PARAMETRIC IDENTIFICATION OF STATIC NONLINEARITIES IN A GENERAL INTERCONNECTED SYSTEM

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Abstract: We are concerned with the identification of static nonlinear maps in a structured interconnected system. Structural information is often neglected in nonlinear system identification methods. In this paper, we exploit *a priori* structural information and use parametric identification methods. We focus on the case where the linear part of the interconnection is known and only the static nonlinear components require identification. We propose an identification algorithm and investigate its convergence properties.

Keywords: system identification, nonlinear systems, structured systems, convergence, parametric nonlinearities

1. INTRODUCTION

This paper is concerned with identification problems in interconnected nonlinear systems. These problems are of considerable importance in the context of control, simulation, and design of complex systems.

There is available limited past work on the identification of such systems on a case-by-case basis. These include studies of Hammerstein and Wiener systems (Billings and Fakhouri, 1978),(Narendra and Gallman, 1966),(Pawlak, 1991). However, many of the simplest problems here remain open. For instance, the systematic inclusion of *a priori* structural information has been limited by the lack of a paradigm that is sufficiently general to incorporate such information.

We believe that the development of generalizations such as linear fractional transformations (LFT's) in the control systems literature (Packard and Doyle, 1993),(Safonov, 1982), together with the advent of powerful, inexpensive computational resources offer the promise of significant advances in system identification for complex nonlinear systems.

In general, both first principle laws and black-box model selection procedures result in only an approximate modeling of the involved phenomena. For example, the identification procedure may be subject to incorrect *a priori* information which can counteract the positive effects of correct known information. Evaluating the overall balance of these two effects on the identification error is a largely open problem for nonlinear systems. These considerations motivate the need for identification methods to incorporate known structural information that is to a large extent considered to be correct. We offer a systematic framework based on linear fractional transformations to incorporate known structural information about the interconnected system. While there is occasional work that incorporates *a priori* structural information (Narendra and Gallman, 1966),(Stoica, 1981),(Vandersteen and Schoukens,

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1997),(Milanese and Novara, 2003), this is not commonly the case.

In this paper, we present an algorithm for the identification of static nonlinear components in interconnected systems. In particular, we are concerned with problems in which the static nonlinear elements to be identified are *parametric*. We prove that the estimated nonlinearity converges asymptotically with probability 1. The proof of convergence follows the presentation in Chapter 8 of (Ljung, 1999). We assume that the linear components of the interconnection are known. Note that the Hammerstein and Wiener systems are special cases of our formulation and under this assumption, the identification of these systems becomes trivial. However, the class of problems we consider involves more complex interconnections that cannot be captured by the Hammerstein and Wiener formulations.

The remainder of this paper is organized as follows. In Section 2, we define the class of model structures under consideration. In Section 3, we motivate and present our identification algorithm. Section 4 contains our main convergence result. Section 5 provides conditions under which our estimate converges to the true nonlinearity. In Section 6, we offer illustrative examples. The proofs of our main results may be found online at `http://jagger.me.berkeley.edu/~ken` or by contacting the authors.

NOTATION

$\mathbb{R}^n$	standard Euclidean space
$u, y, w, ...$	vector-valued discrete-time signals (finite or infinite)
L	length of data record
y_t	value of signal y at time t
$\{u_t, y_t\}_{t=0}^{L-1}$	finite horizon input-output data
e	noise signal
LTI	linear time-invariant
$\mathcal{L}$	linear time-invariant operator
$\mathcal{N}$	static nonlinear operator
$\{\phi^{[k]}(\cdot)\}_{k=1}^{N}$	set of nonlinear basis functions
N	number of basis functions
Θ	compact subset of $\mathbb{R}^N$
$\theta \in \Theta$	vector of parameters to be identified
$\theta^{true} \in \Theta$	"true" vector of parameters
$\hat{\theta}^L$	estimate of θ based on first L samples

2. PROBLEM FORMULATION

We are concerned with the identification of static nonlinear maps in general structured interconnected systems. An example of the class of structured systems we consider is shown in Figure 1. Here, the static nonlinearities $\mathcal{N}_1$ and $\mathcal{N}_2$ are to be identified. The possibly unstable LTI systems $\mathcal{L}_1$ and $\mathcal{L}_2$ are known. We have access to the noisy input-output data $\{u_t, y_t\}_{t=0}^{L-1}$ and e is a noise signal.

