

# IDENTIFICATION OF PHYSICAL PARAMETERS OF A PNEUMATIC SERVOSYSTEM

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Abstract: This paper deals with the estimation of physical parameters of a pneumatic servo positioning system. It consists of two servodistributors and an actuator. This work concerns not only the identification of the mass, viscous and Coulomb non symmetric frictions parameters of the mechanical part but also the polytropic coefficient of the gas considered as a perfect one. At first the mass flow rate characteristic of the servodistributor is determined indirectly by an approximation using several polynomial functions. Next, a dynamic model which is linear in relation to a set of dynamic parameters is proposed. The dynamic parameters are estimated using the weighted least squares solution of an over determined linear system obtained from the sampling of the dynamic model along a closed loop tracking trajectory. An experimental study exhibits good identification results. *Copyright © 2005 IFAC*

Keywords: closed loop identification, pneumatic system, physical parameters, polynomial interpolation, weighted least squares

## 1. INTRODUCTION

Because of increasing performances of pneumatic servo positioning systems, accurate models are needed to check their performances (accuracy and rapidity) by simulation and to improve their design and their control (Isermann, 1996)(Janiszowski, 2004). Some investigations in modelling of pneumatic actuators use linear models or a multi-model structure (Schulte and Hahn, 2001). The authors in (Liu and Bobrow, 1988) have worked on a linearized state space model to implement an optimal regulator for a fixed operating point. These models are well adapted if efforts are made in developing robust control strategy to address the difficulties in modelling pneumatic actuators.

The main difficulties in modelling pneumatic actuators are their highly nonlinear behaviors (Mc Cloy, 1968)(Shearer, 1956)(Blackburn *et al.*, 1960). These ones are associated with the nonlinear dynamic properties of pneumatic systems such as servodistributor flow characteristics, the thermodynamic properties of air compressed in a cylinder and the nonlinear friction between the contacting surfaces of the slider-piston system. The majority of papers in the field of research on identification of physical parameters of pneumatic servo positioning systems focuses on either the identification of mechanical parameters such as frictions or the mass to be moved (Uebing and Vaughan, 2001)(Wang *et al.*, 2004) either the study of model of the flow stage delivered by a servodistributor (Belgharbi *et al.*, 1999)(Richard and Scavarda, 1996). In (Nouri *et al.*, 2000), the

most essential elements of a pneumatic servo positioning system have been modelled/identified separately without constructing an overall simulation model in order to establish conformity behaviour with practice. An identification scheme of pneumatic and mechanical parts is proposed in (Zorlu *et al.*, 2003). But this approach is based on a good compensation of dry and static frictions to obtain a linearized model.

This paper deals with the estimation of physical parameters of a pneumatic servo positioning system. This work concerns not only the identification of mechanical parameters but also pneumatic parameters. At first the mass flow rate characteristic of the servodistributor is determined indirectly by an approximation using several polynomial functions. Next, a dynamic model which is linear in relation to a set of dynamic parameters is proposed. The dynamic parameters are estimated using the weighted least squares solution of an over determined linear system obtained from the sampling of the dynamic model along a closed loop tracking trajectory. An experimental study exhibits identification results.

This paper is organized as follows: Section 2 deals with the modelling of the pneumatic servosystem. Section 3 is devoted to the approximation of the servodistributor flow stage characteristics. Section 4 presents the method of identification by weighted least squares and the practical aspects of the method in term of data acquisition and filtering. Section 5 is dedicated to the experimental results of a closed loop identification.

## 2. MODELLING

### 2.1 Dynamic model

The electropneumatic system (figure 1) uses the following structure: two three-way proportional servodistributors/ actuator/ mass in translation. The actuator under consideration is an in-line electropneumatic cylinder using a simple rod. The electropneumatic system model can be obtained using three physical laws: the mass flow rate through a restriction, the pressure behavior in a chamber with variable volume and the fundamental mechanical equation.

