SYSTEM IDENTIFICATION OF OPEN WATER CHANNELS WITH UNDERSHOT AND OVERSHOT GATES

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Abstract: In this paper we consider system identification of irrigation channels. The results in this paper extend previous work in several important aspects. Both undershot and overshot gates are treated, and the overshot gates operate under both free and submerged flow. The models are estimated and validated on operational data from the Coleambally Main Channel in Australia, and the obtained system identification models are very accurate, being able to predict the real water level more than 12 hours ahead of time. Moreover, the system identification models are computationally inexpensive and ideally suited for control design. The results show that system identification and control have important parts to play in management of water resources.

Keywords: System identification, open water channels, environmental systems, control.

1. INTRODUCTION

Water is a scarce resource in many parts of the world today, and management of the water resources has become an important issue. The water losses in irrigation channels are large, but it is recognised that they can be substantially reduced by employing improved control systems. The flows and water levels in an irrigation channel are regulated by gates located along the channel, and control of open water channels is an active research area, see e.g. de Halleux et. al. (2003), Li et. al. (2004), Litrico et. al. (2003), Malaterre (1998), Schurmans et al (1999), Weyer (2002,2003).

In order to design a good control system, a model which gives an accurate description of the relevant dynamics is needed. Traditionally the St. Venant equations have been used for modelling of irrigation channels, e.g. Chaudry (1993). These equations are hyperbolic partial differential equations which are computationally expensive for simulation purposes, and they are not easy to use for control design, although it is quite possible to use them as a starting point for control design as illustrated in the previously cited papers. Recently (Weyer (2001), Ooi and Weyer (2001)) system identification models have become popular. These models are simple, and they can be used in simulations at low computational cost. Since system identification models are build directly from observed data, they accurately reflect the real behaviour of the irrigation channels. Moreover, they are easy to use for control design, and many currently operating control systems for irrigation channels have been designed using system identification models (Weyer (2002,2003)).

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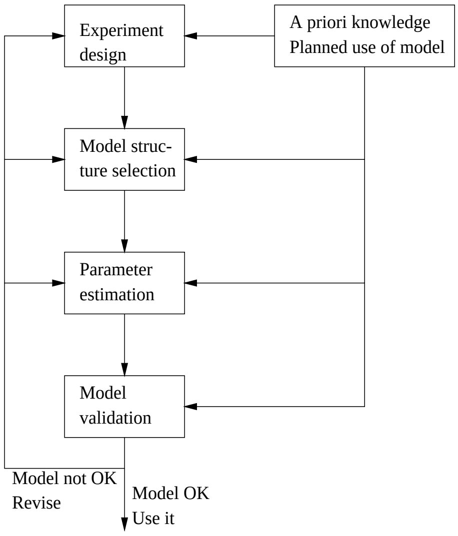

Fig. 1. The system identification procedure

System identification of irrigation channels equipped with overshot gates in free flow (see Section 2) was treated in Weyer (2001). A limitation of system identification models is that they are only confirmed valid for the type of gates and operational conditions for which there are available data. The system identification models developed in this paper are similar to those in Weyer (2001), but they have been modified to take the gate types and the flow conditions into account. The experimental results presented show that system identification models captures the relevant dynamics of a broad range of irrigation channels and operational conditions. The major extensions from Weyer (2001) are as follows

- The irrigation channel in this paper is equipped with both overshot and undershot gates.
- During the experiment the overshot gates were in both free and submerged flow.
- There are four or six gates across the channel at each regulator point. The gates have different gate positions which means that the flow over or under each gate is different.
- The current channel is much larger than the one in Weyer (2001).

The outline of this paper follows the system identification procedure (e.g. Ljung (1999) or Söderström and Stoica (1988)) in Figure 1. In the next section we give a description of the Coleambally Main Channel where we carried out the experiments, and we present the prior information used to derive the model structure. In the following sections we consider experiment design, model structure selection, parameter estimation and model validation before we finish with a discussion and conclusion. This work is part of an ongoing research project between the University of Melbourne and Rubicon Systems Australia on modelling and control of irrigation channels.

