STOCHASTIC SLIDING MODE CONTROL FOR SYSTEMS WITH MARKOVIAN JUMP PARAMETERS

Peng Shi * **Yuanqing Xia** ** **G. P. Liu** **,*** **D. Rees** **

* *School of Technology, University of Glamorgan, Pontypridd, CF37 1DL, United Kingdom. Email: pshi@glam.ac.uk*
** *School of Electronics, University of Glamorgan, Pontypridd, CF37 1DL, United Kingdom. Email: xia_yuanqing@163.net;gpliu@glam.ac.uk;drees@glam.ac.uk*
*** *Lab of Complex Systems and Intelligence Science, Chinese Academy of Sciences, Beijing 100080, China*

Abstract: In this paper, we consider the problems of stochastic stability and sliding mode control for a class of linear continuous-time systems with stochastic jumps, in which the jumping parameters are modelled as a continuous-time, discrete-state homogeneous Markov process with right continuous trajectories taking values in a finite set. By using Linear matrix inequalities (LMI) approach, sufficient conditions are proposed to guarantee the stochastic stability of the underlying system and a reaching motion controller is designed such that the resulting closed-loop system can be driven onto the desired sliding surface in a limited time. It has been shown that the sliding mode control problem for the markovian jump systems is solvable if a set of coupled linear matrix inequalities (LMIs) have solutions. Simulation studies show the effectiveness of the control scheme.

Keywords: Linear matrix inequality, markovian jump parameter, sliding mode control, stochastic stability

1. INTRODUCTION

A large class of physical systems have variable structures subject to random changes, which may result from the abrupt phenomena such as component and interconnection failures, parameters shifting, tracking, and the time required to measure some of the variables at different stages. Systems with this character may be modelled as *hybrid* ones, that is, to the continuous state variable, a discrete random variable called the mode, or regime, is appended. The mode describes the random jumps of the system parameters and the occurrence of discontinuities. One of the most important hybrid systems is the so-called **Markovian jumping system** (MJS), in which the mode-process is a continuous-time discrete-state Markov process taking values in a finite set. In engineering applications, frequently occurring dynamical systems which can be represented by different forms depending on the value of an associated Markov chain process are termed *jump systems*. Research into this class of system and their applications span several decades. For some representative prior work on this general topic, we refer the reader to (Boukas *et al.*, 2002*a*; Boukas *et al.*, 2002*b*; Mahmoud and Shi, 2003; Mahmoud and Shi, 2002; Shi *et al.*, 1999*b*; Shi *et al.*, 1999*a*; Shi and Boukas, 1997) and the references therein.

In another active research area, the so called sliding mode control has attractive features to keep systems insensitive to the uncertainties on the sliding surface. Sliding mode control as a general design tool for robust control systems has been well established, see for example, (Itis, 1976) (Xia and Jia, 2003*b*; Xia and Jia, 2002; Xia and Jia, 2003*a*) and the references therein. The motivation for this research stems from the fact that both sliding mode control systems and markovian jump system (or hybrid system) are quite important in theory investigation and practical applications. The work conducted in this paper not only contributes to the theory development, but also solves the practical problems, which has already been reported on in the literature, such as those occurring in manufacturing systems, telecommunication systems, internet based implement and power systems, etc (see for example, (Boukas *et al.*, 2002*a*; Kushner, 1967; S.Willsky, 1976) and the references therein).

In this paper, we consider the problem of stochastic sliding mode control for a class of linear continuous systems with Markovian jump parameters. The jumping parameters are treated as continuous-time, discrete-state Markov process. Concepts of stochastic stability and stochastic stabilization for the underlying systems are introduced. The sliding surface and reaching motion controller for the system will be designed. The condition for the existence of linear sliding surfaces is derived. The solution to the condition can be used to characterize linear sliding surfaces, and by selecting suitable reaching law, the reaching motion controller is proposed. The above problems are solved in terms of a finite set of coupled linear matrix inequalities (LMIs). Finally, a numerical example is included to demonstrate the effectiveness of the theoretical results obtained.