The two servodistributors are supposed to be identical. This component is a pneumatic flow valve and consists of a matching spool-sleeve assembly and a proportional magnet directly controlling the movement of the spool against a spring. The spool is controlled in position by means of a position sensor. On the contrary of many other valve designs used in automotive or railway applications or in pneumatic circuits, the spool-sleeve technology has been preferred to the

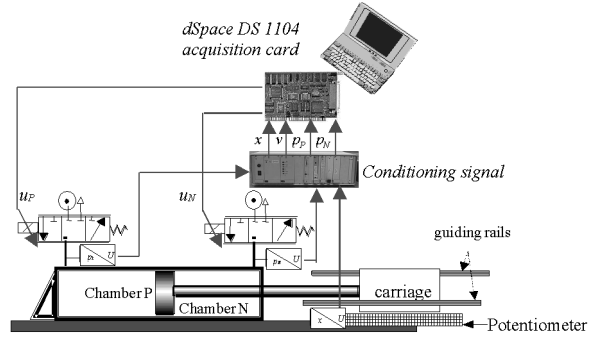


Fig. 1. The electropneumatic system

poppet technology. This means that pressure accuracy around zero opening has been set to the detriment of leakage. So this technology leads to characteristics without dead zone. In our case, the bandwidth of the Servotronic Joucomatic servodistributor and the actuator are respectively about 200 Hz and 1.5 Hz. Using the singular perturbation theory, the dynamics of the servodistributors are neglected and their models can be reduced to a static one described by two relationships  $q_{mP}(u_P, p_P)$  and  $q_{mN}(u_N, p_N)$  between the mass flow rates  $q_{mP}$  and  $q_{mN}$ , the input voltages  $u_P$  and  $u_N$ , and the output pressures  $p_P$  and  $p_N$ . The pressure and temperature evolution laws in a chamber with variable volume are obtained assuming the following assumptions (Shearer, 1956): air is a perfect gas and its kinetic energy is negligible, the pressure and the temperature are homogeneous in each chamber. Moreover the process is supposed to be polytropic and characterized by the coefficient  $k_{(P \text{ or } N)}$ . The following electropneumatic system model is obtained:

$$\begin{cases} \dot{p}_P = \frac{k_P r T_P}{V_P(x)} \left( q_{mP}(u_P, p_P) - \frac{S_P}{r T_P} p_P \dot{x} \right) \\ \dot{p}_N = \frac{k_N r T_N}{V_N(x)} \left( q_{mN}(u_N, p_N) + \frac{S_N}{r T_N} p_N \dot{x} \right) \\ M \ddot{x} = S_P p_P - S_N p_N - F_{ext} - F_f(\dot{x}) \end{cases} \quad (1)$$

Where:

$$F_{ext} = (S_P - S_N) p_{ext} \quad (2)$$

And:

$$\begin{cases} V_P(x) = V_P(0) + S_P x \\ V_N(x) = V_N(0) - S_N x \end{cases} \quad \text{With : } \begin{cases} V_P(0) = V_{DP} + S_P \frac{l}{2} \\ V_N(0) = V_{DN} + S_N \frac{l}{2} \end{cases} \quad (3)$$

being volumes of the chambers for the zero position and  $V_{D(P \text{ or } N)}$  the dead volumes present on each extremities of the cylinder. The term  $F_f(\dot{x})$  in (1) represents all the friction forces which act on the moving part. It was observed during experimental testing that Coulomb friction depends on the direction of motion. Thus the function  $F_f(\dot{x})$  is defined by:

$$F_f(\dot{x}) = \begin{cases} f_v \dot{x} + F_c^+ & \text{if } \dot{x} > 0 \\ f_v \dot{x} - F_c^- & \text{if } \dot{x} < 0 \\ 0 & \text{if } \dot{x} = 0 \end{cases} \quad (4)$$

## 2.2 Identification model

The dynamic model (1) can be written in a relation linear to the dynamic parameters as follows:

$$y = D_s X_s \quad (5)$$

With:

$$y = \begin{bmatrix} q_{mP}(u_P, p_P) \\ q_{mN}(u_N, p_N) \\ S_{PP} - S_{NN} - (S_P - S_N) p_{ext} \end{bmatrix} \quad (6)$$

And:

$$D_s = \begin{bmatrix} \dot{p}_P & x\dot{p}_P & \dot{x}p_P & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \dot{p}_N & -x\dot{p}_N & -\dot{x}p_N & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \ddot{x} & \dot{x} & f(\dot{x}) & -g(\dot{x}) \end{bmatrix} \quad (7)$$