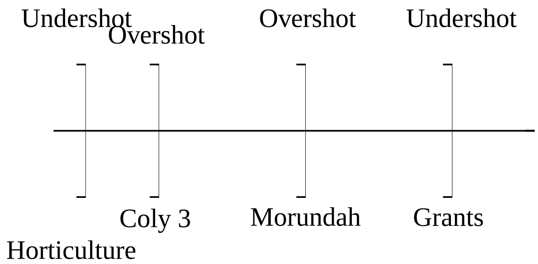

Fig. 2. Top view of a part of the CMC. The flow of water is from the left to the right.

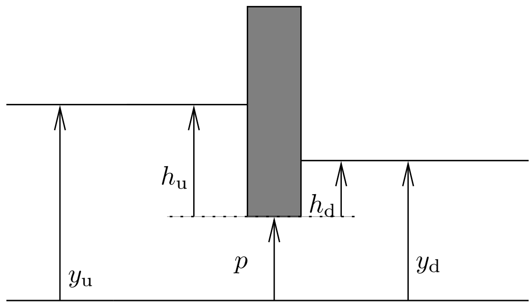

Fig. 3. Undershot gate in submerged flow. y_u - upstream water level, h_u - upstream head over gate, p - gate position, h_d - downstream head over gate, y_d - downstream water level.

Table 1. Pool lengths at the CMC

Pool	Horticulture to Coly 3	Coly 3 to Morundah	Morundah to Grants
Length (km)	2.83	6.53	5.37

2. PRIOR INFORMATION

The data presented in this paper is from the Coleambally Main Channel (CMC) in New South Wales, Australia. A top view of the part of the channel considered in this paper is given in Figure 2. In the CMC both undershot and overshot gates are used for control. At the sites Horticulture and Grants there are four identical undershot gates across the channel, and at Coly 3 and Morundah there are six identical overshot gates across the channel. Schematic figures of undershot and overshot gates are shown in Figures 3 and 4. At each site we have measurements of the upstream and downstream water levels and of the four or six gate positions. The stretch of the channel between two sites are referred to as a pool. The channel is 2.5 to $3m$ deep and $27m$ wide, and maximum flow is about 3000 ML/day ($34m^3/sec$). The pool lengths are given in table 1.

Since there is no pumping, the offtakes of water to farms and secondary channels are relying on gravity. It is therefore important to control the water levels. For this reason we seek models with the water levels as the output signals and the gate positions as the input signals.

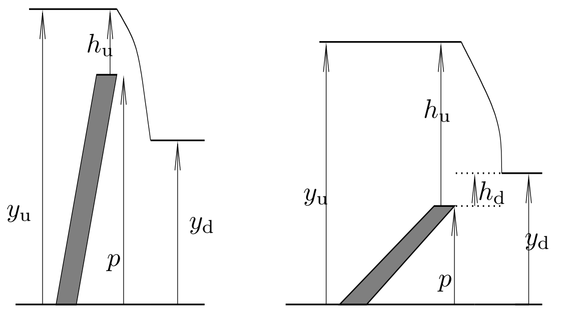

Fig. 4. Overshot gate in free flow (left) and submerged flow (right). The variable names are as for the undershot gate in Figure 3.

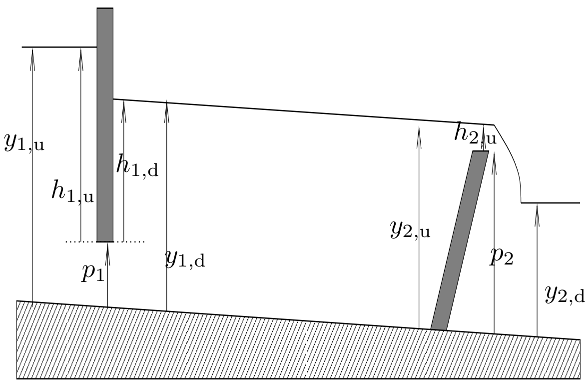

Fig. 5. Side view of the pool between Horticulture (upstream undershot gates) and Coly 3 (downstream overshot gates).