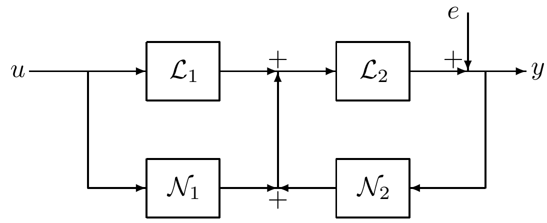

Fig. 1. Example of structured interconnected system

Any general interconnected nonlinear system may be represented through a linear fractional transformation (LFT) framework as shown in Figure 2. The LFT framework allows us to separate the LTI dynamics from the static nonlinearities in an interconnected system. The signals u, y are measured, and the signals z, w will denote the inputs and outputs of the static nonlinear block $\mathcal{N}$, respectively. The signal e is a zero-mean gaussian white noise process. The input signal u is assumed to be uncorrelated with the noise signal e.

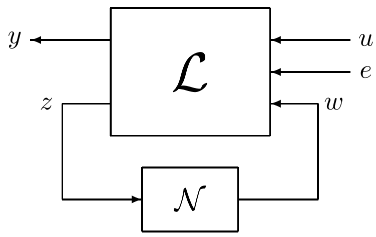

Fig. 2. LFT Model Structure

We gather all the nonlinearities of the interconnection into the multi-input multi-output block $\mathcal{N}$, which is to be identified. In general, the static nonlinear block $\mathcal{N}$ has block diagonal structure (Claassen, 2001).

$$\mathcal{N} = \begin{bmatrix} \mathcal{N}^{[1]} & & \\ & \ddots & \\ & & \mathcal{N}^{[m]} \end{bmatrix}$$

We partition the inputs z and outputs w of $\mathcal{N}$ conformably with its structure. The nonlinear block $\mathcal{N}$ may also have repeated components. This situation arises when a particular nonlinearity appears more than once in the dynamical equations describing the interconnected system.

The frequently studied Hammerstein and Wiener systems are special cases of our formulation. However, under our assumption that the linear components of the interconnection are known, the identification of these classes of systems becomes trivial. Indeed, it is important to note that the class of problems we wish to identify involve *complex* interconnections. For example, consider the system depicted in Figure 3.

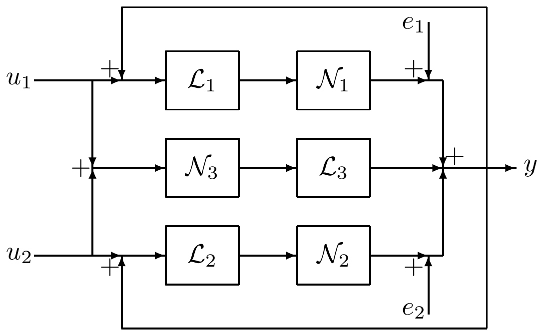

Fig. 3. Complexity in an interconnected system.

Here, the feedback interconnection along with the presence of several multivariable linear and nonlinear blocks suggests the complexity of the system. In addition, the measured signal y contains the sum of the outputs of several nonlinearities and other signals. In order to develop a systematic approach for the identification of these systems, we use the LFT to collect all such systems under a common framework for analysis.

We will refer to the interconnected system of Figure 2 as the *LFT Model Structure*. We assume that the LTI block $\mathcal{L}$ and the dimensions of all signals are known. The components of the nonlinear block $\mathcal{N}$ are to be identified. For this, we have available measured (bounded) input-output data $\{u_t, y_t\}_{t=0}^{L-1}$. We assume that the nonlinear block $\mathcal{N}$ can be parameterized by a finite set of nonlinear basis functions. By this, we mean that the input-output behavior of $\mathcal{N}$ can be described as

$$w = \mathcal{N}(z) = \sum_{k=1}^{N} \theta_k \phi^{[k]}(z), \quad \theta_k \in \mathbb{R}, \tag{1}$$

where $\{\phi^{[k]}(\cdot)\}_{k=1}^{N}$ are vector-valued nonlinear functions.

