

# ON THE STOCHASTIC MODELLING AND SOLVENCY OF BANKING SYSTEMS

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Abstract: This paper investigates issues related to the capital adequacy regulation and philosophy of internationally active banks. We make a technical contribution to this discussion by constructing a stochastic continuous-time model for the dynamics of the capital adequacy ratio of such a bank. This ratio is obtained by dividing the bank's eligible regulatory capital (ERC) by its total risk-weighted assets (TRWAs) from credit, market and operational risk. In the main, our discussions about the ERC and TRWAs conform to the qualitative and quantitative standards prescribed by the Basel II Capital Accord (see BCBS, June 2004).  
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## 1. INTRODUCTION

In recent years, a great deal of research has been done on the soundness of bank systems from a theoretical and empirical point of view. In this regard, a concerted effort has been made to set global qualitative and quantitative standards for banking supervision by the drafting of the Basel Capital Accords and their amendments (see, for instance, BCBS, July 1988, BCBS, January 2001 and BCBS, June 2004). In the latter two publications, the cornerstone of bank supervision and risk management, the CAR, given by

$$\text{Capital Adequacy Ratio (CAR)} = \frac{\text{Eligible Regulatory Capital (ERC)}}{\text{Total Risk-Weighted Assets (TRWAs)}} \quad (1)$$

is a subject of much discussion. The main question in our study is related to the capital adequacy issue and is stated below.

*Can the dynamics of the CAR of internationally active banks be formally expressed ?*

The main novelty of this paper is the determining of a stochastic model for the dynamics of the capital adequacy ratio in continuous-time. Prior to this, stochastic models for the ERC and TRWAs from credit, operational and market risks are constructed.

## 2. ELIGIBLE REGULATORY CAPITAL

A bank's available capital comprises share capital reserves and a series of hybrid capital instruments that can be categorized as Tier 1, 2 and 3 capital as stipulated in BCBS (June 2004) (see, also

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Diamond and Raja (2000)). **Tier 1 (T1) capital** consist of ordinary share capital (or equity) of the bank and audited revenue reserves, e.g., retained earnings less current year's losses, future tax benefits and intangible assets, e.g., goodwill. Tier 1 capital or core capital acts as a buffer against losses without a bank being required to cease trading, e.g., ordinary share capital. **Tier 2 (T2) capital** includes unaudited retained earnings; revaluation reserves; general provisions for bad debts; perpetual cumulative preference shares (i.e., preference shares with no maturity date whose dividends accrue for future payment even if the bank's financial condition does not support immediate payment) and perpetual subordinated debt (i.e., debt with no maturity date which ranks in priority behind all creditors except shareholders). Tier 2 capital or supplementary capital can absorb losses in the event of a wind-up and so provides a lesser degree of protection to depositors, e.g., long term subordinated debt. **Tier 3 (T3) capital** consists of subordinated debt with a term of at least 5 years and redeemable preference shares which may not be redeemed for at least 5 years. T3 capital can be used to provide a hedge against losses caused by market risks if T1 and T2 capital are insufficient for this.

In the sequel, we define the stochastic system for the eligible regulatory capital described above. The dynamics of the three types of tier capital may be represented as

$$dc_{T1}(t) = c_{T1}(t) \quad (2)$$

$$\left[ (r_{T1}(t) + u_{T1}(t))dt + \sum_{j=1}^{m_{w1}} M_{T1,j}dw_{1,j}(t) \right],$$

with  $c_{T1}(t_0) = c_{T1,0}$ ,

$$dc_{T2}(t) = c_{T2}(t)$$

$$\left[ r_{T2}(t)dt + \sum_{j=1}^{m_{w2}} M_{T2,j}dw_{2,j}(t) \right],$$

with  $c_{T2}(t_0) = c_{T2,0}$ ,

$$dc_{T3}(t) = c_{T3}(t)$$

$$\left[ r_{T3}(t)dt + \sum_{j=1}^{m_{w3}} M_{T3,j}dw_{3,j}(t) \right],$$

where  $c_{T3}(t_0) = c_{T3,0}$ , and

$$dc_d(t) = 0, \quad c_d(t_0) = c_{d,0}.$$

Here we have that

$c_{T1} : \Omega \times T \rightarrow \mathbf{R}_+$  T1 Capital,

$r_{T1} : T \rightarrow \mathbf{R}$  Net In- & Outflow Rate of T1  
Captl Except From Equity,