Consider the pool between Horticulture and Coly 3. A schematic side view of this pool is shown in Figure 5 (only one gate is shown at each site). As a starting point for deriving a model structure we consider a mass balance

$$\dot{V}(t) = Q_\mathrm{in}(t) - Q_\mathrm{out}(t) \tag{1}$$

where V is the volume of water in the pool, Q_in is the flow under the gates at Horticulture and, ignoring offtakes to farms and secondary channels, Q_out is the flow over the gates at Coly 3.

In the literature (e.g. Bos (1978)) the flow over an overshot gate in free flow (the top of the gate is above the downstream water level, see Figure 4) is approximated by

$$Q_\mathrm{os,f}(t) = c h_\mathrm{u}^{3/2}(t) \tag{2}$$

while in submerged flow (Webber (1971)), a correction factor $(1 - (h_\mathrm{d}(t)/h_\mathrm{u}(t))^{3/2})^{0.385}$ is used to obtain

$$Q_\mathrm{os,s}(t) = c h_\mathrm{u}^{3/2}(t) \left(1 - \left(\frac{h_\mathrm{d}(t)}{h_\mathrm{u}(t)}\right)^{3/2}\right)^{0.385} \tag{3}$$

In (2) and (3) h_u is the upstream head over gate and h_d the downstream head over gate as shown in Figure 4. c is a proportionality constant which incorporates the geometric dimensions of the gates and the discharge coefficient. The formulas are equal when $h_\mathrm{d} = 0$, and to simplify the presentation we redefine h_d

$$h_\mathrm{d}(t) = \begin{cases} h_\mathrm{d}(t) & \text{if } h_\mathrm{d}(t) > 0 \\ 0 & \text{otherwise} \end{cases}$$

such that we can work with formula (3) only.

For an undershot gate we have that (Bos (1978))

$$Q_\mathrm{us}(t) = c p(t) \sqrt{y_\mathrm{u}(t) - y_\mathrm{d}(t)}$$

where p is the gate opening, $y_\mathrm{u}(t)$ and $y_\mathrm{d}(t)$, the upstream and downstream water levels as sketched in Figure 3, and c is as before a proportionality constant.

Making the further assumptions in (1) that a) the volume of the pool is proportional to the downstream water level, and b) the flow over or under each gate can be added to obtain the total flow at a site, we arrive at the following model structure

$$\dot{y}_{2,\mathrm{u}}(t) = \tilde{c}_1 \sum_{i=1}^{4} p_{1,i}(t-\tau)\sqrt{y_{1,\mathrm{u}}(t-\tau) - y_{1,\mathrm{d}}(t-\tau)} + \tilde{c}_2 \sum_{i=1}^{6} h_{2,i,\mathrm{u}}^{3/2}(t) \left(1 - \left(\frac{h_{2,i,\mathrm{d}}(t)}{h_{2,i,\mathrm{u}}(t)}\right)^{3/2}\right)^{0.385}$$

where we have incorporated a time delay τ to account for the time it takes between a flow passes the gate at the upstream end of the pool and the effect is seen at the downstream end. Subscript i represents gate number. The variable names are explained in Figure 5. Using a simple Euler approximation for the derivative, we arrive at the discrete time model

$$y_{2,\mathrm{u}}(t+1) = y_{2,\mathrm{u}}(t) + c_1 \sum_{i=1}^{4} p_{1,i}(t-\tau)\sqrt{y_{1,\mathrm{u}}(t-\tau) - y_{1,\mathrm{d}}(t-\tau)} + c_2 \sum_{i=1}^{6} h_{2,i,\mathrm{u}}^{3/2}(t) \left(1 - \left(\frac{h_{2,i,\mathrm{d}}(t)}{h_{2,i,\mathrm{u}}(t)}\right)^{3/2}\right)^{0.385} \tag{4}$$

This model contains three unknown parameters: c_1 and c_2 associated with the in- and outflow respectively and the time delay τ.

As a convention, subscript 1 is used for the variables (gate positions, water levels etc.) at the upstream end of a pool and subscript 2 for the variables at the downstream end of the pool.