In this paper we will address the problem of identifying the unknown parameters θ_k. Let us partition $\mathcal{L}$ comformably and realize it as

$$\mathcal{L} = \begin{bmatrix} \mathcal{L}_{yu} & \mathcal{L}_{ye} & \mathcal{L}_{yw} \\ \mathcal{L}_{zu} & \mathcal{L}_{ze} & \mathcal{L}_{zw} \end{bmatrix} \sim \left[\begin{array}{c|ccc} A & B_u & B_e & B_w \\ \hline C_y & D_{yu} & D_{ye} & D_{yw} \\ C_z & D_{zu} & D_{ze} & D_{zw} \end{array}\right]$$

We summarize our principal assumptions below.

A.1 $\mathcal{L}$ has a stabilizable and detectable realization.

A.2 Measurability of z, i.e., there exists an LTI system Ψ_M such that

$$\begin{bmatrix} \mathcal{L}_{ze} & \mathcal{L}_{zw} \end{bmatrix} = \Psi_M \begin{bmatrix} \mathcal{L}_{ye} & \mathcal{L}_{yw} \end{bmatrix}. \tag{2}$$

A.3 Co-measurability of z, i.e., there exists an LTI system Ψ_C such that

$$\begin{bmatrix} \mathcal{L}_{ze} & \mathcal{L}_{zw} \end{bmatrix} = \mathcal{L}_{zu} \Psi_C. \tag{3}$$

A.4 $\mathcal{N}$ is parametric, i.e., $\mathcal{N}$ can be expressed as a finite linear combination of nonlinear basis functions

$$w = \mathcal{N}(z) = \sum_{k=1}^{N} \theta_k \phi^{[k]}(z), \quad \theta_k \in \mathbb{R}.$$

A.5 There is no undermodelling. That is, there exists $\theta^{true} \in \mathbb{R}^n$ such that for all signals z,

$$w^{true} = \mathcal{N}^{true}(z) = \sum_{k=1}^{N} \theta_k^{true} \phi^{[k]}(z).$$

A.6 $\mathcal{L}_{yw} X \mathcal{L}_{ze}$ is strictly proper, where X is a matrix with the same block diagonal structure as $\mathcal{N}$, and $X^{[i]}$ has the same input-output dimensions as $\mathcal{N}^{[i]}$.

A.7 $D_{ye} D_{ye}^*$ is invertible.

A.8 The signals $\phi^{[i]}(z)$, $i = 1, 2, ..., N$ are bounded.

We now make several comments regarding these assumptions.

R.1 Note that we do not require $\mathcal{L}$ to be stable.

R.2 Assumption A.2 is critical to our needs. Observe that

$$\begin{aligned} z &= \mathcal{L}_{zu} u + \mathcal{L}_{ze} e + \mathcal{L}_{zw} w \\ &= \mathcal{L}_{zu} u + \Psi_M \mathcal{L}_{ye} e + \Psi_M \mathcal{L}_{yw} w \\ &= \mathcal{L}_{zu} u + \Psi_M (y - \mathcal{L}_{yu} u). \end{aligned}$$

This is equivalent to requiring that z be measured, i.e., z can be inferred from u, y and $\mathcal{L}$.

R.3 Assumption A.3 is the dual of Assumption A.2. We do not require this assumption for our identification procedure. We require this only for our analysis on persistence of excitation (see Section 5.2).

R.4 Assumption A.4 is made to restrict $\mathcal{N}$ to the class of static nonlinearities that can be represented as a basis function expansion.

R.5 Assumption A.5 ensures that the behavior of the static nonlinear map $\mathcal{N}$ can be fully captured by our particular choice of basis functions.

R.6 A.6 is necessary so that the noise signal e is uncorrelated with the signal $\mathcal{L}_{yw} w$. If $\mathcal{L}_{yw} X \mathcal{L}_{ze}$ is not strictly proper, minimization of our cost function will result in bias in the estimates.