Notations. The notation used in this paper is quite standard. In the sequel, the Euclidean norm is used for vectors. We use W^t, W^{-1}, $\lambda(W)$, $Tr(W)$ and $||W||$ to denote, respectively, the transpose, the inverse, the eigenvalues, the trace and the induced norm of any square matrix W. We use $W > 0$ $(\geq, <, \leq 0)$ to denote a symmetric positive definite (positive semidefinite, negative, negative semidefinite) matrix W with $\lambda_m(W)$ and $\lambda_M(W)$ being the minimum and maximum eigenvalues of W and I to denote the $n \times n$ identity matrix. The Lebesgue space $\mathcal{L}_2[0, T]$ consists of square-integrable functions on the interval $[0, T]$ equipped with the norm $||.||_2$. $\mathcal{E}[\cdot]$ stands for mathematical expectation. $Sgn(\cdot)$ sign function, that is, $sgn(x) = 1, \text{if } x > 0$, $sgn(x) = 0, \text{if } x = 0$, and $sgn(x) = -1, \text{if } x < 0$. Given a probability space $(\Omega, \mathcal{F}, \mathbf{P})$ where Ω is the sample space, $\mathcal{F}$ is the algebra of events and $\mathbf{P}$ is the probability measure defined on $\mathcal{F}$. Sometimes, the arguments of a function will be omitted in the analysis when no confusion can arise.

2. PROBLEM FORMULATION AND PRELIMINARIES

We consider a class of stochastic systems with Markovian jump parameters in a fixed probability space $(\Omega, \mathcal{F}, \mathbf{P})$:

$$\begin{aligned} \dot{x}(t) &= A(\eta_t)x(t) + B(\eta_t)[u(t) + F(\eta_t)w(t)], \\ \eta_o &= i, \ t \geq 0 \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state vector; $u(t) \in R^m$ is the control input, $w \in R^l$ is the disturbance, while $\{\eta_t, t \in [0, \mathcal{T}]\}$ is a finite-state Markovian process having a state space $\mathcal{S} \triangleq \{1, 2, ..., \nu\}$, generator (α_{ij}) with transition probability from mode i at time t to mode j at time $t+\delta$, $i, j \in \mathcal{S}$:

$$\begin{aligned} p_{ij} &= Pr(\eta_{t+\delta} = j \mid \eta_t = i) \\ &= \begin{cases} \alpha_{ij}\delta + o(\delta), & if \ i \neq j, \\ 1 + \alpha_{ij}\delta + o(\delta), & if \ i = j \end{cases} \end{aligned} \tag{2}$$

$$\begin{aligned} \alpha_{ii} &= -\sum_{m=1, m\neq i}^{s} \alpha_{im} \ , \ \alpha_{ii} \geq 0 \\ & \forall\, i, j \in \mathcal{S}, i \neq j \end{aligned} \tag{3}$$

where $\delta > 0$ and $\lim_{\delta \downarrow 0} o(\delta)/\delta = 0$.

For each possible value $\eta_t = i, \ i \in \mathcal{S}$, we will denote the system matrices associated with mode i by

$$A(\eta_t) \triangleq A(i), \quad B(\eta_t) \triangleq B(i), \quad F(\eta_t) \triangleq F(i)$$

where $A(i)$, $B(i)$ and $F(i)$ are known real constant matrices of appropriate dimensions which describe the nominal system. It is assumed that

$$||F(i)w(t)|| \leq f(i), i \in \mathcal{S} \tag{4}$$

where $f(i), i \in \mathcal{S}$ are positive scalars, $||\cdot||$ denotes the Euclidean norm of a vector and its induced norm of a matrix..

Remark 2.1. The model of the form (1) is a hybrid system in which one state $x(t)$ takes values continuously and another state η_t, referred to as the mode or operating form, takes values discretely in $\mathcal{S}$. This kind of system can be used to represent many important physical systems subject to random failures and structure changes, such as electric power systems (S.Willsky, 1976), control systems of a solar thermal central receiver (Sworder and Rogers, 1983), communications systems (Athans, 1987), aircraft flight control (Moerder *et al.*, 1989), control of nuclear power plants (Petkovski, 1987) and manufacturing systems (Boukas *et al.*, 1995; Boukas and Yang, 1996).