The functions  $f$  and  $g$  in (7) are defined by:

$$\begin{cases} f(\dot{x}) = \frac{1 + \text{sign}(\dot{x})}{2} \\ g(\dot{x}) = \frac{1 - \text{sign}(\dot{x})}{2} \end{cases} \quad (8)$$

The vector of unknown parameters  $X_s$  is:

$$X_s^T = (X_{sP}^T \ X_{sN}^T \ M \ f_v \ F_c^+ \ F_c^-) \quad (9)$$

Where  $X_{sP}$  and  $X_{sN}$  are respectively the parameters describing the pressure evolution laws in the chambers P and N:

$$X_{sP}^T = \left( \frac{V_P(0)}{k_P r T_P} \ \frac{S_P}{k_P r T_P} \ \frac{S_P}{r T_P} \right) = (X_s(1) \ X_s(2) \ X_s(3)) \quad (10)$$

$$X_{sN}^T = \left( \frac{V_N(0)}{k_N r T_N} \ \frac{S_N}{k_N r T_N} \ \frac{S_N}{r T_N} \right) = (X_s(4) \ X_s(5) \ X_s(6)) \quad (11)$$

## 3. APPROXIMATION OF THE SERVODISTRIBUTOR FLOW STAGE CHARACTERISTICS

The main difficulty for model (5) is to know the mass flow rates  $q_{mP}(u_P, p_P)$  and  $q_{mN}(u_N, p_N)$ . In order to establish a mathematical model of the power modulator flow stage, many works present approximations based on physical laws (Araki, 1981)(Mo, 1989) by the modelling of the geometrical variations of the restriction areas of the servodistributor as well as by the experimental local characterization (Richard and Scavarda, 1996). These methods are based on approximations of fluid flow through a convergent nozzle in turbulent regime, corrected by a coefficient  $C_q$  (Mc Cloy and Martin, 1980) or on the the norm ISO 6358.

In this paper, we propose to use the results of the global experimental method giving the static characteristics of the flow stage (Sesmat and Scavarda, 1996). The global characterization (figure 2) corresponds to the static measurement of the output mass flow rate  $q_m$  depending on the input control  $u$  and the output pressure  $p$  for a constant source pressure. Figure 2 clearly shows the nonlinear behaviour of the flow rate evolution according to the pressure and the input control. The global characterization has the advantage of obtaining simply, by projection of the characteristics series  $q_m(u, p)$  on different planes:

- the mass flow rate characteristics series (plane "p - q<sub>m</sub>"),
- the mass flow rate gain characteristics series (plane "u - q<sub>m</sub>"),
- the pressure gain characteristics series (plane "u - p").

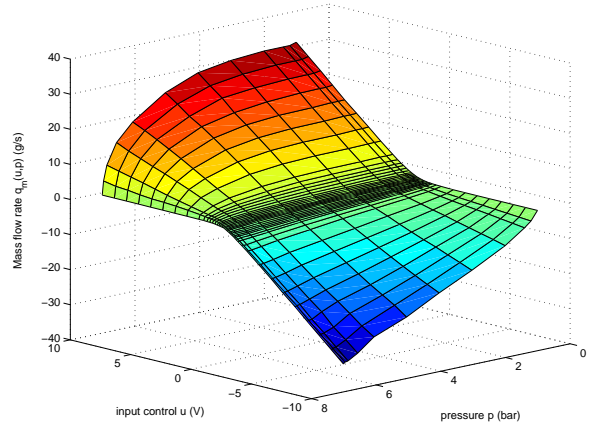


Fig. 2. Global static characteristics

The authors in (Belgharbi *et al.*, 1999) have developed analytical models for both simulation and control purposes. Two cases have been studied to approximate the flow stage characteristics by polynomial functions:

- a multivariable polynomial function,
- a polynomial approximation affine in control such as:

$$q_m(u, p) = \varphi(p) + \psi(p, \text{sign}(u))u \quad (12)$$

In this paper we will use the second approximation because it allows to give a physical significance to the polynomial functions.  $\varphi(p)$  in (12) is a polynomial function whose evolution corresponds to the mass flow rate leakage, it is identical for all input control value  $u$ .  $\psi(p, \text{sign}(u))$  is a polynomial function whose evolution is similar to the one described by the methods based on approximations of mass flow rate through a convergent nozzle in turbulent regime (Mc Cloy and Martin, 1980). It is a function of the input control  $\text{sign}$  because the behaviour of the mass flow rate char-

acteristics is clearly different for the inlet ( $u > 0$ ) and the exhaust ( $u < 0$ ). For a discussion and more details on the choice of functions and their degrees please refer to (Belgharbi *et al.*, 1999). Figure 3 shows the mass flow rate error between the analytical model and the measurements when the polynomial functions  $\varphi(p)$ ,  $\psi(p, u > 0)$  and  $\psi(p, u < 0)$  have respectively degrees equal to seven, seven and four.

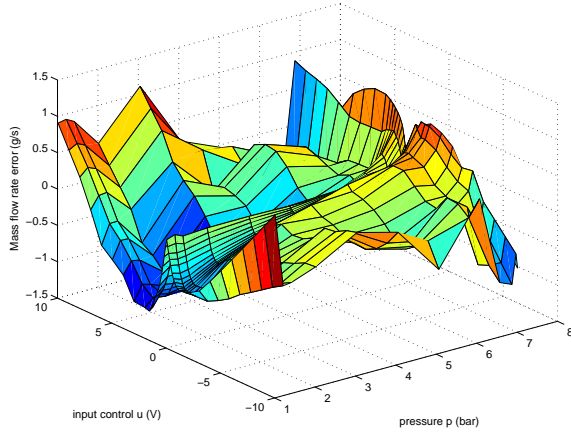


Fig. 3. Mass flow rate error

The error fitted in figure 3 describes a polynomial approximation which fits the actual data extremely closely. This approximation is used to estimate the mass flow rates  $q_{mP}(u_P, p_P)$  and  $q_{mN}(u_N, p_N)$  in (6).

## 4. IDENTIFICATION METHOD

### 4.1 Weighted Least Squares

The vector  $X_s$  is estimated as the solution of the Weighted Least Squares (WLS) of an over determined system obtained from the sampling at the various moments  $t_i$ ,  $i = 1, \dots, r = ne$  of the system (5)(Canudas de Wit *et al.*, 1996):

$$Y = WX_s + \rho \quad (13)$$

where:  $W$  is a ( $r \times Np$ ) observation matrix, which is a sampling of the regressor (7),  $Y$  is a ( $r \times 1$ ) vector which is a sampling of (6),  $\rho$  is a ( $r \times 1$ ) vector of errors due to model error and noise measurements,  $r > Np$  is the number of equations.

The W.L.S. solution minimizes the 2 norm  $\|\rho\|$  of the vector of errors  $\rho$ . The unicity of the W.L.S. solution depends on the rank of the observation matrix  $W$ . The rank deficiency of  $W$  can come from two origins:

- structural rank deficiency which stands for any samples of  $(\dot{x}, \ddot{x}, p_P, p_N, \dot{p}_P, \dot{p}_N)$  in (7). This is the structural parameters identifiability problem

which is solved using base (or minimal) parameters (Gautier, 1991).

- data rank deficiency due to a bad choice of the trajectory  $(\dot{x}, \ddot{x}, p_P, p_N, \dot{p}_P, \dot{p}_N)$  which is sampled in  $W$ . This is the problem of optimal measurement strategies which is solved using closed loop identification to track exciting trajectories (Gautier and Khalil, 1992).

Calculating the W.L.S. solution of (13) from noisy discrete measurements or estimations of derivatives, may lead to bias because  $W$  and  $Y$  may be non independent random matrices. Then it is essential to filter data in  $Y$  and  $W$ , before computing the W.L.S. solution.