R.7 We require A.7 to ease computation of the Kalman Filter.

R.8 Assumption A.8 is made in order to deal with bounded quantities.

3. THE IDENTIFICATION ALGORITHM

In this section, we describe our proposed identification algorithm for general structured interconnected nonlinear systems. In subsequent sections, we will analyze convergence properties of the proposed algorithm, address computational issues, and offer illustrative examples.

For ease of notation, let $B = [B_u \;\; B_e \;\; B_w]$, $D = [D_{yu} \;\; D_{ye} \;\; D_{yw}]$ and $\Phi(z) = [\phi^{[1]}(z) \;\; \cdots \;\; \phi^{[N]}(z)]$.

We now propose the following identification algorithm.

1 Perform stable left coprime factorization

$$\begin{bmatrix}\mathcal{L}_{yu} & \mathcal{L}_{ye} & \mathcal{L}_{yw}\end{bmatrix} = G^{-1}\begin{bmatrix}H_{yu} & H_{ye} & H_{yw}\end{bmatrix}$$

where

$$G \sim \left[\begin{array}{c|c}\hat{A} & F \\ \hline C_y & I\end{array}\right], \quad H \sim \left[\begin{array}{c|c}\hat{A} & \hat{B} \\ \hline C_y & D\end{array}\right]$$

$$\hat{A} = A + FC_y, \quad \hat{B} = B + FD$$

and F is such that G and H are stable

2 Realize the Kalman Filter $\mathcal{K}$ as

$$\mathcal{K} \sim \left[\begin{array}{c|c}F_t & K_t \\ \hline M_t & R_t\end{array}\right]$$

where

$$\begin{aligned}\hat{B}_e &= B_e + FD_e \\ K_t &= (\hat{A}P_tC_y^* + \hat{B}_eD_e^*)\Lambda_t^{-1} \\ F_t &= \hat{A} - K_tC_y \\ M_t &= \Lambda_t^{-\frac{1}{2}}C_y \\ R_t &= -\Lambda_t^{-\frac{1}{2}}\end{aligned}$$

and P_t, Λ_t, Z_t are obtained by solving the Riccati difference equation for $t = 0, \cdots, L-1$ with zero initial conditions

$$\begin{aligned}P_0 &= 0 \\ P_{t+1} &= \hat{A}P_t\hat{A}^* + \hat{B}_e\hat{B}_e^* - Z_t\Lambda_tZ_t^* \\ \Lambda_t &= C_yP_tC_y^* + D_eD_e^* \\ Z_t &= (\hat{A}P_tC_y^* + \hat{B}_eD_e^*)\end{aligned}$$

3 Simulate Kalman Filter for $t = 0, \ldots, L-1$ with zero initial conditions to obtain

$$\begin{aligned}Y &= \mathcal{K}(Gy - H_{yu}u) \\ \mathcal{Q} &= \mathcal{K}H_{yw}\Phi(z)\end{aligned}$$

4 The parameter esitimate $\hat{\theta}^L$ is obtained by solving the convex least squares minimization problem

$$\hat{\theta}^L = \arg\min_{\theta\in\Theta} J^L(\theta)$$

where $J^L(\theta) = \frac{1}{L}\|Y - \mathcal{Q}\theta\|^2$

Fig. 4. Identification algorithm.

The rationale behind the algorithm can be explained as follows. Since e is a white gaussian process, we wish to choose the value of θ that results in the minimum energy signal e. This can be accomplished with the following minimization problem.

$$\begin{gathered}\min_{\theta\in\Theta} \frac{1}{L}\|e\|^2 \quad \text{subject to} \\ \mathcal{L}_{ye}e = y - \mathcal{L}_{yu}u - \mathcal{L}_{yw}\sum_{k=1}^{N}\theta_k\phi^{[k]}(z).\end{gathered} \tag{4}$$

