OPTIMAL INPUT AND DECISION IN MULTIPLE MODEL FAULT DETECTION

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Abstract: A design of detection system using closed loop information processing strategy in multiple model framework is treated. The proposed detection system should simultaneously generate decisions and auxiliary input signal which can improve fault detection. The design of detection system is formulated as an optimization problem based on the assumption that the measurements will be available in the future. Then the dynamic programming can be utilized for the minimization of a considered criterion. A significant special case where the relation between the auxiliary input signal and the decision is given a priori is analyzed.

Keywords: fault detection, optimal experiment design, optimal detection, dynamic programming, loss minimization, detection systems

1. INTRODUCTION

The aim of fault detection and isolation (FDI) is early recognition and determination of undesired or even dangerous behavior of the observed process. The increasing attention to FDI in recent years is caused by economical, environmental and other possible savings. A survey of various design methods can be found within many of others in (Willsky, 1976; Basseville, 1988). New and more complex methods were proposed with growing availability of faster and cheaper computers. The survey can be found e.g. in (Isermann, 2004). The design of detector is usually based on some sort of mathematical model of a real process and measurements which can be obtained from this process. Such detector passively monitors available measurements and makes decisions.

A lot of the approaches consider that the detection consists of two steps. The first step is the generation of residuals which represent inconsistencies between the true and expected behavior of the observed process and the decisions based on the residuals are made in the second step. The large review of methods used for design of residual generators can be found in (Patton *et al.*, 1989). Basseville and Nikiforov (1993) aimed to design both parts of detector and their design is based on statistical approach mainly. In many papers great attention is aimed to robustness problem in the residual generator design (Frank and Ding, 1997). On the other side the problem of process excitation is often either omitted or the request for appropriate excitation is stated even if the change detection methods sensitive to the process excitation are used.

Design of special input signal which can help to improve change detection is well known from system identification (Zarrop, 1979). Zhang (1989) studied a method for design of input signal for change detection in multiple model framework. The auxiliary input is designed there to maximize the Baram's distance between possible models. A method for active fault detection in linear system subject to inequality-bounded perturbations is presented by Nikoukhah (1998). A similar approach where the perturbations are modelled as bounded energy signals is introduced by Nikoukhah *et al.* (2000) where the aim is to find minimum energy auxiliary input signal which enables to decide surely in which mode the process operates.

Most of the known FDI methods do not consider the future information as a mean for minimizing the possible losses caused be wrong decisions. They utilize all available information at the moment of decision but the fact that the additional information will be obtained in the future is not considered. If it is considered that decision can influence the process progression, then the future information can backward affect this decision. This idea is elaborated by Šimandl and Herejt (2003) where three different information processing strategies (open loop, open loop feedback and closed loop) to change detection in multiple model framework are considered. The design procedure based on closed loop information processing strategy is superior to commonly used Bayesian approach e.g. (Berec, 1998) which matches the open loop feedback information processing strategy.

The aim of the paper is to extend the idea presented in (Šimandl and Herejt, 2003), where the generator was given *a priori* and depended on decision in known manner, to more general case for which the generator of the auxiliary input is also searched.

The paper is organized as follows. In Section 2 the model of observed process together with a general description of detection system are presented. The choice of possible criterion is also discussed. The general solution of stated problem utilizing dynamic programming is presented in Section 3. Section 4 is devoted to above mentioned special case, where the generator of the auxiliary input is given *a priori* and depends on the decision. The solution for additive criterion is examined in both cases. The designed approach together with the Bayesian approach are applied and compared in a simple example in Section 5.

