ROBUST H_∞ CONTROL OF UNCERTAIN MARKOVIAN JUMP LINEAR SYSTEMS WITH MODE-DEPENDENT TIME-DELAYS

Shanbin Li, Yong-Yan Cao, Yongqiang Wang, and Youxian Sun

Institute of Modern Control Engineering
National Laboratory of Industrial Control Technology
Zhejiang University, Hangzhou 310027, PRC

Abstract: This paper addresses the robust H_∞ issue of uncertain Markovian jump linear systems (MJLSs) with mode-dependent time-delays. A new delay-dependent condition on the stochastic stability is proposed by a new stochastic Lyapunov-Krasovskii functional. The stability conditions are formulated as bilinear matrix inequalities solvable by an iterative linear matrix inequality (LMI) method. Then a new robust H_∞ feedback controller is developed by solving a set of coupled LMIs. A simple numerical example demonstrates the effectiveness of the proposed method.

Keywords: Markovian jump linear systems (MJLSs), robust control, time-delay system, linear matrix inequality (LMI) .

1. INTRODUCTION

Markovian jump linear system (MJLS), introduced by Krasovskii and Lidskii in (Krasovskii and Lidskii, 1961), is a class of stochastic linear systems subject to abrupt variations governed by a Markov process. The family of systems represents a large variety of processes, including fault-tolerant flight control systems, sudden changes in economic systems, see for instance (Mariton, 1990). Great developments involving the optimal regulator, controllability, observability, stability and stabilization problems have been extensively studied, see for example (Ji and Chizeck, 1990; Costa and Fragoso, 1995; Benjelloun and Boukas, 1998; Cao and Lam, 2000; Boukas and Liu, 2001) and the references therein.

On the other hand, time-delays and parameter uncertainties which are inherent features of many physical processes, are main resources of performance deterioration and even instability. For MJLSs with time-delay, the issues of stability and H_∞ control have been well investigated, see e.g. (Cao and Lam, 2000). However, the above-cited results were obtained under the assumption that the time-delay is constant for all modes. In many engineering applications, random delays are unavoidably encountered. It is noted that a random delay can be modelled by a Markov process with a finite number of states. The H_∞ control issue of discrete-time MJLSs with mode-dependent time-delays was proposed in (Boukas and Liu, 2001).

In this paper, we address the robust stochastic stabilizability and robust H_∞ disturbance attenuation for continuous-time MJLSs with mode-dependent delays by a new stochastic Lyapunov-Krosovskii functional approach. A new delay-dependent stochastic stability condition will be derived by introducing a special equality which involves the state, delayed state and delay-distributed derivative variables of the state with free weighting matrices. The organization of this paper is as follows. In Section 2, definitions and preliminary results are described for uncertain jump systems with mode-dependent time-delays. Sufficient conditions on stochastic stability and the H_∞ disturbance attenuation problem are developed in Section 3 and 4 by a new Lyapunov-Krasovskii functional involving the differential variable of the state. The state feedback controller is also developed by the LMI optimization method. An illustrative example is presented in Section 5 to show the effectiveness of the result. The paper is concluded in Section 6.

2. PROBLEM STATEMENT

In what follows, if not explicitly stated, matrices are assumed to have compatible dimensions. The notation $A > (<)0$ is used to denote a positive (negative)-positive matrix. $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and maximum eigenvalue of the corresponding matrix respectively. $\|\cdot\|$ denotes the Euclidean norm for vectors or the spectral norms of matrices. $\mathbf{E}[\cdot]$ stands for the mathematical expectation. $\text{sym}\{A\}$ is used to denote the expression $A + A^T$.

r_t is a finite state Markov jump process representing the system mode, that is, r_t takes discrete values in a given finite set $\mathcal{S} = \{1, 2, \cdots, s\}$. Let $\Pi = [\pi_{ij}]_{i,j\in\mathcal{S}}$ denote the transition rate matrix with

$$\Pr\{r_{t+\Delta} = j | r_t = i\} = \begin{cases} \pi_{ij}\Delta + o(\Delta), & i \neq j, \\ 1 + \pi_{ii}\Delta + o(\Delta), & i = j. \end{cases} \quad (1)$$

where $\Delta > 0$, $\pi_{ij} \geq 0$ for $i \neq j$, with

$$\sum_{j=1, j\neq i}^{s} \pi_{ij} = -\pi_{ii}, \quad (2)$$

for each mode $i \in \mathcal{S}$, and $\frac{o(\Delta)}{\Delta} \to 0$ as $\Delta \to 0$.