The constraint restricts the signal e to be consistent with our input-output data, and Θ is a compact subset of $\mathbb{R}^N$. Note that if $\mathcal{L}_y = [\mathcal{L}_{yu} \ \ \mathcal{L}_{ye} \ \ \mathcal{L}_{yw}]$ is stable, we can choose

$$G = I \ \text{ and } \ \begin{bmatrix}H_{yu} & H_{ye} & H_{yw}\end{bmatrix} = \mathcal{L}_y$$

in step 1 of the algorithm. In the case that $\mathcal{L}_y$ is unstable, performing the stable coprime factorization allows us to rewrite (4) as

$$\begin{gathered}\min_{\theta\in\Theta} \frac{1}{L}\|e\|^2 \quad \text{subject to} \\ H_{ye}e = Gy - H_{yu}u - H_{yw}\sum_{k=1}^{N}\theta_k\phi^{[k]}(z)\end{gathered} \tag{5}$$

in order to work with stable computations.

We recognize the minimization problem (5) as a Kalman smoothing problem (Kailath *et al.*, 2000). Thus, (5) is equivalent to the minimization problem in step 4 of the identification algorithm.

Note that by posing (5) as a Kalman Smoothing problem, we do not require the invertibility of $\mathcal{L}_{ye}$.

4. CONVERGENCE

We now analyze the convergence properties of our candidate identification algorithm. We are interested in the asymptotic behavior of our estimate $\hat{\theta}^L$ as $L \to \infty$. Note that $\hat{\theta}^L$ is a random sequence because it depends on noisy measurements y and z.

Let us define the set of minimizers

$$\mathcal{M}^\infty = \left\{\hat{\theta} : \hat{\theta} = \arg\min_{\theta\in\Theta} \bar{J}(\theta)\right\}$$

where

$$\bar{J}(\theta) = \lim_{L\to\infty}\frac{1}{L}\|\mathcal{Q}(\theta^{true} - \theta)\|^2.$$

Clearly, θ^{true} is a minimizer of $\bar{J}(\theta)$, i.e., $\theta^{true} \in \mathcal{M}^\infty$. Theorem 4.1 will show that the estimate $\hat{\theta}^L$ will converge to some $\theta \in \mathcal{M}^\infty$.

Theorem 4.1. Let Assumptions A.1,A.2,A.4-A.8 hold. Then,

$$\hat{\theta}^L \longrightarrow \mathcal{M}^\infty \quad \text{w.p. 1 as } L \longrightarrow \infty,$$

i.e., $\lim_{L\to\infty} \inf_{\theta\in\mathcal{M}^\infty} \|\hat{\theta}^L - \theta\| = 0$ with probability 1. □

In the following section, we will provide conditions under which $\mathcal{M}^\infty = \{\theta^{true}\}$, i.e., $\mathcal{M}^\infty$ consists of the singleton θ^{true}.

5. IDENTIFIABILITY, PERSISTENCE OF EXCITATION, AND UNIQUENESS

5.1 *Identifiability*

In this section we discuss the notion of identifiability. Identifiability concepts are of fundamental importance in system identification (Ljung, 1999). Loosely speaking, the nonlinear block $\mathcal{N}$ of the LFT model structure is identifiable if it can be determined uniquely from input-output experiments.

Let $\mathbb{N}$ be the class of static nonlinearities that we consider. We will represent the input-output behavior of the LFT model structure as $y = \Omega(\mathcal{L}, \mathcal{N})u$. We begin with the following definition.

Definition 5.1. Suppose $\mathcal{N}^o \in \mathbb{N}$. The LFT model structure $\Omega(\mathcal{L}, \mathcal{N})$ is *identifiable* at $\mathcal{N}^o$ if for any $\mathcal{N}^1 \in \mathbb{N}$ with $\mathcal{N}^1 \neq \mathcal{N}^o$,

$$\Omega(\mathcal{L}, \mathcal{N}^o) \neq \Omega(\mathcal{L}, \mathcal{N}^1).$$

The LFT model structure is *identifiable everywhere* if it is identifiable at all $\mathcal{N} \in \mathbb{N}$. □

Note that the above definitions are *global* notions of identifiability. For our case of parametric static nonlinearities, we have the following result on identifiability.