### 4.2 Filtering aspects

The derivatives in (13) are obtained without phase shift using a central difference algorithm. A low-pass filter without phase shift and without magnitude distortion into the bandwidth is applied on the measurements to reduce the noise. This lowpass filter is easily obtained with a non causal zero-phase digital filtering by processing the input data through an IIR lowpass Butterworth filter in both the forward and reverse direction using a 'filtfilt' procedure from Matlab (Pham *et al.*, 2001). The cut-off frequency  $\omega_H$  of the lowpass filter should be chosen to avoid any distortion of magnitude on the filtered signals into the bandwidth of the system. A second filter is implemented to eliminate the high frequencies noises. The vector  $Y$  and each column of  $W$  are filtered (parallel filtering) by a lowpass filter and are resampled at a lower rate. This step is not sensitive to filter distortion because error introduced by this filtering process is the same in each member of the linear system (13). The key point of this identification method is to choose the cut-off frequency  $\omega_H$  and the sampling frequency  $\omega_s$  to keep useful signal of the dynamic behavior of the system in the filter bandwidth. In (Gautier, 1996), the author proposes to choose the sampling frequency  $\omega_s$  of measurements in practice, if possible, such as :

$$\omega_s \geq 100\omega_{dyn} \quad (14)$$

Where  $\omega_{dyn}$  is the bandwidth of the position closed loop. A strategy of tuning for the frequency  $\omega_H$  and the sampling frequency  $\omega_s$  is presented in (Pham *et al.*, 2001). This method suggests to bound the distortion of amplitude introduced by the derivative filter and the lowpass filter at a frequency fixed with regard to the dynamics of the system.

## 5. EXPERIMENTAL IDENTIFICATION

An experimental identification is performed on the testing bed. The sampling frequency for the acquisition of measurements is equal to  $5kHz$  in order to satisfy the relation (14). A closed loop identification, using a proportional feedback control, has been performed. A chirp sweeping between 0 Hz and 2 Hz in order to excite the system close to its *a priori* natural frequency which is estimated around 1.5 Hz. Several square trajectories for the desired position with different amplitudes are used to excite the friction parameters. The results of the experimental identification are reported in the table 1. The estimated parameters are given with their confidence interval and their relative standard deviation. Standard deviations  $\sigma_{\hat{X}_{s,i}}$  are estimated using classical and simple results from statistics, considering the matrix  $W$  to be a deterministic one, and  $\rho$  to be a zero mean additive independent noise, with standard deviation  $\sigma_\rho$  such like:

$$C_{\rho\rho} = \sigma_\rho^2 I_{r \times r} \quad (15)$$

Where  $I_{r \times r}$  is the matrix identity ( $r \times r$ ). The covariance matrix of the estimation error and standard deviations can be calculated by:

$$C_{\hat{X}_s \hat{X}_s} = \sigma_\rho^2 [W^T W]^{-1} \quad (16)$$

$\sigma_{\hat{X}_{s,i}}^2 = C_{\hat{X}_s \hat{X}_s, ii}$ , is the  $i^{\text{th}}$  diagonal coefficient of  $C_{\hat{X}_s \hat{X}_s}$ . The relative standard deviation  $\% \sigma_{\hat{X}_{s,r}}$  is given by:

$$\% \sigma_{\hat{X}_{s,r}} = 100 \frac{\sigma_{\hat{X}_{s,i}}}{\hat{X}_{s,i}} \quad (17)$$

A parameter with  $\% \sigma_{\hat{X}_{s,r}} \geq 10\%$  can be removed from the model because it is not identifiable on the given trajectory and it poorly increases the relative error norm.

Table 1. Identification results

Parameters (ISO units)	$\hat{X}_s$	$2\sigma_{\hat{X}_s}$	$\% \sigma_{\hat{X}_{s,r}}$
$X_s(1)$	3.49e-009	8.62e-012	0.1234
$X_s(2)$	9.95e-009	6.41e-011	0.3223
$X_s(3)$	1.04e-008	2.74e-011	0.1278
$X_s(4)$	1.78e-009	2.74e-011	0.7695
$X_s(5)$	7.00e-009	2.89e-010	2.0603
$X_s(6)$	7.17e-009	8.30e-011	0.5784
$M$	1.69e+001	7.40e-002	0.2187
$f_v$	1.10e+001	8.75e-001	3.9679
$F_c^+$	1.03e+001	4.80e-001	2.3245
$F_c^-$	2.38e+001	4.55e-001	0.9565

From the table 1, we notice that the dynamic parameters present a very small relative standard deviation, which translates the good identification of these parameters. Besides the moving

mass  $M$  is close to the manufacturer data (17 kg). In general, concerning the pneumatic part, it is assumed that the polytropic coefficient  $k$  lies between 1 (isothermal evolution) and 1.4 (adiabatic evolution). It is noteworthy that the ratios  $X_s(3)/X_s(2) = 1.04$  and  $X_s(6)/X_s(5) = 1.02$  give a non classical result. They correspond to the polytropic coefficients of the gas  $k_P$  and  $k_N$  in each chamber.