Consider the class of jump linear system with mode-dependent time-delays in a fixed complete probability space $(\Omega, \mathcal{F}, P)$:

$$\begin{aligned} \dot{x}_t &= A_1(r_t,t)x_t + A_2(r_t,t)x_{t-\tau(r_t)} \\ &\quad + B_1(r_t)w_t + B_2(r_t,t)u_t, \quad (3) \\ z_t &= C_1(r_t,t)x_t + C_2(r_t,t)x_{t-\tau(r_t)} \\ &\quad + D_1(r_t)w_t + D_2(r_t,t)u_t, \quad (4) \\ x_t &= \phi_t, t \in [-\tau,\ 0], r(0) = r_0. \quad (5) \end{aligned}$$

where $x,\ u,\ z$ is the state, control and controller output of the system with appropriate dimensions, w is the exogenous disturbance which belongs to $\mathcal{L}_2[0,\infty]$, $r_0 \in \mathcal{S}$ are the initial conditions of the mode,$\phi(t)$ is a smooth vector-valued initial function defined in the Banach space $\mathcal{C}[-\tau, 0]$ of smooth functions

$$\phi : [-\tau, 0] \mapsto \mathcal{R}^n, \text{with } \|\phi\|_\infty := \sup_{-\tau\leq\eta\leq 0} \|\phi(\eta)\|.$$

For each $i \in \mathcal{S}$

$$\begin{aligned} A_1(r_t,t) &= A_1(r_t) + \Delta_1(r_t,t), \\ A_2(r_t,t) &= A_2(r_t) + \Delta_2(r_t,t), \\ B_2(r_t,t) &= B_2(r_t) + \Delta_3(r_t,t), \\ C_1(r_t,t) &= C_1(r_t) + \Delta_4(r_t,t), \\ C_2(r_t,t) &= C_2(r_t) + \Delta_5(r_t,t), \\ D_2(r_t,t) &= D_2(r_t) + \Delta_6(r_t,t), \end{aligned}$$

with $A_1(r_t),\ A_2(r_t),\ B_2(r_t),\ C_1(r_t),\ C_2(r_t)$ and $D_2(r_t)$ are matrix functions of the random jumping process $\{r_t\}$ with appropriate dimensions, $\Delta_j(r_t,t)$ $(j = 1, \cdots, 6)$ are unknown matrices denoting the uncertainties in the system. $\tau(r_t)$ is a constant time-delay when the system is in mode r_t and satisfies $0 < \underline{\tau} \leq \tau(r_t) \leq \overline{\tau}$, where $\underline{\tau} = \min\{\tau(r_t), r_t \in \mathcal{S}\}$, $\overline{\tau} = \max\{\tau(r_t), r_t \in \mathcal{S}\}$.

For the notational simplicity, when the system operates in the mode $r_t = i \in \mathcal{S}$, we will denote $\star(r_t)$ as $\star_i$, where $\star$ is any matrix. For instance, $A(r_t)$ is denoted as A_i.

Furthermore, we assume that the admissible uncertainties satisfy the following:

$$\begin{bmatrix} \Delta_{1i} & \Delta_{2i} & \Delta_{3i} \\ \Delta_{4i} & \Delta_{5i} & \Delta_{6i} \end{bmatrix} = \begin{bmatrix} F_{1i} \\ F_{2i} \end{bmatrix} \Delta_i \begin{bmatrix} G_{1i} & G_{2i} & G_{3i} \end{bmatrix},$$

with $\Delta_i^T \Delta_i \leq I, \forall i \in \mathcal{S}$.

The definitions of stochastic stability and H_∞ disturbance attenuation performance of the jump linear system with $u(t) \equiv 0$ are similar to definitions in (Cao and Lam, 2000). The details are omitted here for the limit of pages.

3. ROBUST STATE FEEDBACK STABILIZATION

In this section, we will establish a delay-dependent sufficient stability condition for MJLSs with mode-dependent time-delays with $w_t \equiv 0$ by applying a new Lyapunov-Krasovskii functional.

Theorem 1. The autonomous jump time-delay system is robust stochastically stable for $\underline{\tau} \leq \tau_i \leq \overline{\tau}$, for each mode $i \in \mathcal{S}$, if there exists a scalar $\varepsilon_i > 0$ and matrices $P_i > 0$, $Q_1 > 0$, $Q_2 > 0$, $T_i = \begin{bmatrix} T_{1i} & T_{2i} & T_{3i} \end{bmatrix}$, H_i, N_i satisfying the following matrix inequalities

$$\begin{bmatrix} \Theta_i + \varepsilon_i T_i^T F_{1i} F_{1i}^T T_i & G_i^T \\ * & -\varepsilon_i I \end{bmatrix} < 0, \quad (6)$$

$$\begin{bmatrix} H_i & N_i \\ * & Q_2 \end{bmatrix} \geq 0, \quad (7)$$

where the $*$ represent block that is readily inferred by symmetry and

$$\Theta_i = M_i + \text{sym}\{T_i^T A_i\} + \text{sym}\{N_i \Gamma_1\} + \tau_i H_i,$$

$$M_i = \begin{bmatrix} \sum_{j=1}^{s} \pi_{ij} P_j + \mu Q_1 & 0 & P_i \\ * & -Q_1 & 0 \\ * & * & \varrho_i Q_2 \end{bmatrix},$$

$$A_i = \begin{bmatrix} A_{1i} & A_{2i} & -I \end{bmatrix}, \quad \Gamma_1 = \begin{bmatrix} I & -I & 0 \end{bmatrix},$$

$$\mu = 1 + (\overline{\tau} - \underline{\tau})\pi_m, \quad \varrho_i = \tau_i + \frac{\pi_m}{2}(\overline{\tau}^2 - \underline{\tau}^2),$$

$$\pi_m = \max\{|\pi_{ii}|, \ i \in \mathcal{S}\}, \quad G_i = \begin{bmatrix} G_{1i} & G_{2i} & 0 \end{bmatrix}.$$

