^T & Q_{11}^i & * & * \\ C_i V_i & 0 & Q_{21}^i & Q_{22}^i & * \\ V_i & 0 & 0 & 0 & -Y_{i_k} \end{pmatrix} < 0 \tag{20}$$

$$\begin{pmatrix} -Y_{i_k} & * \\ G_i^T & Q_{11}^i \end{pmatrix} < 0$$

is feasible for all $k = 1, \ldots, L$ and $i = 1, \ldots, N$. Then, the system is stable with Lyapunov function (18) and

$$\begin{cases} K_i = N_i V_i^{-1} \\ X_i(\alpha) = \sum_{k=1}^{L} \alpha_k Y_{i_k}^{-1} \end{cases}$$

PROOF. The proof is an immediate application of Theorem 6, for each k. □

If we use a Lyapunov function parameter independent, we can consider Y_i instead of Y_{i_k} and the same expression is obtained.

3.4 Reliable guaranteed cost control

By Definition 3, the system (10) has reliable guaranteed cost control $u_i = K_i x_i$ if there exist symmetric variable matrices $\mathbf{X}_i$ and gain matrices K_i such that the following inequality

$$\begin{pmatrix} \Xi_i & * & * \\ G_i^T \mathbf{X}_i & -I & * \\ \mathbf{B}_i^T \mathbf{X}_i + R_i \Lambda_i K_i & 0 & R_i - I \end{pmatrix} < 0, \qquad (21)$$

is feasible, where

$$\Xi_i = (\mathbf{A}_i + \mathbf{B}_i \Lambda_i K_i)^T \mathbf{X}_i + \mathbf{X}_i (\mathbf{A}_i + \mathbf{B}_i \Lambda_i K_i) + \\ + W_i + Q_i + K_i^T \Gamma_i^2 K_i + K_i^T \Lambda_i R_i \Lambda_i K_i \; .$$

This characterization involves tedious manipulations where (4) and (8) are used. The details are omitted here, but can be found in (Pujol, 2004). By Schur complement, inequality (21) is equivalent to (22), defined in figure 1.

In order to obtain an LMI characterization, it is necessary to separate the terms in $\mathbf{X}_i$ and K_i in Ξ_i from (21) to be linear. After some manipulations, we obtain that this is possible if $R_i - Id$ is a negative definite matrix and invertible. In this way, we obtain a restriction on the cost function. This means that we may loose some freedom in prescribing the control performance to achieve a reliable control. In accordance with this result, we introduce the following assumption before constructing the reliable guaranteed cost control.

Assumption 3. The cost control matrix R_i must be invertible and verify $R_i - Id < 0$.

Taking this assumption into account, we obtain that (22) is equivalent to:

$$\begin{pmatrix} (\mathbf{A}_i + \hat{\mathbf{B}}_i K_i)^T \mathbf{X}_i + (*) & * & * & * \\ \mathbf{E}_i^T \mathbf{X}_i & Q_{11}^i & * & * \\ C_i & 0 & Q_{22}^i & * \\ F_i K_i & 0 & 0 & Q_{33}^i \end{pmatrix} < 0 \qquad (23)$$

where

$$\hat{\mathbf{B}}_i = \mathbf{B}_i (I + (Id - R_i)^{-1} R_i) \Lambda_i$$

$$F_i = (\Lambda_i, \Gamma_i, R_i \Lambda_i) \; , \; \mathbf{E}_i = (G_i, \mathbf{B}_i) \; , \; C_i = (I, I)^T$$

$$Q_{11}^i = \begin{pmatrix} -I & * \\ 0 & R_i - I \end{pmatrix}, \quad Q_{22}^i = \begin{pmatrix} -Q_i^{-1} & * \\ 0 & -W_i^{-1} \end{pmatrix}$$

$$Q_{33}^i = \begin{pmatrix} -R_i^{-1} & * & * \\ 0 & -I & * \\ 0 & 0 & R_i - I \end{pmatrix}.$$

We present now the main result.