Using the same approach for the Morundah-Grants pool where there are six overshot gates at the upstream end and four undershot gates at the downstream end, we obtain the model structure.

$$y_{2,\mathrm{u}}(t+1) = y_{2,\mathrm{u}}(t) + c_1 \sum_{i=1}^{6} h_{1,i,\mathrm{u}}^{3/2}(t-\tau) \left(1 - \left(\frac{h_{1,i,\mathrm{d}}(t-\tau)}{h_{1,i,\mathrm{u}}(t-\tau)}\right)^{3/2}\right)^{0.385} + c_2 \sum_{i=1}^{4} p_{2,i}(t)\sqrt{y_{2,\mathrm{u}}(t) - y_{2,\mathrm{d}}(t)}$$

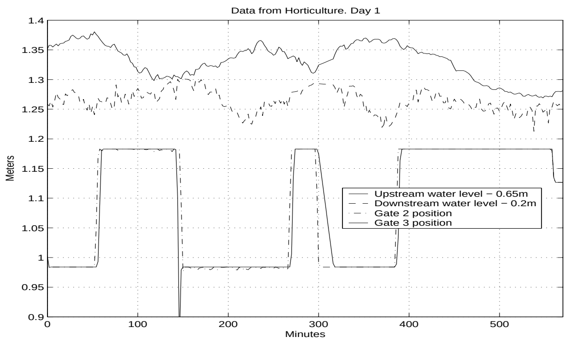

Fig. 6. Data from Horticulture. Day 1. 0.65m has been subtracted from the upstream water level and 0.2m from the downstream level.

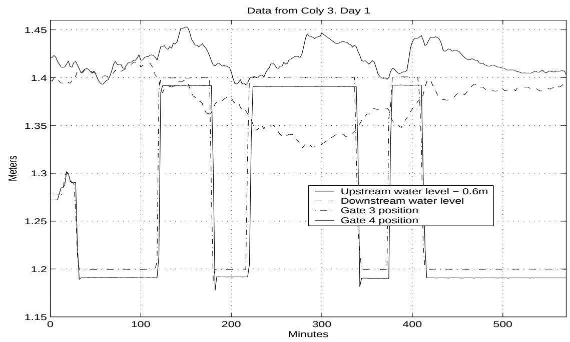

Fig. 7. Data from Coly 3. Day 1. 0.6m has been subtracted from the upstream water level.

3. EXPERIMENTS

For the experiments we were allowed to operate the two middle gates at each site. The other gates were in fixed positions. In order to get informative data we let the gates alternate between two positions following a binary signal. Based on step tests (Eurén (2004)) and experience with other channels (Weyer (2001)) we let the gate positions be constant in intervals which were multiple of 30 minutes for the pool Horticulture-Coly 3 and multiples of 60 minutes for the pool Morundah-Grants. The difference between the two gate positions were in the range $0.2\ m$ to $0.3\ m$. The variations in gate positions were limited by operational constraints since the channel was fully operational when the experiments were carried out. The flows during the experiments were in the range 1200 ML/day to 2000ML/day ($14m^3/sec$ to $23m^3/sec$). The offtakes to farms and secondary channels were small compared to the flow in the main channel, and they were therefore ignored.

The experiments were carried out over two days, each of them giving about 12 hours worth of data. The data were regular sampled at two minutes intervals, and the data for the first day for the Horticulture-Coly 3 pool are shown in Figures 6 and 7. In order to fit all graphs onto the same figure we have subtracted a constant value from the upstream and downstream water levels. Note that in Figure 7 the flow over the two middle overshot gates varies between free flow (the gate positions are above the downstream water level) and submerged flow (the gate positions are below the downstream water level). The measurements are relative to a site specific reference level, and they do not represent the actual water depth. The data for the first day for the Morundah-Grants pool are shown in Figure 8 to 9.

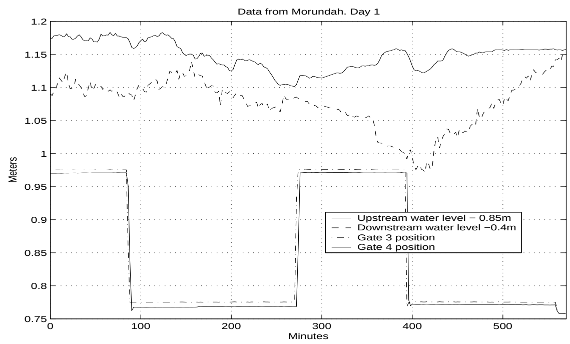

Fig. 8. Data from Morundah. Day 1. 0.85m has been subtracted from the upstream water level and 0.4m from the downstream level.