Lemma 5.2. The LFT model structure is identifiable everywhere if and only if

$$H_{yw}\Phi(z)\theta = 0 \ \ \forall z \quad \Longrightarrow \quad \theta = 0.$$

□

5.2 Persistence of Excitation

We now focus on the notion of persistence of excitation. The identifiability and persistence of excitation conditions are both necessary for the uniqueness of the solution to the parameter estimation problem. However, the two conditions apply to different aspects of the identification procedure. Identifiability concerns the structure of the model while persistence of excitation is a condition on whether the input to the system generates an input-output data set informatively rich enough for convergence of our estimate. The latter notion can be captured by the following definition.

Definition 5.3. The input z to the nonlinear block $\mathcal{N}$ is *persistently exciting* if there exists $L_0 > 0$ such that for any $L > L_0$,

$$\frac{1}{L}\mathcal{Q}^*\mathcal{Q} \succ 0.$$

□

Note that the persistence of excitation condition depends on the LFT model structure. A signal that is persistently exciting for a given LFT model structure may be not persistently exciting for another. This is unusual since persistence of excitation conditions are commonly independent of the model to identify. However, in the case of general interconnected systems, a notion of persistence of excitation that considers the model structure is more appropriate.

The identifiability of a system is also necessary for the signal z to be persistently exciting. To illustrate this, suppose that the LFT model structure is not identifiable. It then follows from Lemma 5.2 that the matrix $H_{yw}\Phi(z)$ has linearly dependent columns for all z. Since $\mathcal{K}$ is a linear operator, $\mathcal{Q} = \mathcal{K}H_{yw}\Phi(z)$ also has linearly dependent columns for all z. As a result, $\lim_{L\to\infty} \frac{1}{L}\mathcal{Q}^*\mathcal{Q} \not\succ 0$ for all z, and *no* persistently exciting signal exists.

One should note that signals usually considered non-informative may be persistently exciting for a particular LFT model structure under the above notion. For example, consider the simple system in Figure 5.

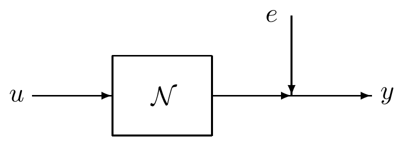

Fig. 5. Single static nonlinear block system.

Here $\mathcal{L}_{yu} = \mathcal{L}_{ze} = \mathcal{L}_{zw} = 0$, $\mathcal{L}_{ye} = \mathcal{L}_{zu} = \mathcal{L}_{yw} = 1$, $\mathcal{N}(z) = \theta^{true}\phi(z)$, $\mathcal{Q}(z) = \mathcal{Q}(u) = \exp(-u^2)$ and $\theta^{true} \neq 0$. Let $u = 0$ be the input to the system. We then have that $\mathcal{Q}(z) = [1\ 1\ \ldots]^*$, $\mathcal{Q}^*(z)\mathcal{Q}(z) = L$, and $\frac{1}{L}\mathcal{Q}^*(z)\mathcal{Q}(z) = 1 > 0$, $\forall L > 0$. As a result, the signal $u = 0$ is persistently exciting for the given LFT model structure. We will later show that together with the identifiability of the system, this choice of input u will lead to convergence of the estimate $\hat{\theta}^L$ to the true value θ^{true}.

Note that the co-measurability assumption (A.3) guarantees that for any persistently exciting signal z, there exists an input u that could have generated z. This can be shown by the following argument. From Assumption A.3, we have that

$$\begin{aligned} z &= \mathcal{L}_{zu}u + \begin{bmatrix} \mathcal{L}_{ze} & \mathcal{L}_{zw} \end{bmatrix} \begin{bmatrix} e \\ w \end{bmatrix} \\ &= \mathcal{L}_{zu}u + \mathcal{L}_{zu}\Psi_C \begin{bmatrix} e \\ w \end{bmatrix} \\ z &\in Range(\mathcal{L}_{zu}). \end{aligned}$$

That is, for any signals e, z, there always exists an input u that can generate z through $\mathcal{L}$ and $\mathcal{N}$.