Proof. Let the mode at time t be i, that is $r_t = i \in \mathcal{S}$. Then the autonomous jump time-delay system is

$$\begin{cases} \dot{x}_t = A_1(i,t)x_t + A_2(i,t)x_{t-\tau_i}, \\ x_t = \phi_t, t \in \left[-\tau, \ 0 \right], r(0) = r_0. \end{cases} \tag{8}$$

Since x_t is continuously differentiable for $t \geq 0$, Leibnitz-Newton formula gives $x_{t-\tau_i} = x_t - \int_{t-\tau_i}^{t} \dot{x}_\alpha \mathrm{d}\alpha$. From (8), we have

$$0 = [A_1(i,t) + A_2(i,t)]x_t - \dot{x}_t - A_2(i,t) \int_{t-\tau_i}^{t} \dot{x}_\alpha \mathrm{d}\alpha.$$

In the sequel, we will use the following notations

$$A_{si} = A_1(i,t) + A_2(i,t), \ y_{\tau_i} = \int_{t-\tau_i}^{t} \dot{x}_\alpha \mathrm{d}\alpha,$$

$$\xi_t = \begin{bmatrix} x_t^T & x_{t-\tau_i}^T & \dot{x}_t^T \end{bmatrix}^T.$$

So, we have

$$2\xi_t^T T_i^T (A_{si} x_t - \dot{x}_t - A_2(i,t) y_{\tau_i}) = 0. \tag{9}$$

Choose the Lyapunov-Krasovskii functional candidate as

$$V(x_t, i) \triangleq \sum_{j=1}^{4} V_j(x_t, i), \tag{10}$$

where

$$V_1(x_t, i) \triangleq x_t^T P_i x_t,$$

$$V_2(x_t, i) \triangleq \int_{t-\tau_i}^{t} x_\alpha^T Q_1 x_\alpha \mathrm{d}\alpha,$$

$$V_3(x_t, i) \triangleq \int_{-\tau_i}^{0} \int_{t+\beta}^{t} \dot{x}_\alpha^T Q_2 \dot{x}_\alpha \mathrm{d}\alpha \mathrm{d}\beta,$$

$$V_4(x_t, i) \triangleq \pi_m \int_{-\overline{\tau}}^{-\underline{\tau}} \int_{t+\beta}^{t} \{ x_\alpha^T Q_1 x_\alpha + \dot{x}_\alpha^T Q_2 \dot{x}_\alpha (\alpha - t - \beta) \} \mathrm{d}\alpha \mathrm{d}\beta.$$

The weak infinitesimal operator $\mathcal{A}$ of the stochastic process $\{r_t, x_t\}$, $t \geq 0$, is given by

$$\mathcal{A}V(x_t, i) = \lim_{\Delta \to 0} \frac{1}{\Delta} [\mathbf{E}\{V(x_{t+\Delta}, r_{t+\Delta}) | x_t, r_t\} - V(x_t, r_t)]. \tag{11}$$

By (9), we have

$$\begin{aligned} \mathcal{A}V_1(x_t, i) &= 2x_t^T P_i \dot{x}_t + x_t^T (\sum_{j=1}^{s} \pi_{ij} P_j) x_t \\ &= 2x_t^T P_i \dot{x}_t + x_t^T (\sum_{j=1}^{s} \pi_{ij} P_j) x_t \\ &\quad + 2\xi_t^T T_i^T (A_{si} x_t - \dot{x}_t - A_2(i,t) y_{\tau_i}) \\ &= \xi_t^T \left\{ \begin{bmatrix} \sum_{j=1}^{s} \pi_{ij} P_j & 0 & P_i \\ * & 0 & 0 \\ * & * & 0 \end{bmatrix} + \mathrm{sym}\{T_i^T \bar{A}_i\} \right\} \\ &\quad \times \xi_t - 2\xi_t^T T_i^T A_2(i,t) y_{\tau_i}, \end{aligned} \tag{12}$$

where $\bar{A}_i = \begin{bmatrix} A_{si} & 0 & -I \end{bmatrix}$.

By the inequality in (Moon *et al.*, 2001), we have

$$\begin{aligned} &-2\xi_t^T T_i^T A_2(i,t) y_{\tau_i} \\ &\quad \leq \tau_i \xi_t^T H_i \xi_t + 2\xi_t^T (N_i - T_i^T A_2(i,t))(x_t - x_{t-\tau_i}) \\ &\quad + \int_{-\tau_i}^{0} \dot{x}_{t+\alpha}^T Q_2 \dot{x}_{t+\alpha} \mathrm{d}\alpha, \end{aligned} \tag{13}$$

for any matrices H_i, N_i and Q_2 satisfying (7).