$u_{T1} : T \rightarrow \mathbf{R}$  Equity T1 Captl Inflow Rate,

$\sum_{j=1}^{m_{w1}} M_{T1,j}dw_{1,j}(t)$  T1 Capital Diffusion Term;

$c_{T2} : \Omega \times T \rightarrow \mathbf{R}_+$  T2 Capital,

$r_{T2} : T \rightarrow \mathbf{R}$  T2 Net In- & Outflow Rate,

$\sum_{j=1}^{m_{w2}} M_{T2,j}dw_{2,j}(t)$  T2 Capital Diffusion Term;

$c_{T3} : \mathbf{T3}$  Capital,

$r_{T3} : \mathbf{T3}$  Net In- & Outflow Rate,

$\sum_{j=1}^{m_{w3}} M_{T3,j}dw_{3,j}(t)$  T3 Capital Diffusion Term;

$c_d \in \mathbf{R}$ , Regulatory Deductions from  
Available Capital.

An interesting feature of the SDE for Tier 1 capital given by (2) is that it can actually be considered to be a control system. Here the rate of inflow of Tier 1 capital from shareholders,  $u_{T1}$ , is the control variable. In principle, the form of (2) affords us the opportunity to solve an optimal stochastic control problem that involves determining an optimal inflow rate,  $u_{T1}^*$ , of shareholder capital. To what extent shareholders are prepared to contribute to the capital inflow of a bank that may not be operating optimally or that is in danger of insolvency is always a thorny issue. This situation is the subject of a follow-up study.

The stochastic control system for the ERC can be deduced from the above models and, for  $c_{er} : \Omega \times T \rightarrow \mathbf{R}_+$ , can be expressed in the form

$$c_{er}(t) = c_{T1}(t) + c_{T2}(t) + c_{T3}(t) - c_d(t),$$

$$dc_{er}(t) = c_{er}(t) \left( \frac{c_{T1}(t)}{c_{er}(t)} \right) \quad (3)$$

$$\left[ (r_{T1}(t) + u_{T1}(t))dt + \sum_{j=1}^{m_{w1}} M_{T1,j}dw_{1,j}(t) \right]$$

$$+ \frac{c_{T2}(t)}{c_{er}(t)} \left[ r_{T2}(t)dt + \sum_{j=1}^{m_{w2}} M_{T2,j}dw_{2,j}(t) \right]$$

$$+ \frac{c_{T3}(t)}{c_{er}(t)} \left[ r_{T3}(t)dt + \sum_{j=1}^{m_{w3}} M_{T3,j}dw_{3,j}(t) \right],$$

where  $c_{er}(t_0) = c_{er,0}$ . For ease of computation, we choose to express the dynamics of the ERC,  $c_{er}$ , given in (3) in the simplified form

$$dc_{er}(t) = c_{er}(t)[r_{c_{er}}(t)dt + \sigma_{c_{er}}(t)dW_{c_{er}}(t)],$$

where

$$r_{c_{er}}(t) = \frac{c_{T1}(t)}{c_{er}(t)}(r_{T1}(t) + u_{T1}(t)) + \frac{c_{T2}(t)}{c_{er}(t)}r_{T2}(t) + \frac{c_{T3}(t)}{c_{er}(t)}r_{T3}(t).$$

and

$$\begin{aligned} \sigma_{c_{er}}(t)dW_{c_{er}}(t) &= \frac{c_{T1}(t)}{c_{er}(t)} \sum_{j=1}^{m_{w1}} M_{T1,j}dw_{1,j}(t) \\ &+ \frac{c_{T2}(t)}{c_{er}(t)} \sum_{j=1}^{m_{w2}} M_{T2,j}dw_{2,j}(t) \\ &+ \frac{c_{T3}(t)}{c_{er}(t)} \sum_{j=1}^{m_{w3}} M_{T3,j}dw_{3,j}(t). \end{aligned}$$

### 3. CREDIT RISK-WEIGHTED ASSETS

In our paper, credit risk capital is determined by using the internal ratings-based (IRB) approach. The measurement of credit risk exposures (CREs) requires that amendments be made to the value of assets displayed on a bank's balance sheet. In this regard, the different categories of loans a bank has issued are weighted according to their general degree of riskiness. Off-balance sheet contracts, such as guarantees and foreign exchange contracts, also carry credit risks. The IRB approach identifies 15 CRE types that may be listed as follows.