In order to obtain a regular form of systems (1), we can choose a nonsingular matrix $T(\eta_t)$ such that

$$T(\eta_t)B(\eta_t) = \begin{bmatrix} 0_{(n-m)\times m} \\ B_2(\eta_t) \end{bmatrix}$$

where $B_2(\eta_t) \in R^{m\times m}$ is nonsingular. For convenience, let us partition

$$T(\eta_t) = \begin{bmatrix} U_2^T(\eta_t) \\ U_1^T(\eta_t) \end{bmatrix}$$

where $U_1(\eta_t) \in R^{n\times m}$ and $U_2(\eta_t) \in R^{n\times(n-m)}$ are two sub-blocks of a unitary matrix resulting from the singular value decomposition of $B(\eta_t)$, that is,

$$B(\eta_t) = \begin{bmatrix} U_1(\eta_t) & U_2(\eta_t) \end{bmatrix} \begin{bmatrix} \Sigma(\eta_t) \\ 0_{(n-m)\times m} \end{bmatrix} V^T(\eta_t)$$

where $\Sigma(\eta_t) \in R^{m\times m}$ is a diagonal positive-definite matrix and $V(\eta_t) \in R^{m\times m}$ is a unitary matrix((Kim *et al.*, 2000)). By the state transformation $z = T(\eta_t)x$, system (1) has the regular form

$$\begin{aligned} \dot{z}(t) &= \bar{A}(\eta_t)z(t) + \begin{bmatrix} 0_{(n-m)\times m} \\ B_2(\eta_t) \end{bmatrix} [u(t) \\ &\quad + F(\eta_t)w(t)] \end{aligned} \tag{5}$$

where $\bar{A}(\eta_t) = T(\eta_t)A(\eta_t)T^{-1}(\eta_t)$. System (5) can be written as:

$$\dot{z}_1(t) = \bar{A}_{11}(\eta_t)z_1(t) + \bar{A}_{12}(\eta_t)z_2(t) \tag{6}$$

$$\begin{aligned} \dot{z}_2(t) &= \bar{A}_{21}(\eta_t)z_1(t) + \bar{A}_{22}(\eta_t)z_2(t) + \\ &\quad B_2(\eta_t)[u(t) + F(\eta_t)w(t)] \end{aligned} \tag{7}$$

where $z_1 \in R^{n-m}$, $z_2 \in R^m$ and

$$\begin{aligned} B_2(\eta_t) &= \Sigma(\eta_t)V^T(\eta_t), \\ \bar{A}_{11}(\eta_t) &= U_2^T(\eta_t)A(\eta_t)U_2(\eta_t), \\ \bar{A}_{12}(\eta_t) &= U_2^T(\eta_t)A(\eta_t)U_1(\eta_t) \\ \bar{A}_{21}(\eta_t) &= U_1^T(\eta_t)A(\eta_t)U_2(\eta_t) \\ \bar{A}_{22}(\eta_t) &= U_1^T(\eta_t)A(\eta_t)U_1(\eta_t). \end{aligned}$$

It is obvious that the first equation of system (6) represents the sliding motion dynamics of the system (5), and hence the corresponding sliding surface can be chosen as follows:

$$\begin{aligned} s(t) &= \begin{bmatrix} C_1(\eta_t) & C_2(\eta_t) \end{bmatrix} z \\ &= C_1(\eta_t)z_1 + C_2(\eta_t)z_2 = 0 \end{aligned} \tag{8}$$

where $C_1(i)$ is invertble for any $i \in \{1, 2, ..., s\}$. Let $C(\eta_t) = C_2^{-1}(\eta_t)C_1(\eta_t) \in R^{m\times(n-m)}$ and substitute $z_2 = -C(\eta_t)z_1$ to (6) gives the sliding motion

$$\dot{z}_1(t) = [\bar{A}_{11}(\eta_t) - \bar{A}_{12}(\eta_t)C(\eta_t)]z_1(t) \tag{9}$$

Let us recall the definition of stochastic stability for system (9).

Definition 2.1. For system (9), the equilibrium point 0 is *stochastically stable*, if for any $z_1(0)$ and $\eta_0 \in \mathcal{S}$

$$\int_0^\infty \mathcal{E}\left\{\|z_1(t, z_1(0))\|^2\right\} dt < +\infty.$$

The following result shows that the stochastic stability of system (9) is equivalent to a set of ν intercoupled algebraic Lyapunov-type equations have solutions.