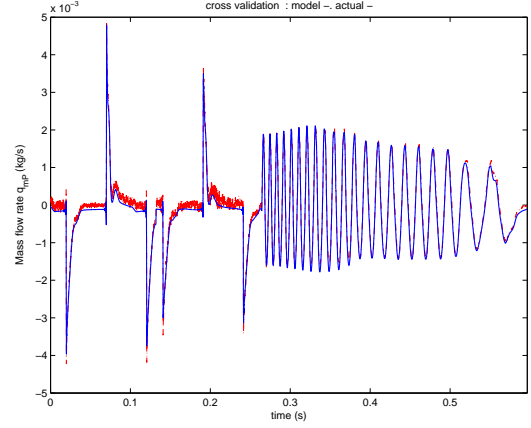


Fig. 4. Mass flow rate  $q_m$

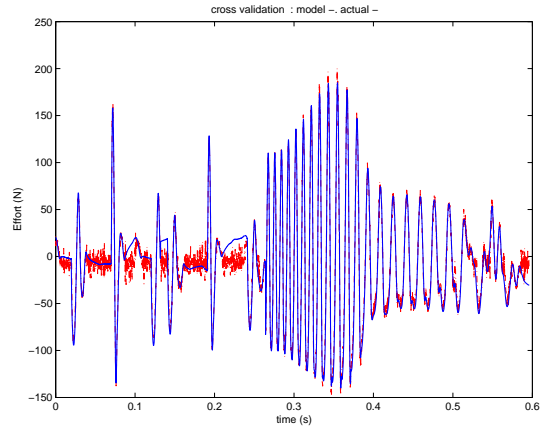


Fig. 5. Effort :  $S_{PPP} - S_{NPN} - F_{ext}$

A cross-validation of the identification is performed to test the model. It consists in comparing the estimations of the mass flow rates and the effort of the model with experimental signals which had not been used in the identification process. On figures 4 and 5, we present a comparison between the simulated and the actual mass flow rate and effort. These figures show that the simulation and the measurements are very close, this means a good identification of the parameters for the testing bed.

## 6. CONCLUSION

This paper deals with the estimation of physical parameters of a pneumatic servo positioning system. At first the mass flow rate characteristic of

the servodistributor is determined indirectly by a polynomial approximation. Next, the dynamic parameters are estimated using the weighted least squares solution of an over determined linear system obtained from the sampling of the dynamic model along a closed loop tracking trajectory. An first experimental study exhibits good identification results. Nevertheless, these results should be viewed with some degree of reservation because more investigations should be address in order to check the sensivity of identified parameters in relation to the static mass flow rate approximation.

## NOMENCLATURE

$f_v$  : viscous friction coefficient (N/m/s)  
 $k$  : polytropic coefficient  
 $M$  : total load mass (kg)  
 $p$  : pressure in the cylinder chamber (Pa)  
 $q_m$  : mass flow rate provided from servodistributor to cylinder chamber (kg/s)  
 $r$  : perfect gas constant related to unit mass (J/kg/K)  
 $S$  : area of the piston cylinder (m<sup>2</sup>)  
 $T$  : temperature (K)  
 $V$  : volume (m<sup>3</sup>)  
 $x, \dot{x}, \ddot{x}$  : position (m), velocity (m/s), acceleration(m/s<sup>2</sup>)  
 $u$  : spool position (v)  
 $\omega$  : pulsation (rad/s)  
 $\varphi(\cdot)$  : leakage polynomial function (kg/s)  
 $\psi(\cdot)$  : polynomial function (kg/s/V)  
 $l$  : length of stroke (m)

## Subscript

$c$  Coulomb friction,  $D$  dead volume,  $ext$  external,  $f$  friction,  $N$  chamber N,  $P$  chamber P

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