- $i = 1$  : Project Finance (PF);
- $i = 2$  : Object Finance (OF);
- $i = 3$  : Commodities Finance (CF);
- $i = 4$  : Income Producing Real Estate (IPRE);
- $i = 5$  : Specialized Lending High Volatility Commercial Real Estate (SLHVCRE);
- $i = 6$  : Specialized Lending Not Including High Volatility Commercial Real Estate (SLNIHVCRE);
- $i = 7$  : Bank Exposure (BE);
- $i = 8$  : Sovereign Exposure (SE);
- $i = 9$  : Retail Residential Mortgage (RRM);
- $i = 10$  : Home Equity Line of Credit (HELOC);
- $i = 11$  : Other Retail Exposure (ORE);
- $i = 12$  : Qualifying Revolving Retail Exposure (QRRE);
- $i = 13$  : Small to Medium Size Enterprises with Corporate Treatment (SMECT);
- $i = 14$  : Small to Medium Size Enterprises with Retail Treatment (SMERT);
- $i = 15$  : Equity Exposure Not Held in the

### Trading Book (EENHTB)

with  $i = 1-6$  and  $i = 9-12$  constituting corporate and retail exposures, respectively. The derivation of RWAs for these categories is dependent on estimates of the probability of default (PD), loss given default (LGD), exposure at default (EAD) and, in some cases, effective maturity (EM). In the sequel, the actual values of PD, LGD, EAD and EM are denoted by  $p_d$ ,  $l_d$ ,  $e_d$  and  $m$ , respectively. Throughout we have that

$$0 \leq p_d \leq 1, \quad 0 \leq l_d \leq 1$$

and  $e_d$  is measured in a monetary unit. Also, the unit of measurement of the effective maturity,  $m$ , is years. Furthermore, for EENHTB, we recall that the **trading book** consists of positions in financial instruments and commodities held either with trading intent or in order to hedge other elements of the trading book.

Next, we discuss the UL capital requirements for CREs that are not in default and the cases where they are. The former situation is treated by considering a risk-weighted function that provides the means by which risk components are transformed into RWAs and ultimately capital requirements.

For *CREs not in default*, seven categories of UL RWFs for calculating RWAs can be distinguished. The first component is the **weighted correlation for the exposure** given by

$$R = c_1w + c_2(1 - w), \quad (4)$$

where the **weight for the exposure**,  $w$ , is given by

$$w = \frac{1 - \exp\{Jp_d\}}{1 - \exp\{J\}}.$$

Furthermore, for SMECT and EENHTB, a **firm-size adjustment** can be made by subtracting

$$0,04 \left[ 1 - \frac{s-5}{45} \right], \quad s_1 = 5 \leq s \leq s_2 = 50,$$

from (4). The **maturity adjustment for the exposure** may be represented as

$$b = (p_A + p_B \times \ln(p_d))^2.$$

In this case, the **capital requirement for the exposure** has the form

$$\begin{aligned} & \left[ N \left[ G(p_d) \sqrt{\frac{1}{1-R}} + G(0,999) \sqrt{\frac{R}{1-R}} \right] - p_d \right] \\ & \times \left[ \frac{1 + (m-2,5)b}{1-1,5b} \right], \end{aligned}$$

where  $N(x)$  denotes the cumulative distribution function for a standard normal random variable while  $G(z)$  denotes the inverse cumulative function for a standard normal random variable. Finally, we have that the **value of the RWAs for the exposure**, is given by

$$a_c = 12,5k_e d.$$

Choices of values for the RWF parameters  $c_1$ ,  $c_2$ ,  $J$ ,  $p_A$ ,  $p_B$  and  $s$  per CRE type are made in BCBS (June 2004).

The capital requirement,  $k_i^{def}$ ,  $i = 1, \dots, 15$ , for *defaulted CREs* is subject to the following condition:

$$k_i^{def} = \max\{0, l_{d_i}^{def} - l_{e_i}^{def\epsilon}\},$$

where

$l_{d_i}^{def}$  : Value of LGD of CREs in Default;

$l_{e_i}^{def\epsilon}$  : Best Estimate of ELs for Defaulted CREs.