Lemma 2.1. (Feng *et al.*, 1992; **?**) Consider system (9), then the following statements are equivalent:

(a) System (9) is stochastically stable;

(b) For any given positive definite matrices $N(k)$, $k \in \mathcal{S}$, there exist positive definite matrices $M(k)$, $k \in \mathcal{S}$, satisfying

$$\begin{aligned} \tilde{A}^T(k)M(k) &+ M(k)\tilde{A}(k) + \sum_{j=1}^{\nu} \alpha_{kj}M(j) \\ &+ N(k) = 0, \ k \in \mathcal{S}. \end{aligned} \tag{10}$$

where $\tilde{A}(k) = [\bar{A}_{11}(k) - \bar{A}_{12}(k)C(k)]$.

Remark 2.2. In (Ji and Chizeck, 1990; Shi and Boukas, 1997), it has been proved that for system (9), all the concepts of *stochastically stable, asymptotically mean square stable* and *exponentially mean square stable* are equivalent, and any of them can imply *almost surely (asymptotically) stable.* Lemma 2.1 also provides the necessary and sufficient conditions for asymptotically mean square stability and exponentially mean square stability, and sufficient conditions for almost surely (asymptotically) stability of system (9). Also, note that the left hand side of (2.1) being less than zero also implies the statement (a) in Lemma 2.1.

In the following, attention is focused on the design of gain $C(k) \in R^{m\times(n-m)}$ and a reaching motion control law $u(t)$ for each $k \in \mathcal{S}$ such that

1) The sliding motion (9) is stochastically stable; and

2) The system (6)-(7) is stochastically stable with the reaching control law $u(t)$.

3. MAIN RESULTS

The first result of designing sliding surface can be stated as follows.

Theorem 1: The reduced order system (9) is stochastically stable if there exists symmetric positive-definite matrices $P(k) \in R^{m\times m}, k \in \mathcal{S}$ and general matrix $Q(k) \in R^{m\times(n-m)}, k \in \mathcal{S}$ such that

$$\begin{bmatrix} \Pi(k) & * & * & \cdots & * & * & \cdots & * & * \\ \alpha_{k1}^{\frac{1}{2}}P(k) & a_1 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \alpha_{k2}^{\frac{1}{2}}P(k) & 0 & a_2 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \alpha_{k(k-1)}^{\frac{1}{2}}P(k) & 0 & 0 & \cdots & a_3 & 0 & \cdots & 0 & 0 \\ \alpha_{k(k+1)}^{\frac{1}{2}}P(k) & 0 & 0 & \cdots & 0 & a_4 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \alpha_{k(\nu-1)}^{\frac{1}{2}}P(k) & 0 & 0 & \cdots & 0 & 0 & \cdots & a_5 & 0 \\ \alpha_{k\nu}^{\frac{1}{2}}P(k) & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & a_6 \end{bmatrix} < 0 \quad (11)$$

Moreover, the sliding surface of the system (6) is

$$s(t) = \overline{C}_1(k)z_1(t) + \overline{C}_2(k)z_2(t) = 0,\ k \in \mathcal{S} \quad (12)$$

where $a_1 = -P(1), a_2 = -P(2), a_3 = -P(k-1), a_4 = -P(k+1), a_5 = -P(\nu-1), a_6 = -P(\nu)$, $\overline{C}_1(k)$ and $\overline{C}_2(k)$ are appropriately factorization of $\overline{Q}(k)P^{-1}(k)$, that is, $\overline{C}_2^{-1}(k)\overline{C}_1(k) = \overline{Q}(k)P^{-1}(k), k \in \mathcal{S}$, $\Pi(k) = P(k)\bar{A}_{11}^T(k) + \bar{A}_{11}(k)P(k)+\alpha_{kk}P(k)-\bar{A}_{12}(k)Q(k)-Q^T(k)\bar{A}_{12}^T(k)$.