Also, we have

$$\begin{aligned} \mathcal{A}V_2(x_t, i) &= x_t^T Q_1 x_t - x_{t-\tau_i}^T Q_1 x_{t-\tau_i} \\ &\quad + \sum_{j=1}^{s} \pi_{ij} \int_{t-\tau_j}^{t} x_\alpha^T Q_1 x_\alpha \mathrm{d}\alpha. \end{aligned} \tag{14}$$

Notice that (2), we have

$$\begin{aligned} &\sum_{j=1}^{s} \pi_{ij} \int_{t-\tau_j}^{t} x_\alpha^T Q_1 x_\alpha \mathrm{d}\alpha \\ &= -\pi_{ii} \left\{ \int_{t-\tau_j}^{t} x_\alpha^T Q_1 x_\alpha \mathrm{d}\alpha - \int_{t-\tau_i}^{t} x_\alpha^T Q_1 x_\alpha \mathrm{d}\alpha \right\}. \end{aligned}$$

Furthermore, we have

$$\begin{aligned} \int_{t-\tau_j}^{t} x_\alpha^T Q_1 x_\alpha \mathrm{d}\alpha &= \int_{t-\tau_j}^{t-\underline{\tau}} x_\alpha^T Q_1 x_\alpha \mathrm{d}\alpha \\ &\quad + \int_{t-\underline{\tau}}^{t} x_\alpha^T Q_1 x_\alpha \mathrm{d}\alpha, \\ \int_{t-\tau_j}^{t-\underline{\tau}} x_\alpha^T Q_1 x_\alpha \mathrm{d}\alpha &\leq \int_{t-\overline{\tau}}^{t-\underline{\tau}} x_\alpha^T Q_1 x_\alpha \mathrm{d}\alpha, \\ \int_{t-\underline{\tau}}^{t} x_\alpha^T Q_1 x_\alpha \mathrm{d}\alpha &\leq \int_{t-\tau_i}^{t} x_\alpha^T Q_1 x_\alpha \mathrm{d}\alpha. \end{aligned}$$

Hence,

$$\begin{aligned} \mathcal{A}V_2(x_t, i) &\leq x_t^T Q_1 x_t - x_{t-\tau_i}^T Q_1 x_{t-\tau_i} \\ &\quad + \pi_m \int_{t-\overline{\tau}}^{t-\underline{\tau}} x_\alpha^T Q_1 x_\alpha \mathrm{d}\alpha. \end{aligned} \tag{15}$$

Similarly

$$\begin{aligned} \mathcal{A}V_3(x_t, i) &\leq \tau_i \dot{x}_t^T Q_2 \dot{x}_t - \int_{t-\tau_i}^{t} \dot{x}_\alpha^T Q_2 \dot{x}_\alpha \mathrm{d}\alpha \\ &\quad + \pi_m \int_{-\overline{\tau}}^{-\underline{\tau}} \mathrm{d}\beta \int_{t+\beta}^{t} \dot{x}_\alpha^T Q_2 \dot{x}_\alpha \mathrm{d}\alpha. \end{aligned} \tag{16}$$

Also,

$$\begin{aligned} \mathcal{A}V_4(x_t, i) &= \frac{\pi_m}{2}(\overline{\tau}^2 - \underline{\tau}^2) \dot{x}_t^T Q_2 \dot{x}_t \\ &\quad - \pi_m \int_{-\overline{\tau}}^{-\underline{\tau}} \mathrm{d}\beta \int_{t+\beta}^{t} \dot{x}_\alpha^T Q_2 \dot{x}_\alpha \mathrm{d}\alpha \\ &\quad + \pi_m (\overline{\tau} - \underline{\tau}) x_t^T Q_1 x_t \end{aligned}$$

$$- \pi_m \int_{t-\overline{\tau}}^{t-\underline{\tau}} x_\alpha^T Q_1 x_\alpha \mathrm{d}\alpha. \quad (17)$$

Combining (12)-(17), we have

$$\mathcal{A}V(x_t, i) \leq \xi_t^T(\Theta_i + \mathrm{sym}\{T_i^T F_{1i} \Delta_i G_i\})\xi_t.$$

By (Wang *et al.*, 1992) and using Schur complement, it is easy to see that $\mathcal{A}V(x_t, i) < 0$ if LMI (6) holds.

Hence we have $\mathcal{A}V(x_t, i) \leq -\beta_1 \xi_t^T \xi_t$, where $\beta_1 = \min_{i\in\mathcal{S}}(\lambda_{\min}(-(\Theta_i + \varepsilon_i T_i^T F_{1i} F_{1i}^T T_i + \varepsilon_i^{-1} G_i^T G_i))) > 0$. By Dynkin's formula, we have

$$\begin{aligned} &\mathbf{E}\{V(x_t, i)\} - V(\phi, r_0) \\ &= \mathbf{E}\left\{\int_0^t \mathcal{A}V(x_\alpha, r_\alpha)\mathrm{d}\alpha\right\} \\ &\leq -\beta_1 \int_0^t \mathbf{E}\left\{\xi_\alpha^T \xi_\alpha \mathrm{d}\alpha\right\}. \end{aligned} \quad (18)$$

On the other hand, we can show that

$$\mathbf{E}\{V(x_t, i)\} \geq \beta_2 \mathbf{E}\left\{\xi_t^T \xi_t\right\}, \quad (19)$$

where $\beta_2 = \min_{i\in\mathcal{S}}(\lambda_{\min}(P_i)) > 0$.