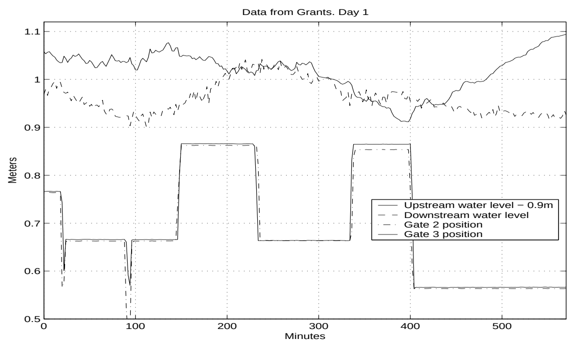

Fig. 9. Data from Grants. Day 1. 0.9m has been subtracted from the upstream water level.

4. MODEL STRUCTURE SELECTION

The natural choice of a model structure for the Horticulture-Coly 3 pool is (4). From a system identification point of view the most important aspect of a model structure is that it can be used to define predictors. Here we have used a simulation model type predictor, i.e.

$$\begin{aligned}\hat{y}_{2,\mathrm{u}}(t+1) &= \hat{y}_{2,\mathrm{u}}(t) + \\ &c_1 \sum_{i=1}^{4} p_{1,i}(t-\tau)\sqrt{y_{1,\mathrm{u}}(t-\tau) - y_{1,\mathrm{d}}(t-\tau)} + \\ &c_2 \sum_{i=1}^{6} \hat{h}_{2,i,\mathrm{u}}^{3/2}(t)\left(1 - \left(\frac{h_{2,i,\mathrm{d}}(t)}{\hat{h}_{2,i,\mathrm{u}}(t)}\right)^{3/2}\right)^{0.385} \qquad (5)\end{aligned}$$

where $\hat{h}_{2,i,\mathrm{u}}(t) = \hat{y}_{2,\mathrm{u}}(t) - p_{2,i}(t)$, see Figure 4. In (5) the previously predicted value of the water level $\hat{y}_{2,\mathrm{u}}(t)$ is used when the predicted value at time $t+1$ is calculated. The reason for using this type of predictor instead of one which utilises the measured value at time t, is that the resulting model will generally have a better long range predictive capability and give a better representation of the slow to medium range dynamics of the system which are important for control and simulations. This difference is similar to the difference between ARX and OE models in the linear case.

From a simulation point of view (5) is not entirely satisfactory since it depends on the water level $y_{1,\mathrm{d}}(t-\tau)$ downstream of the upstream gate. This water level is internal to the pool and should be incorporated in the model. In order to model this water level we followed the same approach as for $y_{2,\mathrm{u}}$, apart from that the time delay is now associated with the flow over the downstream gate. This lead to the following predictors for the two water levels

$$\begin{aligned}
&\hat{y}_{2,\mathrm{u}}(t+1,\theta) = \hat{y}_{2,\mathrm{u}}(t,\theta)+ \\
&\quad c_1 \sum_{i=1}^{4} p_{1,i}(t-\tau)\sqrt{y_{1,\mathrm{u}}(t-\tau) - \hat{y}_{1,\mathrm{d}}(t-\tau,\theta)}+ \\
&\quad c_2 \sum_{i=1}^{6} \hat{h}_{2,i,\mathrm{u}}^{3/2}(t,\theta)\left(1-\hat{r}_{2,i}^{3/2}(t,\theta)\right)^{0.385} \qquad (6)
\end{aligned}$$