The *value of the RWAs for defaulted CREs* is

$$a_{c_i}^{def} = 12,5k_i^{def} e_{d_i}^{def}, \quad i = 1, \dots, 15.$$

For each defaulted asset, the bank's best estimates of expected losses are based on prevailing economic circumstances and institutional status.

#### 4. MARKET AND OPERATIONAL RISK WEIGHTS

In this section, we consider the market RWAs that are determined via the internal model approach that involves Value-at-Risk (VaR) models. We consider the capital requirement for *operational risk* from the viewpoint of the *standardized approach*.

##### 4.1 Market Risk Capital Charges

**Market risk** is defined as the risk of losses in on- and off-balance sheet positions arising from movements in market prices. Market risks include risks of losses on foreign exchange and interest rate contracts caused by changes in foreign exchange rates and interest rates.

In our paper, a version of the well-known Value-at-Risk (VaR) model is used to describe the capital charge for *market risk*. A VaR model that is used by many banks in G10 countries is

$$a_{m_p}(t) = \max[VaR(t_-) + d(t)ASR^{VaR}(t_-), M(t)\frac{1}{60}\sum_{p=1}^{60}VaR((t-p)_-) + d(t)\frac{1}{60}\sum_{p=1}^{60}ASR^{VaR}((t-p)_-)], \quad (5)$$

where

$VaR(s)$  : Value-at-Risk at Time  $s$ ;

$VaR(s_-)$  : Value-at-Risk 24-Hrs Before Time  $s$ ;

$d(t)$  : 0-1 Ind. Fn. Related to Estimation of Specific Risk Measured Through VaR Addl Spec. Risk (ASR) Measure;

$M(t)$  : Stress Factor Multiplier,  $M(t) \geq 3$ ;

$p$  : Days,  $1 \leq p \leq 60$ .

The choice of VaR formula in (5) satisfies the qualitative standards for the model approach to market risk outlined in BCBS (June 2004). We also note that (5) falls within the class of VaR models that depend on random changes in the prices of the underlying instruments, like, for instance, equity indices, interest rates, foreign exchange rates, commodity indices.

##### 4.2 Operational Risk Capital Charges

**Operational risk** is defined as the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events. For the standardized approach the activities of bank's are categorized into eight business lines, viz., corporate finance, trading and sales, retail banking, commercial banking, payment and settlement, agency services, asset management and retail brokerage. The *capital charge for each business line* is determined by multiplying the business line gross income by a weighting term known as a **beta factor**. This beta factor is an indication of the correlation between the operational risk loss experience and the aggregate level of gross income for that business line taking the whole industry into account. The total capital charge for operational risk under the standardized approach is expressed as

$$a_{occ} = \max\left[\sum_{k=1}^8 \beta_k g_k, 0\right],$$

where

$a_{occ}$  : Tot. Capital Charge for Operational Risk under the Standardized Approach;

$g_{1-8}$  : Three-Year Average of Gross Income  
for Each of Eight Business Lines;

$\beta_{1-8}$  : Fixed % Relating Level of Required  
Capital to Level of Gross Income  
for Each of Eight Business Lines.

## 5. TOTAL RISK-WEIGHTED ASSETS

According to the Basel II Capital Accord, the TRWAs of an internationally active bank are determined by multiplying the capital charges for market and operational risk by 12,5 and adding the resulting value to the sum of RWAs for credit risk. In the sequel, we denote the value of the TRWAs by  $a$ , where, for  $i = 1, \dots, 15$  and  $k = 1, \dots, 8$ , we have

$$\begin{aligned} a(t) &= \sum_{i=1}^{15} a_{c_i}^{tot}(t) + 12,5a_{occ}(t) + 12,5a_{m_p}(t) \\ &= a_c(t) + a_o(t) + a_m(t). \end{aligned} \quad (6)$$