From (18) and (19), we have

$$\mathbf{E}\{\xi_t^T \xi_t\} \leq \lambda_1 \exp(-\lambda_2 t)V(\phi, r_0),$$

where $\lambda_1 = \beta_1\beta_2^{-1}, \lambda_2 = \beta_2^{-1}$. Therefore,

$$\begin{aligned} &\mathbf{E}\left\{\int_0^t \xi_\alpha^T \xi_\alpha \mathrm{d}\alpha | \phi, r_0\right\} \\ &\leq \lambda_1^{-1}\lambda_2[1 - \exp(-\lambda_1 t)]V(\phi, r_0). \end{aligned}$$

Taking limit as $t \to \infty$, we have

$$\begin{aligned} &\lim_{t\to\infty} \mathbf{E}\left\{\int_0^t x_\alpha^T x_\alpha \mathrm{d}\alpha | \phi, r_0\right\} \\ &\leq \lim_{t\to\infty} \mathbf{E}\left\{\int_0^t \xi_\alpha^T \xi_\alpha \mathrm{d}\alpha | \phi, r_0\right\} \\ &\leq \lambda_1^{-1}\lambda_2 V(\phi, r_0). \end{aligned}$$

Noting that there always exists a scalar $c > 0$, such that $\lambda_1^{-1}\lambda_2 V(\phi, r_0) \leq c \sup_{-\tau\leq s\leq 0} \|\phi(s)\|^2$, it then follows that the autonomous jump time-delay system is stochastically stable. Hence the theorem holds. □

Remark 1. It's noted that the above results are obtained under the assumption that for each mode $i \in \mathcal{S}$, τ_i is known in advance.

Remark 2. It's also noted that the condition (6) is not an LMI but a bilinear matrix inequality because the term $\varepsilon_i T_i F_{1i} F_{1i}^T T_i$. By Schur complement, the condition (6) will be converted to a nonlinear matrix inequality with term $\varepsilon_i^{-1} I$. By an LMI-based iterative algorithm developed by (Moon *et al.*, 2001), a feasible solution of (6) and (7) will be obtained, the details are omitted here.

In this paper, we consider the following state feedback controller design, for each $i \in \mathcal{S}$

$$u_i = K_{1i} x_t + K_{2i} x_{t-\tau_i}. \quad (20)$$

The above control law is the general formula of the state feedback. When $K_{2i} = 0$, it is just the instantaneous state feedback, while it is the mode-dependent delayed state feedback when $K_{1i} = 0$. It should be noticed that the time-delay in the control law (20) is assumed to be the same as for the system (3)-(5).

By Theorem 1, we know that the closed-loop system is robust stochastically stable for $\underline{\tau} \leq \tau_i \leq \overline{\tau}$, for each mode $i \in \mathcal{S}$, if there exists a scalar $\varepsilon_i > 0$ and matrices $P_i > 0$, $Q_1 > 0$, $Q_2 > 0$, T_i, H_i, N_i satisfying (7) and

$$\begin{bmatrix} \bar{\Theta}_i + \varepsilon_i T_i^T F_{1i} F_{1i}^T T_i & \bar{G}_i^T \\ * & -\varepsilon_i I \end{bmatrix} < 0, \quad (21)$$

where

$$\begin{aligned} \bar{\Theta}_i &= M_i + \mathrm{sym}\{T_i^T \bar{A}_i\} + \mathrm{sym}\{N_i \Gamma_1\} + \tau_i H_i, \\ \bar{A}_i &= \left[\, A_{1i} + B_{2i}K_{1i} \;\; A_{2i} + B_{2i}K_{2i} \;\; -I \,\right], \\ \bar{G}_i &= \left[\, G_{1i} + G_{3i}K_{1i} \;\; G_{2i} + G_{3i}K_{2i} \;\; 0 \,\right]. \end{aligned}$$

In what follows, we denote $S_{1i} = T_{1i}^{-1}$, $S_{2i} = T_{2i}^{-1}$, $S_{3i} = T_{3i}^{-1}$. $S_{1i} = X_i$, $S_{2i} = \delta_1 X_i$, $S_{3i} = \delta_2 X_i$, where δ_1, $\delta_2 > 0$ are known tuning parameters. Furthermore, $S_i = diag\{X_i, \delta_1 X_i, \delta_2 X_i\}$, $\hat{P}_i = X_i^T P_i X_i$, $\hat{H}_i = S_i^T H_i S_i$, $\hat{N}_i = S_i^T N_i X_i$, $Y_{1i} = K_{1i} X_i$, $Y_{2i} = K_{2i} X_i$, $\Gamma_2 = \left[I \; I \; I\right]$. Pre- and post-multiplying $diag\{S_i^T,\ I\}$, $diag\{S_i,\ I\}$ to (21) respectively. Pre- and post-multiplying $diag\{S_i^T,\ \delta_1 X_i^T\}$ and $diag\{S_i,\ \delta_1 X_i\}$ to (7) respectively. If we constrain X_i to be same for all i and let $\hat{Q}_1 = X_i^T Q_1 X_i$ and $\hat{Q}_2 = X_i^T Q_2 X_i$, the following theorem is obvious.