Proof. From Lemma 2.1, System (9) is stochastically stable if only if for any given positive definite matrices $N(k),\ k \in \mathcal{S}$, there exists positive definite matrices $M(k),\ k \in \mathcal{S}$, satisfying

$$\tilde{A}^T(k)M(k) + M(k)\tilde{A}(k) + \sum_{j=1}^{\nu}\alpha_{kj}M(j) + N(k) = 0, k \in \mathcal{S}. \quad (13)$$

which is equivalent to the following inequalities

$$\tilde{A}^T(k)M(k) + M(k)\tilde{A}(k) + \sum_{j=1}^{\nu}\alpha_{kj}M(j) < 0 \quad (14)$$

for $k \in \mathcal{S}$. Pre- and post-multiplying inequality (14) by $M^{-1}(k)$ gives

$$\begin{aligned} &M^{-1}(k)\tilde{A}^T(k) + \tilde{A}(k)M^{-1}\tilde{A}(k)+ \\ &M^{-1}(k)(\sum_{j=1}^{\nu}\alpha_{kj}M(j))M^{-1}(k) < 0,\ k \in \mathcal{S}. \end{aligned} \quad (15)$$

Let $P(k) = M^{-1}(k), k \in \mathcal{S}$ yields

$$\begin{aligned} &P(k)\tilde{A}^T(k) + \tilde{A}(k)P(k) + \alpha_{kk}P(k) \\ &+P(k)(\sum_{j=1,j\neq k}^{\nu}\alpha_{kj}P^{-1}(j))P(k) < 0 \end{aligned} \quad (16)$$

Applying Schur complement formula gives

$$\begin{bmatrix} \Delta(k) & * & * & \cdots & * & * & \cdots & * & * \\ \alpha_{k1}^{\frac{1}{2}}P(k) & a_1 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \alpha_{k2}^{\frac{1}{2}}P(k) & 0 & a_2 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \alpha_{k(k-1)}^{\frac{1}{2}}P(k) & 0 & 0 & \cdots & a_3 & 0 & \cdots & 0 & 0 \\ \alpha_{k(k+1)}^{\frac{1}{2}}P(k) & 0 & 0 & \cdots & 0 & a_4 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \alpha_{k(\nu-1)}^{\frac{1}{2}}P(k) & 0 & 0 & \cdots & 0 & 0 & \cdots & a_5 & 0 \\ \alpha_{k\nu}^{\frac{1}{2}}P(k) & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & a_6 \end{bmatrix} < 0 \quad (17)$$

where $k \in \mathcal{S}$, $\Delta(k) = P(k)\tilde{A}^T(k) + \tilde{A}(k)P(k) + \alpha_{kk}P(k)$. Let $Q(k) = C(k)P(k)$, then (17) is equivalent to (11).

Next the result of designing reaching motion controller is given.

Theorem 2: Assume the condition in Theorem 1 holds, i.e., inequalities (11) have solutions $P(k) \in R^{m\times m}, k \in \mathcal{S}$, $Q(k) \in R^{m\times(n-m)}, k \in \mathcal{S}$, the linear sliding surface is given by (12), and there exist $\Omega(k) and \Theta(k)$ satisfying the following inequalities:

$$-\Omega(k)\Theta(k) - \Theta^T(k)\Omega(k) + \sum_{j=1}^{\nu}\alpha_{kj}\Omega(k) < 0 \quad (18)$$

where $\Theta(k), k \in \mathcal{S}$ are selected such that (18) have feasible definite positive matrix solutions $\Omega(k)$.

Then the following control makes the sliding surface $s(z(t)) = 0$ stochastically stable and globally attractive in finite time.

$$\begin{aligned} u(t) = &-(C_2(k)B_2(k))^{-1}[\begin{bmatrix} C_1(k) & C_2(k) \end{bmatrix}\bar{A}(k)z(t) \\ &+\Theta(k)s(t) + (\epsilon(k) + f(k))sgn(\Omega(k)s(t))] \end{aligned} \quad (19)$$

where $\epsilon(k), k \in \mathcal{S}$ are given positive constants.