5.3 Uniqueness

Theorem 5.4. Let Assumptions A.1,A.2,A.4-A.8 hold. Suppose that the LFT model structure is identifiable everywhere. If z is persistently exciting, then

(1) There exists $L_0 > 0$ such that for any $L > L_0$, the minimization problem

$$\hat{\theta}^L = \arg\min_{\theta\in\Theta} \frac{1}{L}\|Y - \mathcal{Q}\theta\|^2$$

yields the unique solution

$$\hat{\theta}^L = (\mathcal{Q}^*\mathcal{Q})^{-1}\mathcal{Q}^*Y.$$

(2) The set of minimizers $\mathcal{M}^\infty$ consists of the singleton θ^{true}, i.e.,

$$\mathcal{M}^\infty = \{\theta^{true}\}.$$

□

Note that without Assumption A.6, the cost function $J^L(\theta)$ will contain a correlation component, leading to bias in the estimate.

6. EXAMPLES

We now present two simulation examples demonstrating our identification algorithm. The input u was chosen to be a random sequence. Zero-mean white gaussian noises have been used.

6.1 Example 1

Consider the interconnected system in Figure 6. Here, the nonlinearities $\mathcal{N}_1$ and $\mathcal{N}_2$ are to be identified. The LTI systems $\mathcal{L}_1$ and $\mathcal{L}_2$ are known, and the signals u, y are measured. The noise signal e is a white noise process acting on the output.

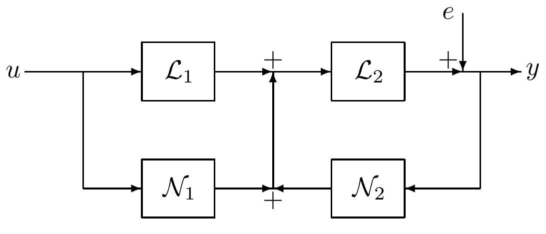

Fig. 6. Interconnected System for Example 1

The nonlinear block $\mathcal{N}$ of the LFT model structure is block diagonal, with $\mathcal{N}_1(u) = 2\arctan(u)$ and $\mathcal{N}_2(y) = -0.3y + 0.1\sin(y) + 1$. It is easy to verify that the input to the nonlinear block $z = [u \quad y]^T$ meets our measurability assumptions. Figure 7 illustrates the convergence behavior of our estimate. Here, the estimated parameters converge quickly to their true values as the size of the data sample increases.

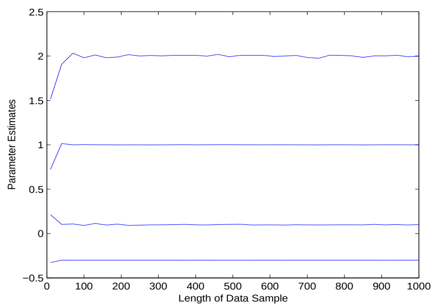

Fig. 7. Convergence of estimates for Example 1

6.2 Example 2

We now present an example where $\mathcal{L}$ is unstable. Consider the system depicted in Figure 8.

Here, $\mathcal{L}$ is an unstable LTI system and we wish to identify $\mathcal{N}$. In this example, $\mathcal{N}(u) = 0.25u - 5\arctan(u)$. Our identification algorithm allows us to consider systems where the linear block is unstable. Figure 9 illustrates the convergence behavior of our estimate for this example.

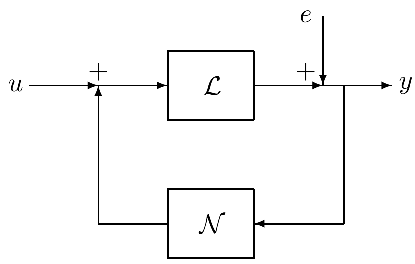

Fig. 8. Interconnected System for Example 2

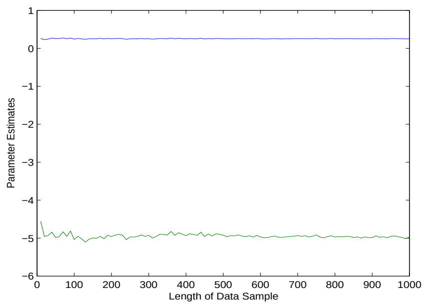

Fig. 9. Convergence of Estimates for Example 2

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