Theorem 2. Given tuning parameters δ_1, $\delta_2 > 0$, there exists a state feedback controller (20) such that the uncertain jump time-delay system is robust stochastically stabilized for all $\underline{\tau} \leq \tau_i \leq \overline{\tau}$, for each mode $i \in \mathcal{S}$, if there exists a scalar $\varepsilon_i > 0$ and matrices $\hat{P}_i > 0$, $\hat{Q}_1 > 0$, $\hat{Q}_2 > 0$, $X > 0$, $\hat{H}_i$, $\hat{N}_i$ and Y_{1i}, Y_{2i} satisfying the following LMIs

$$\begin{bmatrix} \hat{\Theta}_i + \varepsilon_i \Gamma_2^T F_{1i} F_{1i}^T \Gamma_2 & \hat{G}_i^T \\ * & -\varepsilon_i I \end{bmatrix} < 0, \quad (22)$$

$$\begin{bmatrix} \hat{H}_i & \delta_1 \hat{N}_i \\ * & \delta_1^2 \hat{Q}_2 \end{bmatrix} \geq 0, \quad (23)$$

where

$$\hat{\Theta}_i = \hat{M}_i + \mathrm{sym}\{\Gamma_2^T \hat{A}_i\} + \mathrm{sym}\{\hat{N}_i \hat{\Gamma}_1\} + \tau_i \hat{H}_i,$$

$$\hat{M}_i = \begin{bmatrix} \sum_{j=1}^{s} \pi_{ij} \hat{P}_j + \mu \hat{Q}_1 & 0 & \delta_2 \hat{P}_i \\ * & -\delta_1^2 \hat{Q}_1 & 0 \\ * & * & \delta_2^2 \varrho_i \hat{Q}_2 \end{bmatrix},$$

$$\begin{aligned} \hat{A}_i &= \left[\, A_{1i}X + B_{2i}Y_{1i} \;\; \delta_1 A_{2i}X + \delta_1 B_{2i}Y_{2i} \;\; -\delta_2 X \,\right], \\ \hat{G}_i &= \left[\, G_{1i}X + G_{3i}Y_{1i} \;\; \delta_1 G_{2i}X + \delta_1 G_{3i}Y_{2i} \;\; 0 \,\right], \\ \hat{\Gamma}_1 &= \left[I \;\; -\delta_1 I \;\; 0\right], \quad \Gamma_2 = \left[I \;\; I \;\; I\right]. \end{aligned}$$

Then the control law can be constructed as $K_{1i} = Y_{1i}X^{-1}$ and $K_{2i} = Y_{2i}X^{-1}$.

However, the above theorem may be too conservative. In this sequel, we propose a less conservative theorem by not constraining X_i to be same for all $i \in \mathcal{S}$. Let $\hat{Q}_1 = Q_1^{-1}$ and $\hat{Q}_2 = Q_2^{-1}$. Also, we note that

$$\begin{aligned} \delta_1^2 X_i^T \hat{Q}_j^{-1} X_i &\geq \mathrm{sym}\{\delta_1 X_i\} - \hat{Q}_j, \ j = 1, 2 \\ X_i^T \hat{P}_i^{-1} X_i &\geq \mathrm{sym}\{X_i\} - \hat{P}_i. \end{aligned}$$

Using the Schur complement, we have the following theorem.

Theorem 3. Given tuning parameter $\delta_1, \ \delta_2 > 0$, there exists a state feedback controller (20) such that the uncertain jump time-delay system is robust stochastically stabilized for all $\underline{\tau} \leq \tau_i \leq \overline{\tau}$, for each mode $i \in \mathcal{S}$, if there exists a scalar $\varepsilon_i > 0$ and matrices $\hat{P}_i > 0, \ \hat{Q}_1 > 0, \ \hat{Q}_2 > 0, \ X_i > 0, \ \hat{H}_i, \ \hat{N}_i$ and $Y_{1i}, \ Y_{2i}$ satisfying the following LMIs

$$\begin{bmatrix} \hat{\Theta}_i + \varepsilon_i \Gamma_2^T F_{1i} F_{1i}^T \Gamma_2 & \hat{G}_i^T & \mu \Pi_1 X_i^T & \varrho_i \Pi_2 X_i^T & \Pi_1 \Xi_i \\ * & -\varepsilon_i I & 0 & 0 & 0 \\ * & * & -\mu \hat{Q}_1 & 0 & 0 \\ * & * & * & -\varrho_i \hat{Q}_2 & 0 \\ * & * & * & * & -\Upsilon_i \end{bmatrix} < 0, \quad (24)$$

$$\begin{bmatrix} \hat{H}_i & \delta_1 \hat{N}_i \\ * & \mathrm{sym}\{\delta_1 X_i\} - \hat{Q}_2 \end{bmatrix} \geq 0, \quad (25)$$