2. PROBLEM FORMULATION

Time-discrete models will be considered due to using digital devices to fault detection. Description of observed process is considered in multiple model framework. Therefore, it is assumed that the process can be described at each time $k \in \mathcal{T} = \{0, 1, \dots, F\}$ by a model chosen from *a priori* known and complete set of possible models and the process model can change at each sampling time. Similarly as in the area of FDI, it is supposed that one of models defines the safety behavior and the others describe individual faulty modes. Then the system can be described by the known transition probability density function (pdf)

$$f(\mathbf{x}_k, \theta_k | \mathbf{x}_{k-1}, \theta_{k-1}, \mathbf{u}_{k-1}) \tag{1}$$

and the measurement pdf

$$g(\mathbf{y}_k | \mathbf{x}_k, \theta_k), \tag{2}$$

with given *a priori* pdf

$$f(\mathbf{x}_0, \theta_0 | \mathbf{x}_{-1}, \theta_{-1}, \mathbf{u}_{-1}) = f(\mathbf{x}_0, \theta_0), \tag{3}$$

where $\mathbf{u}_k \in \mathbf{U}_k \subset \mathcal{R}^{n_u}$ and $\mathbf{y}_k \in \mathcal{R}^{n_y}$ are inputs and measurements, respectively. The pair $[\mathbf{x}_k, \theta_k]$ creates hybrid state of the system. The first part $\mathbf{x}_k \in \mathcal{R}^{n_x}$ denotes continuous part of the hybrid state, the second part $\theta_k \in \mathcal{M} = \{1, 2, \dots, N\}$ is a scalar variable denoting index of the model which represents process at time k and N is number of possible models. If the state space representation of process is given the conditional pdf's (1) and (2) can be obtained from state equation and measurement equation, respectively. Now, some useful assumptions on the pdf (1) which lead to computational simplification will be introduced. The parts $\mathbf{x}_k$ and θ_k of the state are considered to be mutually independent and the transition pdf (1) can be factorized as

$$f_1(\mathbf{x}_k | \mathbf{x}_{k-1}, \theta_{k-1}, \mathbf{u}_{k-1}) f_2(\theta_k | \mathbf{x}_{k-1}, \theta_{k-1}, \mathbf{u}_{k-1}), \tag{4}$$

where f_1 and f_2 are transition pdf's describing evolution of $\mathbf{x}_k$ and process of model changes, respectively. The sequence of models is considered to be a finite-state Markov chain with known transition probabilities $P(\theta_k|\theta_{k-1})$. Thus, the second factor of (4) is

$$f_2(\theta_k | \mathbf{x}_{k-1}, \theta_{k-1}, \mathbf{u}_{k-1}) = P(\theta_k | \theta_{k-1}). \tag{5}$$

Now, the first factor of (4) and the pdf (2) can be considered in the following form

$$f_1 = \sum_{i=1}^{N} \delta_{i,\theta_{k-1}} \, {}^{i}f_1(\mathbf{x}_k | \mathbf{x}_{k-1}, \mathbf{u}_{k-1}), \tag{6}$$

$$g = \sum_{i=1}^{N} \delta_{i,\theta_{k-1}} \, {}^{i}g(\mathbf{y}_k | \mathbf{x}_k), \tag{7}$$

where ${}^{i}f_1(\mathbf{x}_k|\mathbf{x}_{k-1}, \mathbf{u}_{k-1})$ and ${}^{i}g(\mathbf{y}_k|\mathbf{x}_k)$ are pdf's of the particular model i and δ_{i,θ_k} is the Kronecker delta defined as

$$\delta_{i,j} = \begin{cases} 1, & i = j \\ 0, & i \neq j. \end{cases} \tag{8}$$

The class of considered problems is restricted only by assumption (5) which does not allow the dependence of transition probabilities f_2 on the part $\mathbf{x}_{k-1}$ of the state and the input $\mathbf{u}_{k-1}$.