$$\begin{aligned}
&\hat{y}_{1,\mathrm{d}}(t+1,\theta) = \hat{y}_{1,\mathrm{d}}(t,\theta)+ \\
&\quad c_3 \sum_{i=1}^{4} p_{1,i}(t)\sqrt{y_{1,\mathrm{u}}(t) - \hat{y}_{1,\mathrm{d}}(t,\theta)}+ \qquad (7) \\
&\quad c_4 \sum_{i=1}^{6} \hat{h}_{2,i,\mathrm{u}}^{3/2}(t-\tau,\theta)\left(1-\hat{r}_{2,i}^{3/2}(t-\tau,\theta)\right)^{0.385}
\end{aligned}$$

where we have introduced the variable $\hat{r}_{2,i}(t,\theta) = \frac{h_{2,i,\mathrm{d}}(t)}{\hat{h}_{2,i,\mathrm{u}}(t,\theta)}$, and $\theta = [c_1,\ c_2,\ c_3,\ c_4]^T$ are the unknown parameters to be estimated together with the time delay τ.

For the Morundah-Grants pool the following predictors were obtained from the same considerations

$$\begin{aligned}
&\hat{y}_{2,\mathrm{u}}(t+1,\theta) = \hat{y}_{2,\mathrm{u}}(t,\theta)+ \qquad (8) \\
&\quad c_1 \sum_{i=1}^{6} h_{1,i,\mathrm{u}}^{3/2}(t-\tau)\left(1-\hat{r}_{1,i}^{3/2}(t-\tau,\theta)\right)^{0.385} \\
&\quad + c_2 \sum_{i=1}^{4} p_{2,i}(t)\sqrt{\hat{y}_{2,\mathrm{u}}(t,\theta) - y_{2,\mathrm{d}}(t)}
\end{aligned}$$

$$\begin{aligned}
&\hat{y}_{1,\mathrm{d}}(t+1,\theta) = \hat{y}_{1,\mathrm{d}}(t,\theta)+ \qquad (9) \\
&\quad c_3 \sum_{i=1}^{6} h_{1,i,\mathrm{u}}^{3/2}(t)\left(1-\hat{r}_{1,i}^{3/2}(t,\theta)\right)^{0.385} + \\
&\quad c_4 \sum_{i=1}^{4} p_{2,i}(t-\tau)\sqrt{\hat{y}_{2,\mathrm{u}}(t-\tau,\theta) - y_{2,\mathrm{d}}(t-\tau)}
\end{aligned}$$

where $\hat{h}_{1,i,\mathrm{d}}(t,\theta) = \hat{y}_{1,\mathrm{d}}(t,\theta) - p_{1,i}(t)$ and $\hat{r}_{1,i}(t,\theta) = \frac{\hat{h}_{1,i,\mathrm{d}}(t,\theta)}{h_{1,i,\mathrm{u}}(t)}$.

5. PARAMETER ESTIMATION

The data from day 1 were used for estimation while the data from day 2 were used for validation. The parameters $\theta = [c_1,\ c_2,\ c_3,\ c_4]^T$ were estimated for a number of time delays τ using the squared prediction error criterion

$$\hat{\theta}_\tau = \arg\min_\theta \frac{1}{N}\sum_{t=1}^{N}(y_{2,\mathrm{u}}(t) - \hat{y}_{2,\mathrm{u}}(t,\theta))^2 + (y_{1,\mathrm{d}}(t) - \hat{y}_{1,\mathrm{d}}(t,\theta))^2$$

where $y_{2,\mathrm{u}}(t)$ and $y_{1,\mathrm{d}}(t)$ are the measured water levels and $\hat{y}_{2,\mathrm{u}}(t,\theta)$ and $\hat{y}_{1,\mathrm{d}}(t,\theta)$ are the water levels predicted according to (6) and (7) for he Horticulture-Coly 3 pool and according to (8) and (9) for the Morundah-Grants pool. The number of data points were $N = 286$. The results are shown in Table 2.

Table 2. Parameter estimates ($\cdot 10^{-3}$)

Pool	τ (samples)	$\hat{c}_1$	$\hat{c}_2$	$\hat{c}_3$	$\hat{c}_4$
H-C	5	10.54	-6.61	8.81	-5.56
M-G	10	3.58	-5.92	3.19	-5.25

Table 3. Average squared prediction errors.