where

$$\begin{aligned} \hat{\Theta}_i =& \hat{M}_i + \mathrm{sym}\{\Gamma_2^T \hat{A}_i\} + \mathrm{sym}\{\hat{N}_i \hat{\Gamma}_1\} + \tau_i \hat{H}_i, \\ \hat{M}_i =& \begin{bmatrix} \pi_{ii} \hat{P}_i & 0 & \delta_2 \hat{P}_i \\ * & -\mathrm{sym}\{\delta_1 X_i\} + \hat{Q}_1 & 0 \\ * & * & 0 \end{bmatrix}, \\ \hat{A}_i =& \left[A_{1i} X_i + B_{2i} Y_{1i} \ \ \delta_1 A_{2i} X_i + \delta_1 B_{2i} Y_{2i} \ \ -\delta_2 X_i \right], \\ \hat{G}_i =& \left[G_{1i} X_i + G_{3i} Y_{1i} \ \ \delta_1 G_{2i} X_i + \delta_1 G_{3i} Y_{2i} \ \ 0 \right], \\ \Pi_1 =& \left[I \ 0 \ 0 \right]^T, \ \ \Pi_2 = \left[0 \ 0 \ \delta_2 I \right]^T, \\ \hat{\Gamma}_1 =& \left[I \ -\delta_1 I \ 0 \right], \ \ \Gamma_2 = \left[I \ I \ I \right], \\ \Xi_i =& \left[\sqrt{\pi_{i,1}} X_i \ \cdots \ \sqrt{\pi_{i,i-1}} X_i \ \sqrt{\pi_{i,i+1}} X_i \right. \\ & \left. \cdots \ \sqrt{\pi_{i,s}} X_i \right], \\ \Upsilon_i =& diag \left\{ \mathrm{sym}\{X_1\} - \hat{P}_1 \cdots \mathrm{sym}\{X_{i-1}\} - \hat{P}_{i-1} \right. \\ & \left. \mathrm{sym}\{X_{i+1}\} - \hat{P}_{i+1} \ \cdots \ \mathrm{sym}\{X_s\} - \hat{P}_s \right\}. \end{aligned}$$

Then the control law can be constructed as $K_{1i} = Y_{1i}X_i^{-1}$ and $K_{2i} = Y_{2i}X_i^{-1}$.

4. H_∞ DISTURBANCE ATTENUATION ANALYSIS

In this section, we will analyze the H_∞ disturbance attenuation performance of the uncertain jump time-delay systems.

Theorem 4. For the autonomous uncertain jump time-delay systems and a given disturbance attenuation level γ, it is said to be robust stochastically stable with γ-disturbance attenuation property for all $\underline{\tau} \leq \tau_i \leq \overline{\tau}, \ w \in \mathcal{L}_2[0, \infty), \ w \neq 0$, for each mode $i \in \mathcal{S}$, if there exists a scalar $\varepsilon_i > 0$ and matrices $P_i > 0, \ Q_1 > 0, \ Q_2 > 0, \ T_i = \left[T_{1i} \ T_{2i} \ T_{3i} \right], \ H_i, \ N_i$ satisfying the LMIs (7) and

$$\begin{bmatrix} \Theta_i + \varepsilon_i T_i^T F_{1i} F_{1i}^T T_i & T_i^T B_{1i} & C_i^T + \varepsilon_i T_i^T F_{1i} F_{2i}^T & G_i^T \\ * & -\gamma I & D_{1i}^T & 0 \\ * & * & -\gamma I + \varepsilon_i F_{2i} F_{2i}^T & 0 \\ * & * & * & -\varepsilon_i I \end{bmatrix} < 0, \quad (26)$$

where $C_i = \left[C_{1i} \ C_{2i} \ 0 \right]$.

Proof. Choose the stochastic Lyapunov-Krasovskii functional candidate $V(x_t, i)$ as (10). Replacing (9) by equation: $2\xi_t^T T_i^T (A_{si} x_t - \dot{x}_t - A_2(i,t) y_{\tau_i} + B_{1i} w_t) = 0$. Similar to the proof of Theorem 1 we can obtain the theorem, the details are omitted here for the limit of pages. □

Pre- and post-multiplying $diag\{S_i^T, \ I, \ I, \ I\}$ and $diag\{S_i, \ I, \ I, \ I\}$ to (26) respectively. Pre- and post-multiplying $diag\{S_i^T, \ \delta_1 X_i^T\}$ and $diag\{S_i, \ \delta_1 X_i\}$ to (7) respectively. Similar to the proof of Theorem 3, we have the following theorem.