Now, the description of a detection system will be presented. The aim of design is to find the detection system which generates the optimal decision and the optimal auxiliary input at each time $k \in \mathcal{T}$. It is obvious that the best possible behavior of such system at time k can be reached by utilization of all information available at time k. Thus, the general description of the detection system by means of the joint pdf should be supposed in the following form

$$\rho_k(\mathbf{u}_k, d_k|\mathbf{I}_0^k), \tag{9}$$

where $d_k \in \mathcal{M}$ denotes decision of detector at time k, $\mathbf{I}_0^k = [{\mathbf{y}_0^k}^T, {\mathbf{u}_0^{k-1}}^T, {d_0^{k-1}}^T]^T$ denotes all available information and the notation $\mathbf{y}_0^k = [{\mathbf{y}_0}^T, {\mathbf{y}_1}^T, \ldots, {\mathbf{y}_k}^T]^T$ is used for description of whole history of considered variable or pdf. The joint pdf (9) can be factorized in two different ways

$$\rho_k(\mathbf{u}_k, d_k|\mathbf{I}_0^k) = \gamma_k(\mathbf{u}_k|\mathbf{I}_0^k, d_k)\sigma_k(d_k|\mathbf{I}_0^k), \tag{10}$$

$$\rho_k(\mathbf{u}_k, d_k|\mathbf{I}_0^k) = \sigma_k(d_k|\mathbf{I}_0^k, \mathbf{u}_k)\gamma_k(\mathbf{u}_k|\mathbf{I}_0^k), \tag{11}$$

but the stress will be laid only on the factorization (10) in this paper, because it covers the situations in which the relation γ_k between the auxiliary input and the decision is given.

The goal of the design is to choose such detection system which makes statistically least mistakes. So, a loss function in the criterion should penalize wrong decisions and the value of the criterion can be assumed as expectation of the loss function over the finite detection horizon. The general criterion is following

$$J(\rho_0^F) = E\left\{L(\theta_0^F, d_0^F)\right\}, \tag{12}$$

where $E\{\cdot\}$ denotes expectation operator over all included random variables and the loss function $L(\theta_0^F, d_0^F)$ is a real-valued non-negative function which is chosen with respect to real costs caused by the decisions.

3. GENERAL SOLUTION

In this section the main concern is to find the best pdf's ρ_0^F describing detection system. The optimal detection system (9) has to minimize the general criterion (12) or some additive criterion (24) for given system (1), (2). The general solution is found by means of dynamic programming (Bertsekas, 1995; Žampa *et al.*, 2004). It will be shown that the optimal decision d_k^* and input $\mathbf{u}_k^*$ are functions of all available information $\mathbf{I}_0^k$ at time k and hence the pdf (9) is zero everywhere except at $[\mathbf{u}_k^*, d_k^*]$. Next a recursive solution will be presented and the case of an additive criterion will be examined.

Firstly, some important relations which will be used throughout the paper are introduced

$$p(\theta_0^j|\mathbf{I}_0^k, \mathbf{u}_k, d_k) = p(\theta_0^j|\mathbf{y}_0^k, \mathbf{u}_0^{k-1}), \tag{13}$$

$$p(y_{k+1}|\mathbf{I}_0^k, \mathbf{u}_k, d_k) = p(y_{k+1}|\mathbf{y}_0^k, \mathbf{u}_0^k), \tag{14}$$

where $0 \leq j \leq k$. The criterion (12) can be written in the following form

$$J(\rho_0^F) = \int L(\theta_0^F, d_0^F)p(\theta_0^F, \mathbf{I}_0^F, \mathbf{u}_F, d_F)d(\theta_0^F, \mathbf{I}_0^F, \mathbf{u}_F, d_F) = \int W_k(\mathbf{I}_0^k; \rho_k^F)p(\mathbf{I}_0^k)d\mathbf{I}_0^k \tag{15}$$

and the following factorization of joint pdf is supposed

$$p(\theta_0^F, \mathbf{I}_0^F, \mathbf{u}_F, d_F) = p(\theta_0^F|\mathbf{I}_0^F, \mathbf{u}_F, d_F) \prod_{i=0}^{F}\Big[p(\mathbf{u}_{F-i}, d_{F-i}|\mathbf{I}_0^{F-i}) \, p(\mathbf{y}_{F-i}|\mathbf{I}_0^{F-1-i}, \mathbf{u}_{F-1-i}, d_{F-1-i})\Big], \tag{16}$$