Theorem 8. Under Assumptions 1, 2 and 3, consider the system (10) with the polytopic structure (2) and (3). Suppose that, for each $i = 1, \ldots, N$, there exist symmetric positive definite matrices $\{Y_{i_k}\}_{k=1,\ldots,L}$ and matrices V_i and N_i that make the LMI system

$$\begin{pmatrix} -(V_i + V_i^T) & * & * & * & * & * \\ A_{i_k} V_i + Y_{i_k} + \hat{B}_{i_k} N_i & -Y_{i_k} & * & * & * & * \\ 0 & E_{i_k}^T & Q_{11}^i & * & * & * \\ C_i V_i & 0 & 0 & Q_{22}^i & * & * \\ F_i^T N_i & 0 & 0 & 0 & Q_{33}^i & * \\ V_i & 0 & 0 & 0 & 0 & -Y_{i_k} \end{pmatrix} < 0 \qquad (24)$$

$$\begin{pmatrix} -Y_{i_k} & * \\ E_{i_k}^T & Q_{11}^i \end{pmatrix} < 0$$

feasible for all $k = 1, \ldots, L$ and all $i = 1, \ldots, N$. Then, the set of state feedback controls $u_i = K_i x_i$ is a reliable guaranteed cost control with Lyapunov function $V(x, \alpha) = \sum_{i=1}^{N} x_i^T X_i(\alpha) x_i$, where

$$\begin{cases} K_i = N_i V_i^{-1} \\ X_i(\alpha) = \sum_{k=1}^{L} \alpha_k Y_{i_k}^{-1}. \end{cases} \qquad (25)$$

Moreover, for any $x_i(0) = x_{i0}$, the cost function satisfies

$$J < \sum_{i=1}^{N} x_{i0}^T \; X_i(\alpha) \; x_{i0} \; . \qquad (26)$$

PROOF. Applying the Theorem 6, we obtain (23) from (24). □

The cost bound (26) depends on the initial conditions. In order to eliminate this dependence, the mean value of the cost function is sought over all possible values x_{i0}. This is equivalent to:

$$\mathcal{E}(J) < \sum_{i=1}^{N} tr(X_i(\alpha)) = \sum_{i=1}^{N} \sum_{k=1}^{L} tr(X_{i_k}) \alpha_k,$$

where tr denotes the trace of a matrix. This equation allows to find an optimum value. Considering $\bar{J} := \max_k \left(\sum_{i=1}^{N} tr(X_{i_k}) \right)$, we have the cost function bounded by $\mathcal{E}(J) < \bar{J}$.

4. CONCLUSIONS

In this work, a solution for a reliable decentralized guaranteed cost control problem for interconnected systems with polytopic uncertainties has been presented. Failures are described by a model which considers possible outage or partial failures in every actuator of each decentralized

$$
\begin{pmatrix}
(\mathbf{A}_i + \mathbf{B}_i \Lambda_i K_i)^T \mathbf{X}_i + (*) & * & * & * & * & * & * \\
G_i^T \mathbf{X}_i & -I & * & * & * & * & * \\
\mathbf{B}_i^T \mathbf{X}_i + R_i \Lambda_i K_i & 0 & R_i - I & * & * & * & * \\
\Lambda_i K & 0 & 0 & -R_i^{-1} & * & * & * \\
\Gamma_i K_i & 0 & 0 & 0 & -I & * & * \\
I & 0 & 0 & 0 & 0 & -Q_i^{-1} & * \\
I & 0 & 0 & 0 & 0 & 0 & -W_i^{-1}
\end{pmatrix} < 0 \, . \qquad (22)
$$

Fig. 1.

controller. The control design involves two steps. First, an LMI characterization is presented. Second, a sufficient condition is given for the existence of a decentralized reliable guaranteed cost control set. A key point in the control design has been the formulation of a new LMI characterization, which uses parameter-dependent Lyapunov functions and slack variables. The obtained LMI separates the unknown variables from the system parameter data, which smoothes the numerical solution. This characterization can be useful for different class of problems, such as guaranteed cost control, H_2 or H_∞ control design. In the paper, this type of LMI has been exploited to proof that the proposed decentralized control scheme guarantees the quadratic stability and a cost bound for a class of failure model which considers outage or partial degradation of any independent specific actuator. The design presented in this paper shows that the admission of control failures imposes some restrictions in the definition of the cost function to be bounded. Specifically, some freedom is lost in the selection of the control weighting matrices.

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