Pool	$V_2(\hat{\theta})$ Est	$V_2(\hat{\theta})$ Val
H-C	$5.57 \cdot 10^{-5}$	$7.32 \cdot 10^{-5}$
M-G	$1.53 \cdot 10^{-4}$	$1.07 \cdot 10^{-4}$

6. MODEL VALIDATION AND DISCUSSION

We notice that the parameter estimates $\hat{c}_1$ and $\hat{c}_3$ are positive while the estimates $\hat{c}_2$ and $\hat{c}_4$ are negative. This is as expected since c_1 and c_3 are associated with in-flows and c_2 and c_4 with out-flows. Moreover, the time delay for Morundah-Grants is twice as large as for Horticulture-Coly 3 which is again expected since the pool Morundah-Grants is about twice as long as the pool Horticulture-Coly 3.

The obtained models were validated on the data sets from day 2. The average squared prediction errors

$$V_2(\hat{\theta}) = \frac{1}{N}\sum_{t=1}^{N}(y_{2,\mathrm{u}}(t) - \hat{y}_{2,\mathrm{u}}(t,\hat{\theta}))^2$$

are shown in Table 3. The values are similar for the estimation and the validation set and indicates that the models capture the inherent features of the systems.

The models were simulated against the validation data from day 2, and the results are shown in Figures 10 and 11. The results are excellent. The models for $y_{2,\mathrm{u}}$ (the upstream water levels of the downstream gates in the pools) follow the actual water levels closely for more than 12 hours, that is, for the whole duration of the validation set. $y_{2,\mathrm{u}}$ is the most important water level in a pool and the one we want to control since most offtakes to farms and secondary channels are located near the downstream gates. Note that these are simulations, and the models only had access to the initial water levels in the pools.

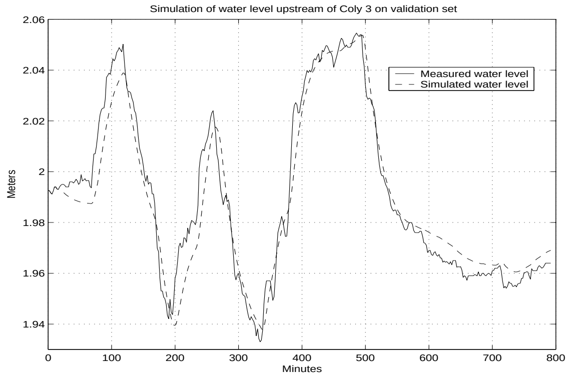

Fig. 10. Simulation of the models against the validation set. Water level upstream of Coly 3.

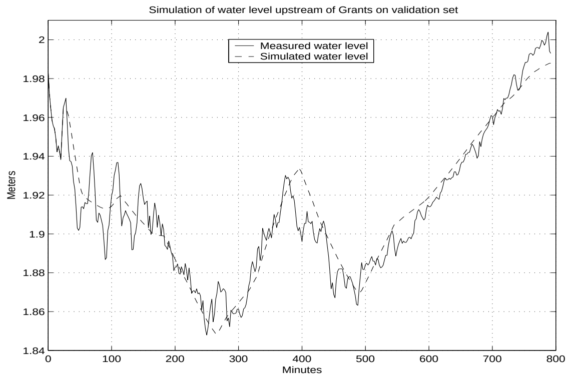

Fig. 11. Simulation of the models against the validation set. Water level upstream of Grants.

7. CONCLUSIONS

System identification of irrigation channels with both overshot and undershot gates has been considered in this paper. All the tasks in the system identification procedure from experiment design to model validation have been treated, and prior information has been taken into account. This work confirms the results in Weyer (2001) and extends them in several important aspects: 1) The irrigation channel was equipped with both overshot and undershot gates, 2) The overshot gates operated in both submerged and free flow 3) There were several gates at each regulator structure and they have different positions and 4) The flows and pools were larger.

The simple system identification models show a remarkable predictive accuracy. They are able to accurately simulate the water levels for more than 12 hours ahead of time. Moreover, the models are very simple, and they are ideally suited for control design. The results further illustrate that system identification and control design have an important part to play in water resource management. The environmental benefits of this work are very large since, with well designed controllers, one can divert less water from rivers for irrigation purposes while maintaining the same level of service to the farmers.

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