where $W_k(\mathbf{I}_0^k; \rho_k^F)$ denotes the estimate of partial losses at time k based on available information $\mathbf{I}_0^k$ and the future sequence of pdf's ρ_k^F. It is supposed for a while that all future optimal pdf's ρ_{k+1}^{F*} are known and the searched pdf ρ_k has to minimize the estimate of the partial losses $W_k(\mathbf{I}_0^k; \rho_k, \rho_{k+1}^{F*})$ for given $\mathbf{I}_0^k$. In order to determine optimal pdf ρ_k the estimate $W_k(\mathbf{I}_0^k; \rho_k, \rho_{k+1}^{F*})$ can be written as

$$W_k(\mathbf{I}_0^k; \rho_k, \rho_{k+1}^{F*}) = \int C_{k+1}^*(\mathbf{I}_0^k, \mathbf{u}_k, d_k; \rho_{k+1}^{F*})\rho_k(\mathbf{u}_k, d_k|\mathbf{I}_0^k)d(\mathbf{u}_k, d_k) \tag{17}$$

and from (15) it is obvious that the following relation holds

$$C_{k+1}^*(\mathbf{I}_0^k, \mathbf{u}_k, d_k; \rho_{k+1}^{F*}) = \int W_{k+1}^*(\mathbf{I}_0^{k+1}; \rho_{k+1}^{F*})p(\mathbf{y}_{k+1}|\mathbf{I}_0^k, \mathbf{u}_k, d_k)d\mathbf{y}_{k+1}, \tag{18}$$

where $W_{k+1}^*(\mathbf{I}_0^{k+1}; \rho_{k+1}^{F*})$ represents minimum of the best estimate of partial losses based on all available information at time $k+1$. Thus the real-valued non-negative function C_{k+1}^* is independent of future behavior and it is assumed that it has a global minimum with given constraints on $\mathbf{u}_k$ and d_k for every given $\mathbf{I}_0^k$. For such pair $\mathbf{u}_k^*$, d_k^* it holds that

$$C_{k+1}^*(\mathbf{I}_0^k, \mathbf{u}_k^*, d_k^*; \rho_{k+1}^{F*}) \leq C_{k+1}^*(\mathbf{I}_0^k, \mathbf{u}_k, d_k; \rho_{k+1}^{F*}). \tag{19}$$

Now, it is obvious that the best pdf (9) is the product of Dirac function and Kronecker delta for $\mathbf{u}_k$ and d_k, respectively. If there is multiple global minimum it does not matter which will be chosen. In general, the optimal decision d_k^* depends on the auxiliary input $\mathbf{u}_k$ and vice versa. If it can be assumed that the optimal input $\mathbf{u}_k^*$ and the optimal decision d_k^* will be used at all times, the both values can be written only as some functions of available information $\mathbf{I}_0^k$. Thus, the optimal detection system is deterministic and the resulting optimal pdf (9) is

$$\rho_k(\mathbf{u}_k, d_k|\mathbf{I}_0^k) = \delta(\mathbf{u}_k - \mathbf{u}_k^*)\delta_{d_k,d_k^*}, \tag{20}$$

where $\delta(\cdot)$ is the Dirac function. The recursive solution can be easily derived from equations (17) and (18) as

$$W_{F+1}^* = L(\theta_0^F, d_0^F), \tag{21}$$

$$W_k^*(\mathbf{I}_0^k) = \min_{\substack{\mathbf{u}_k\in\mathbf{U}_k\\ d_k\in\mathcal{M}}} E\left\{W_{k+1}^*(\mathbf{I}_0^{k+1})|\mathbf{I}_0^k, \mathbf{u}_k, d_k\right\}, \tag{22}$$

$$\mathbf{u}_k^*, d_k^* = \arg\min_{\substack{\mathbf{u}_k\in\mathbf{U}_k\\ d_k\in\mathcal{M}}} E\left\{W_{k+1}^*(\mathbf{I}_0^{k+1})|\mathbf{I}_0^k, \mathbf{u}_k, d_k\right\}, \tag{23}$$

where the computation starts from the end of the detection horizon F $(k = F, F-1, \ldots, 1, 0)$.