Theorem 5. Given tuning parameters $\delta_1, \ \delta_2 > 0$, there exists a state feedback controller (20) such that the uncertain jump time-delay systems is robust stochastically stable with γ-disturbance attenuation property for all $\underline{\tau} \leq \tau_i \leq \overline{\tau}, \ w \in \mathcal{L}_2[0, \infty), \ w \neq 0$, for each mode $i \in \mathcal{S}$, if there exist a scalar $\varepsilon_i > 0$ and matrices $\hat{P}_i > 0, \ \hat{Q}_1 > 0, \ \hat{Q}_2 > 0, \ X_i > 0, \ \hat{H}_i, \ \hat{N}_i$ and $Y_{1i}, \ Y_{2i}$ satisfying the LMIs (25) and

$$\begin{bmatrix} \hat{\Theta}_i + \varepsilon_i \Gamma_2^T F_{1i} F_{1i}^T \Gamma_2 & \Gamma_2^T B_{1i} & \hat{C}_i^T + \varepsilon_i \Gamma_2^T F_{1i} F_{2i}^T \\ * & -\gamma I & D_{1i}^T \\ * & * & -\gamma I + \varepsilon_i F_{2i} F_{2i}^T \\ * & * & * \\ * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$$

$$\begin{bmatrix} \hat{G}_i^T & \mu\Pi_1 X_i^T & \varrho_i \Pi_2 X_i^T & \Pi_1 \Xi_i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\varepsilon_i I & 0 & 0 & 0 \\ * & -\mu \hat{Q}_1 & 0 & 0 \\ * & * & -\varrho_i \hat{Q}_2 & 0 \\ * & * & * & -\Upsilon_i \end{bmatrix} < 0, \tag{27}$$

where $\hat{C}_i = \left[\, C_{1i} X_i + D_{2i} Y_{1i} \;\; \delta_1 C_{2i} X_i + \delta_1 D_{2i} Y_{2i} \;\; 0 \,\right]$. Then the control law providing γ-disturbance attenuation can be constructed as $K_{1i} = Y_{1i} X_i^{-1}$ and $K_{2i} = Y_{2i} X_i^{-1}$.

5. ILLUSTRATIVE EXAMPLE

In this section, we present a simple example to illustrate the usefulness of the proposed method. We borrow the example with two modes from (Benjelloun and Boukas, 1998). The nominal dynamics in each mode is described as follows

$$A_{11} = \begin{bmatrix} 0.5 & -1 \\ 0 & -3 \end{bmatrix}, \quad A_{21} = \begin{bmatrix} 0.5 & -0.2 \\ 0.2 & 0.3 \end{bmatrix},$$
$$A_{12} = \begin{bmatrix} -5 & 1 \\ 1 & 0.2 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} -0.3 & 0.5 \\ 0.4 & -0.5 \end{bmatrix},$$
$$B_{21} = B_{22} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

The initial condition is assumed to be $x_t = \left[\, -1 \;\; 1 \,\right]^T$ and $r_t = 1$ for $t \in [-\tau, \;\; 0]$. $\underline{\tau}$ is assumed to be 0. The generator matrix of the stochastic process r_t is $\Pi = \begin{bmatrix} -\pi_1 & \pi_1 \\ \pi_2 & -\pi_2 \end{bmatrix}$, where $\pi_1 = 7$, $\pi_2 = 3$. $\tau_1 = 1$, $\tau_2 = 0.2$. When $\delta_1 = 10$, $\delta_2 = 1$, by theorem 2, we can obtain a feasible solution when $\overline{\tau} \leq 1.648$, and hence the stability of the system is guaranteed when $\overline{\tau} \leq 1.648$. We want to design the control law (20) such that the closed-loop system is stochastically stable for the time-delay as large as possible. Let $\overline{\tau} = 1.648$, we construct the feedback matrices as

$$K_{11} = \left[\, -71.5046 \;\; -37.4365 \,\right],$$
$$K_{12} = \left[\, -64.2378 \;\; -39.2132 \,\right],$$
$$K_{21} = \left[-0.4107 \;\; 0.1669\right], K_{22} = \left[0.4074 \;\; -0.0546\right].$$

The simulation also shows that the system is stable when $\overline{\tau} = 1.648$. Figure 1 gives the state and mode trajectories.

6. CONCLUSIONS

In this paper, a new Lyapunov-Krasovskii functional for the stochastic stability analysis and H_∞ control design issues of the uncertain jump systems with mode-dependent time-delay were proposed. Sufficient conditions on robust stochastic stability and robust γ-disturbance attenuation were also presented on coupled LMI's. A numerical example was presented to illustrate the usefulness of the proposed method.

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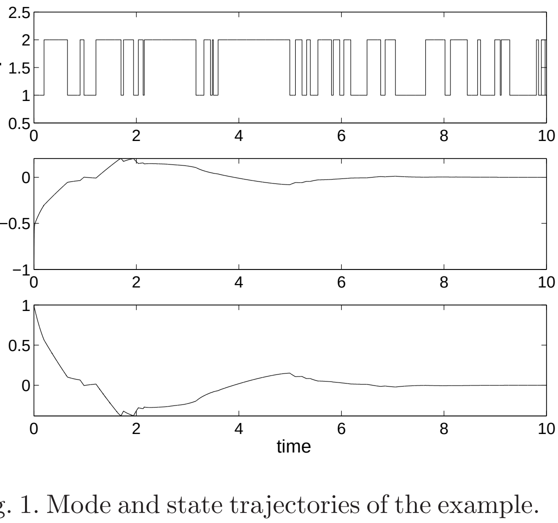

Fig. 1. Mode and state trajectories of the example.