Here we specify that

- $a$  : TRWAs for Credit, Operl & Market RWAs;
- $a_{c_i}$  : UL RWAs for  $i$ -th CRE Not In Default,  
 $a_{c_i}(t) = 12,5k_i(t)e_{d_i}(t)$ ;
- $k_i$  : UL Captl Reqt for  $i$ -th CRE Not In Default;
- $e_{d_i}$  : UL EAD for  $i$ -th CRE Not In Default;
- $a_{c_i}^{def}$  : UL RWAs for  $i$ -th CRE In Default,  
 $a_{c_i}^{def}(t) = 12,5k_i^{def}(t)e_{d_i}^{def}(t)$ ;
- $k_i^{def}$  : UL Captl Reqt for  $i$ -th Defaulted CRE;
- $a_{c_i}^{tot}$  : Value of TRWAs for  $i$ -th CRE,  
 $a_{c_i}^{tot}(t) = a_{c_i}(t) + a_{c_i}^{def}(t)$ ;
- $a_c$  : TRWAs for Credit Risk,  
 $a_c(t) = \sum_{i=1}^{15} a_{c_i}^{tot}(t)$ ;
- $a_{o_k}$  : RWAs for  $k$ -th Business Line for Operl Risk,  
 $a_{o_k}(t) = 12,5\beta_k g_k(t)$ ;
- $\beta_k$  : Fixed % Relating Level of Required Capital  
to Level of Gross Inc. for  $k$ -th Bus. Line;
- $g_k$  : 3-yr Gross Av. Inc. for  $k$ -th Bus. Line;
- $a_o$  : TRWAs for Operational Risk,  
 $a_o(t) = \sum_{k=1}^8 a_{o_k}(t)$ ;
- $a_{m_p}$  : Capital Charge for Market Risk;
- $a_m$  : TRWAs for Market Risk,  
 $a_m(t) = 12,5a_{m_p}(t)$ .

In the sequel, the stochastic process  $e : \Omega \times T \rightarrow \mathbf{R}$  is the **rate of capital outflow from RWAs** whose value at time  $t$  is denoted by  $e(t)$ . Inflows to RWAs arise from such sources as deposits, loan repayments, bank borrowing and bank capital. For the sake of our subsequent analysis, we denote the **rate of capital inflows to RWAs** by  $u(t)$ . From formula (6), we have that the bank's TRWAs consist of assets weighted for credit, operational and market risk. We denote the capital requirements for market risk by  $y_m(t)$ , while categories for credit and operational risk are denoted by  $y_{c_1}(t), \dots, y_{c_{15}}(t)$  and  $y_{o_1}(t), \dots, y_{o_8}(t)$ , respectively. In this case, we represent the stochastic dynamics of the **RWAs for market risk** by the SDE

$$dy_m(t) = y_m(t)[r_m dt + \sigma_m dW_m(t)], \quad (7)$$

where  $y_m(0) = 1$ . Also,  $r_m$ , the rate of change of the market RWAs described in (7) may be stochastic and be modelled as a one-factor diffusion process. For  $i, j = 1, \dots, 15$ , the evolution of the **credit RWAs** may be described by

$$dy_{c_i}(t) = y_{c_i}(t)[r_{c_i} dt + \sum_{j=1}^{15} \sigma_{c_{ij}} dW_{c_j}(t)], \quad (8)$$

where  $y_{c_i}(0) = y_{c_0}$  and  $r_{c_i}$  and  $\sigma_{c_{ij}}$  are positive constants. For  $k, l = 1, \dots, 8$ , we represent the dynamics of the **operational RWAs** by

$$dy_{o_k}(t) = y_{o_k}(t)[r_{o_k} dt + \sum_{l=1}^8 \sigma_{o_{kl}} dW_{o_l}(t)], \quad (9)$$

where  $y_{o_k}(0) = y_{o_0}$  and  $r_{o_k}$  and  $\sigma_{o_{kl}}$  are positive constants. In this case, the vector

$$\begin{aligned} &(W_m(t), W_{c_1}(t), \dots, \\ &W_{c_{15}}(t), W_{o_1}(t), \dots, W_{o_8}(t))^T \end{aligned}$$

is an 24-dimensional Brownian motion defined on the probability space  $(\Omega, \mathcal{G}, \mathbf{P})$ , where  $\{\mathcal{G}_t\}_{t \geq 0}$  represents the completion of the filtration

$$\sigma\{(W_m(s), W_{c_1}(s), \dots, W_{c_{15}}(s), W_{o_1}(s), \dots, W_{o_8}(s))^T : 0 \leq s \leq t\}.$$