In the following text the additive criterion will be considered. The main idea of solution is to divide partial losses into the present and the future. The major contribution of an additive criterion is a simplifications of computations and complete mutually independency of the optimal decision d_k^* and the optimal auxiliary input $\mathbf{u}_k^*$. Let the loss function has the following special form

$$L(\theta_0^F, d_0^F) = \sum_{i=0}^{F} L_i(\theta_i, d_i). \tag{24}$$

The function $W_F^*(\mathbf{I}_0^F)$ can be written with regard to additivity of the criterion as

$$W_F^*(\mathbf{I}_0^F) = \min_{\substack{\mathbf{u}_F\in\mathbf{U}_F\\ d_F\in\mathcal{M}}} \left[E\left\{\sum_{i=0}^{F-1} L_i(\theta_i, d_i)|\mathbf{I}_0^F, \mathbf{u}_F, d_F\right\} + E\left\{L_F(\theta_F, d_F)|\mathbf{I}_0^F, \mathbf{u}_F, d_F\right\}\right]. \tag{25}$$

With respect to equation (13) only the second term on right hand side of equation (25) can be minimized by decision d_F and such a minimal value will be denoted $V_F^*(\mathbf{y}_0^F, \mathbf{u}_0^{F-1})$ and called Bellman function. The function $W_{F-1}^*(\mathbf{I}_0^{F-1})$ can be decomposed in a similar way and it can be rewritten using Bellman function $V_F^*(\mathbf{y}_0^F, \mathbf{u}_0^{F-1})$ as

$$\begin{aligned} W_{F-1}^*(\mathbf{I}_0^{F-1}) = \min_{\substack{\mathbf{u}_{F-1}\in\mathbf{U}_{F-1}\\ d_{F-1}\in\mathcal{M}}} \Bigg[E\Bigg\{ & E\left\{\sum_{i=0}^{F-2} L_i(\theta_i, d_i)|\mathbf{I}_0^F, \mathbf{u}_F, d_F\right\} + \\ & E\left\{L_{F-1}(\theta_{F-1}, d_{F-1})|\mathbf{I}_0^F, \mathbf{u}_F, d_F\right\} + \\ & V_F^*(\mathbf{y}_0^F, \mathbf{u}_0^{F-1})|\mathbf{I}_0^{F-1}, \mathbf{u}_{F-1}, d_{F-1}\Bigg\}\Bigg]. \end{aligned} \tag{26}$$

Denoting the last two terms in (26) by

$$V_{F-1}(\mathbf{I}_0^{F-1}, \mathbf{u}_{F-1}, d_{F-1}) \tag{27}$$

and using relation

$$E\left\{E\left\{\varphi(a)|bc\right\}|c\right\} = E\left\{\varphi(a)|c\right\}, \tag{28}$$

the (26) can be written as

$$\begin{aligned} W_{F-1}^*(\mathbf{I}_0^{F-1}) = \min_{\substack{\mathbf{u}_{F-1}\in\mathbf{U}_{F-1}\\ d_{F-1}\in\mathcal{M}}} \Bigg[& E\left\{\sum_{i=0}^{F-2} L_i(\theta_i, d_i)|\mathbf{I}_0^{F-1}, \mathbf{u}_{F-1}, d_{F-1}\right\} + \\ & V_{F-1}(\mathbf{I}_0^{F-1}, \mathbf{u}_{F-1}, d_{F-1})\Bigg], \end{aligned} \tag{29}$$

where only the second term V_{F-1} on right hand side can be minimized by $\mathbf{u}_{F-1}$ and d_{F-1} if the relations (13) and (14) are considered. The minimal value of V_{F-1} has the following form

$$\begin{aligned} & V_{F-1}^*(\mathbf{y}_0^{F-1}, \mathbf{u}_0^{F-2}) = \\ & \min_{d_{F-1}\in\mathcal{M}} E\left\{L_{F-1}(\theta_{F-1}, d_{F-1})|\mathbf{y}_0^{F-1}, \mathbf{u}_0^{F-2}, d_{F-1}\right\} \\ & \quad + \min_{\mathbf{u}_{F-1}\in\mathbf{U}_{F-1}} E\left\{V_F^*(\mathbf{y}_0^F, \mathbf{u}_0^{F-1})|\mathbf{y}_0^{F-1}, \mathbf{u}_0^{F-1}\right\}. \end{aligned} \tag{30}$$

The presented procedure of derivation repeated at all time steps leads to the resulting backward recursive solution starting at time F

$$\begin{aligned} & V_{F+1}^* = 0, \\ & V_k^*(\mathbf{y}_0^k, \mathbf{u}_0^{k-1}) = \\ & \min_{d_k\in\mathcal{M}} E\left\{L_k(\theta_k, d_k)|\mathbf{y}_0^k, \mathbf{u}_0^{k-1}, d_k\right\} + \\ & \min_{\mathbf{u}_k\in\mathbf{U}_k} E\left\{V_{k+1}^*(\mathbf{y}_0^{k+1}, \mathbf{u}_0^k)|\mathbf{y}_0^k, \mathbf{u}_0^k\right\}, \end{aligned} \tag{31}$$

where the minimum $V_k^*(\mathbf{y}_0^k, \mathbf{u}_0^{k-1})$ in (31) is achieved using the optimal decision and the optimal auxiliary input

$$d_k^* = \arg\min_{d_k\in\mathcal{M}} E\left\{L_k(\theta_k, d_k)|\mathbf{y}_0^k, \mathbf{u}_0^{k-1}, d_k\right\}, \tag{32}$$

$$\mathbf{u}_k^* = \arg\min_{\mathbf{u}_k\in\mathbf{U}_k} E\left\{V_{k+1}^*(\mathbf{y}_0^{k+1}, \mathbf{u}_0^k)|\mathbf{y}_0^k, \mathbf{u}_0^k\right\}. \tag{33}$$

The proposed design procedure provides the general solution to active FDI formulated as optimization problem. The additivity of criterion caused that the optimal detection system could be divided into two independent parts. The first one is the optimal detector and the second is the optimal auxiliary input generator. The optimal decision d_k^* in (32) minimizes conditional mean of loss at time k. The optimal auxiliary input signal $\mathbf{u}_k^*$ in (33) handles the fact that the future measurements will be used for detection and tries to provide sufficiently excited signals. This design procedure has some common features with optimal stochastic control problem and it is the general case of the problem studied by Šimandl and Herejt (2003). Unfortunately, the closed loop solution is intractable in most cases and a numerical approximation must be used.

4. A SPECIAL CASE

The situations where the generator of auxiliary signal depends on the decision is considered in this section. Only the main results will be presented because the derivation is analogical to the ones in previous section. The factorization (10) of the pdf is considered and it is assumed that the generator of auxiliary input $\mathbf{u}_k$ is given by a known function $\mathbf{h}(\mathbf{I}_0^k, d_k)$ of available information and current decision. Thus, the conditional pdf γ_k can be written as

$$\gamma_k(\mathbf{u}_k|\mathbf{I}_0^k, d_k) = \delta(\mathbf{u}_k - \mathbf{h}(\mathbf{I}_0^k, d_k)), \quad (34)$$

where it is required that the range of function $\mathbf{h}(\mathbf{I}_0^k, d_k)$ is a subset of $\mathbf{U}_k$. With mentioned factorization (10) and the known description of generator (34) the equation (17) is

$$W_k(\mathbf{I}_0^k; \sigma_k, \sigma_{k+1}^{F*}) = \int C_{k+1}^*(\mathbf{I}_0^k, \mathbf{h}(\mathbf{I}_0^k, d_k), d_k; \sigma_{k+1}^{F*})\sigma_k(d_k|\mathbf{I}_0^k)d(d_k). \quad (35)$$