From formula (6), the *value of the RWAs for market risks* is given by

$$a(t) - \sum_{i=1}^{15} a_{c_i}^{tot}(t) - \sum_{k=1}^8 a_{o_k}(t).$$

**Proposition 1. (TRWA Dynamics of an Internationally Active Bank)** Suppose that the changes in the value of the bank's TRWAs is solely

determined by the changes in capital requirements for credit, operational and market risk and the rate of inflows to and outflows from RWAs. Then the dynamics of the *value of the bank's TRWAs* may be represented as

$$\begin{aligned}
da(t) = & \left( r_m a(t) + \sum_{i=1}^{15} a_{c_i}^{tot}(t)(r_{c_i} - r_m) \right. \\
& + \sum_{k=1}^8 a_{o_k}(t)(r_{o_k} - r_m) + u(t) - e(t) \Big) dt \\
& + \sum_{i=1}^{15} \sum_{j=1}^{15} a_{c_i}^{tot}(t) \sigma_{c_{ij}} dW_{c_j}(t) \\
& + \sum_{k=1}^8 \sum_{l=1}^8 a_{o_k}(t) \sigma_{o_{kl}} dW_{o_l}(t) \\
& + a_m(t) \sigma_m dW_m(t), \tag{10}
\end{aligned}$$

with initial condition  $a(0) = a_0$ .

**PROOF.** The proof depends on the observation that the dynamics of the bank's TRWAs follows directly from equations (7), (8) and (9).

## 6. CAPITAL ADEQUACY RATIO

We determine the **capital adequacy ratio** denoted by  $x(t)$ , by using (1). The Basel II Capital Accord recommends minimum CARs to ensure that banks can absorb a reasonable level of losses before becoming insolvent (see also von Thadden (2004) and references therein). A bank is expected to compute the value of the CAR and to report it to the national supervisory organization. In the case where the CAR drops below 8 %, the national supervisory organization (depending on the local legal rules) can order the bank to take action which ultimately can include the closure of the bank. Applying minimum capital adequacy standards serves to protect depositors and promote the stability and efficiency of the bank.

### 6.1 Stochastic Modelling of CARs for Active Banks

In this subsection, we define a stochastic model for the CAR dynamics of an internationally active bank.

**Theorem 2. (Capital Adequacy Ratio of an Internationally Active Bank)** Suppose that the ERC and the credit, operational and market RWAs are as given above. A system that describes the stochastic dynamics of the CAR of an internationally active bank may be represented by the stochastic differential equation

$$dx(t) = x(t)[\mu(t)dt + \sigma(t)dW(t)], \tag{11}$$

where  $x(t_0) = x(0)$ ,

$$\begin{aligned}
\mu(t) = & -r_m - a^{-1}(t) \sum_{i=1}^{15} a_{c_i}^{tot}(t)(r_{c_i} - r_m) + r_{c_{er}} \\
& - a^{-1}(t) \sum_{k=1}^8 a_{o_k}(t)(r_{o_k} - r_m) \\
& - a^{-1}(t)[u(t) - e(t)] \\
& + a^{-2}(t) \sum_{i=1}^{15} \sum_{j=1}^{15} \sum_{k=1}^{15} (a_{c_i}^{tot} a_{c_j}^{tot})(\sigma_{c_{ij}} \sigma_{c_{jk}}) \\
& + a^{-2}(t) \sum_{k=1}^8 \sum_{l=1}^8 \sum_{m=1}^8 (a_{o_k} a_{o_l})(\sigma_{o_{kl}} \sigma_{o_{lm}}) \\
& + a^{-2}(t) a_m(t)^2 \sigma_m^2
\end{aligned}$$

and

$$\begin{aligned}
\sigma(t)dW(t) = & -a^{-1}(t) \sum_{i=1}^{15} \sum_{j=1}^{15} a_{c_i}^{tot}(t) \sigma_{c_{ij}} dW_{c_j}(t) \\
& - a^{-1}(t) \sum_{k=1}^8 \sum_{l=1}^8 a_{o_k}(t) \sigma_{o_{kl}} dW_{o_l}(t) \\
& + \sigma_{c_{er}} dW_{c_{er}}(t) \\
& - a^{-1}(t) a_m(t) \sigma_m dW_m(t).
\end{aligned}$$

**PROOF.** In this proof we derive (11) by mainly using the general Ito formula. In fact, the said formula is useful in calculating both the dynamics of the inverse of the TRWAs,  $da^{-1}(t)$ , and the CAR,  $dx(t)$ .

If banks apply control to their credit (lending) operations in such a way that the CAR remains high they will remain out of the zone in which insolvency may be a possibility.

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