Similarly to the general case the pdf σ_k of optimal detector is again Dirac function at the point where the function C_{k+1}^* for given $\mathbf{I}_0^k$ has its minimum with respect to d_k. The recursive solution for the general criterion (12) can be found in the following form

$$W_{F+1}^* = L(\theta_0^F, d_0^F), \quad (36)$$

$$W_k^*(\mathbf{I}_0^k) = \min_{d_k \in \mathcal{M}} E\{W_{k+1}^*(\mathbf{I}_0^{k+1})|\mathbf{I}_0^k, d_k\}, \quad (37)$$

$$d_k^* = \arg \min_{d_k \in \mathcal{M}} E\{W_{k+1}^*(\mathbf{I}_0^{k+1})|\mathbf{I}_0^k, d_k\} \quad (38)$$

and the recursive solution for additive criterion (24) is

$$\begin{aligned} &V_{F+1}^* = 0, \\ &V_k^*(\mathbf{y}_0^k, \mathbf{u}_0^{k-1}) = \min_{d_k \in \mathcal{M}} \\ &E\left\{L_k(\theta_k, d_k) + V_{k+1}^*(\mathbf{y}_0^{k+1}, \mathbf{u}_0^k)|\mathbf{y}_0^k, \mathbf{u}_0^k, d_k\right\}, \end{aligned} \quad (39)$$

where the minimum V_k^* in (39) is obtained by the optimal decision

$$\begin{aligned} &d_k^* = \arg \min_{d_k \in \mathcal{M}} \\ &E\left\{L_k(\theta_k, d_k) + V_{k+1}^*(\mathbf{y}_0^{k+1}, \mathbf{u}_0^k)|\mathbf{y}_0^k, \mathbf{u}_0^k, d_k\right\}. \end{aligned} \quad (40)$$

The optimal decision d_k^* minimizes sum of two terms in (40). The first term represents the estimate of current loss and the second term is estimate of future losses, both caused by this decision. Standard approaches based on Bayesian technique do not consider existence of the second term and minimize only the first one. Comparing (31) and (39) it can be seen that given relationship between optimal decision and auxiliary input leads to higher value of criterion and thus to worse behavior of the detection system. Of course, it still provides better results than standard Bayesian approaches in cases where the relation (34) exists.

5. ILLUSTRATIVE EXAMPLE

Both the general and the special case presented in this paper are compared with commonly used Bayesian approach in a numerical example. It should demonstrate an influence of individual approaches on value of the criterion. In all cases the additive criterion (24) is assumed and the detection horizon $F = 1$ is chosen. The considered values of auxiliary input are kept the same in all three situations to provide comparability. The set of models is $\mathcal{M} = \{1, 2\}$ and the process is described by one of the following models at each time k

$$\begin{aligned} \theta_k = 1: x_{k+1} &= 0.3x_k + u_k + {}^1w_k, \\ y_k &= -2x_k + {}^1v_k, \end{aligned} \quad (41)$$

$$\begin{aligned} \theta_k = 2: x_{k+1} &= 0.5x_k + 1.5u_k + {}^2w_k, \\ y_k &= 1.5x_k + {}^2v_k, \end{aligned} \quad (42)$$

where $\{{}^1w_k\}$, $\{{}^1v_k\}$, $\{{}^2w_k\}$, $\{{}^2v_k\}$ are mutually independent random sequences. Their distribution is zero mean Gaussian with variance $\sigma^2 = 0.25$. The initial condition x_0 has also Gaussian distribution with mean 0 and variance $\sigma_x^2 = 0.1$ and is independent of processes $\{{}^1w_k\}$, $\{{}^1v_k\}$, $\{{}^2w_k\}$, $\{{}^2v_k\}$. *A priori* probabilities of models are $P_2(\theta_0 = 1|\theta_{-1}) = P_2(\theta_0 = 2|\theta_{-1}) = 0.5$. The transition probabilities are defined for all times k as

$$P(\theta_k|\theta_{k-1}) = \begin{bmatrix} 0.2 & 0.8 \\ 0.8 & 0.2 \end{bmatrix}. \quad (43)$$

In both cases the loss function is chosen as zero-one loss function

$$\begin{aligned} \theta_k = d_k &\Rightarrow L_k(\theta_k, d_k) = 0, \\ \theta_k \neq d_k &\Rightarrow L_k(\theta_k, d_k) = 1. \end{aligned} \quad (44)$$

Unknown detector and generator (CL1)

The main diagnostic tools are digital devices as mentioned. Thus, the auxiliary input can be considered in discrete values from bounded interval only. The possible values of input are chosen as

$$u_k \in \mathbf{U}_k = \{1.5, -0.1\}. \quad (45)$$

The Bellman function at time $k = F = 1$ is

$$V_1^*(y_0^1, u_0) = \min_{d_1 \in \mathcal{M}} \sum_{\theta_1} L_1(\theta_1, d_1)p(\theta_1|y_0^1, u_0) \quad (46)$$

and the optimal decision d_1^* is found as

$$d_1^* = \arg \min_{d_1 \in \mathcal{M}} \sum_{\theta_1} L_1(\theta_1, d_1)p(\theta_1|y_0^1, u_0). \quad (47)$$

Any admissible auxiliary input u_1^* can be used because it does not influence the value of criterion. The optimal decision d_0^* and the optimal auxiliary input u_0^* at time $k = 0$ are

$$d_0^* = \arg \min_{d_0 \in \mathcal{M}} \sum_{\theta_0} L_0(\theta_0, d_0)p(\theta_0|y_0), \quad (48)$$

$$u_0^* = \arg \min_{u_0 \in \mathbf{U}_0} \int V_1^*(y_0^1, u_0)p(y_1|y_0, u_0)dy_1. \quad (49)$$

Unknown detector and known generator (CL2)
The description of the auxiliary input generator is supposed to be

$$u_k = h(d_k) = \begin{cases} 1.5, & d_k = 1 \\ -0.1, & d_k = 2. \end{cases} \quad (50)$$

The rule for the optimal decision d_1^* and Bellman function $V_1^*(y_0^1, u_0)$ at time 1 are the same as in the previous case and the optimal decision d_0^* at time $k = 0$ is

$$d_0^* = \arg\min_{d_0 \in \mathcal{M}} \Bigg[\sum_{\theta_0} L_0(\theta_0, d_0) p(\theta_0|y_0) + \int V_1^*(y_0^1, h(d_0)) p(y_1|y_0, h(d_0)) dy_1 \Bigg]. \quad (51)$$

The inputs u_0^* and u_1^* are determined by the corresponding optimal decisions according to (50).

Unknown detector and known generator (BA)
It was shown in (Šimandl and Herejt, 2003) that such approach matches open loop feedback strategy. The dependency of the auxiliary input on decision is same as in the previous situation and the optimal decisions for $k \in \{0, 1\}$ are found as

$$d_k^* = \arg\min_{d_k \in \mathcal{M}} \sum_{\theta_k} L_k(\theta_k, d_k) p(\theta_k|y_0^k, u_0^{k-1}).$$

A bank of Kalman filters is used to compute needed probabilities and the integrals in (49) and (51) must be computed numerically using trapezoidal rule. Mean and variance of the estimate $\hat{J}$ of the criterion J presented in Table 1 were obtained by 1000 Monte Carlo simulations.

Table 1. Values of the criterion

	CL1	CL2	BA
$E\{\hat{J}\}$	0.4659	0.4997	0.7411
$VAR\{\hat{J}\}$	$3.02 \cdot 10^{-4}$	$2.23 \cdot 10^{-4}$	$4.99 \cdot 10^{-4}$

The value of criterion slightly increases with restrictive assumption on the auxiliary input generator but Bayesian approach provides considerably worse value of criterion.

6. CONCLUSION

The active fault detection approach respecting availability of future information was formulated as an optimization problem. The dynamic programming was utilized to minimization of a proposed criterion on finite detection horizon. It was shown that the optimal decision and the optimal auxiliary input are independent in general case of additive criterion. The presented formulation allows to derive easily some special cases and the case with a priori given input generator was introduced. Both general and considered special case produce better decisions comparing to the Bayesian approach.

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