Proof: We will complete the proof by showing that the control law (19) can not only make the system trajectory stochastically stable but also globally attractive in finite time. From the sliding surface

$$s(t) = \begin{bmatrix} C_1(\eta_t) & C_2(\eta_t) \end{bmatrix} z(t) = \overline{C}(\eta_t)z(t)$$

let us consider the function

$$V(t) = s^T(z(t))\Omega(k)s(z(t)) \quad (20)$$

The weak infinitesimal operator $\Im_a^z[\cdot]$ of the process $\{z(t), \eta_t, t \geq 0\}$ for (5) at the point $\{t, z, k\}$ is given by :

$$\Im_a^z[V] = \partial V/\partial t + \dot{z}^T(t)\partial V/\partial x|_{\eta_t=k} + \sum_{m=1}^{s} \alpha_{im} V(t, z, k, m) \quad (21)$$

From (12), differentiating the function along the solutions of (5) yields:

$$\dot{s}(t) = \left[\, C_1(k) \;\; C_2(k) \,\right] (\bar{A}(k)z(t) + \begin{bmatrix} 0_{(n-m)\times m} \\ B_2(k) \end{bmatrix} \times [u(t) + F(k)w(t)]) \quad (22)$$

then,

$$\begin{aligned} \Im_a^z[V] = \; & s^T(t)\Omega(k)[[\, C_1(k) \;\; C_2(k) \,]\, \bar{A}(k)z(t) + \\ & C_2(k)B_2(k)(u(t) + F(k)w(t))] + \\ & [[\, C_1(k) \;\; C_2(k) \,]\, \bar{A}(k)z(t) + \\ & C_2(k)B_2(k)(u(t) + F(k)w(t))]^T \\ & \Omega(k)s(t) + s^T(t)(\sum_{j=1}^{\nu} \alpha_{kj}\Omega(k))s(t) \end{aligned} \quad (23)$$

with control (19) then,

$$\begin{aligned} \Im_a^z[V] = \; & s^T(t)\Omega(k)[-\Theta(k)s(t) + F(k)w(t) - (\epsilon(k) \\ & +f(k))sgn(\Omega(k)s(t))] + [-\Theta(k)s(t) + \\ & (\epsilon(k) + F(k)w(t) + f(k))sgn(\Omega(k)s(t))]^T \\ & \Omega(k)s(t) + s^T(t)(\sum_{j=1}^{\nu} \alpha_{kj}\Omega(k))s(t) \\ = \; & s^T(t)(-\Omega(k)\Theta(k) - \Theta^T(k)\Omega(k) + \\ & \sum_{j=1}^{\nu} \alpha_{kj}\Omega(k))s(t) + 2s^T(t)\Omega(k)[F(k)w(t) \\ & -(\epsilon(k) + f(k))sgn(\Omega(k)s(t))] \end{aligned}$$

From (4) and (18), we have

$$\begin{aligned} \Im_a^z[V] \leq \; & 2s^T(t)\Omega(k)[F(k)w(t) - (\epsilon(k) \\ & +f(k))sgn(\Omega(k)s(t))] \\ \leq \; & -2\epsilon(k)||\Omega(k)s(t)|| \end{aligned} \quad (24)$$

Note that

$$\begin{aligned} ||\Omega(k)s(t)||^2 = \; & (P^{\frac{1}{2}}s^T(t))^T P(P^{\frac{1}{2}}s^T(t)) \\ \geq \; & \lambda_{min}(P)||P^{\frac{1}{2}}s^T(t)||^2 \end{aligned} \quad (25)$$

and

$$V(t) = s^T(t)\Omega(k)s(t) = ||\Omega^{\frac{1}{2}}(k)s^T(t)||^2 \quad (26)$$

we have

$$\Im_a^z[V] \leq -2\epsilon(k)(\lambda_{min}(\Omega(k)))^{\frac{1}{2}}(V(t, z, i))^{\frac{1}{2}} \quad (27)$$

Then, it follows from (Kushner, 1967), by letting $z(t = 0, i) = z_0$, that

$$E[V(t, z, i)|\eta_0] \leq -2\epsilon(k)(\lambda_{min}(\Omega(k)))^{\frac{1}{2}}t + V(z_0, \eta_0)^{\frac{1}{2}} \quad (28)$$

Since the left side of (28) is non-negative, and $t \leq \frac{V(z_0,\eta_0)^{\frac{1}{2}}}{2\epsilon(k)(\lambda_{min}(\Omega(k)))^{\frac{1}{2}}}$ then $V(x(t))$ reaches zero in finite time, which means the state stochastically converges to sliding surface in finite-time.

4. A NUMERICAL EXAMPLE

In this section, an illustrative example is constructed to verify the design method developed in this paper. Let generator for the Markov process governing the mode switching be

$$\Im = \begin{bmatrix} -4 & 4 \\ 3 & -3 \end{bmatrix}$$

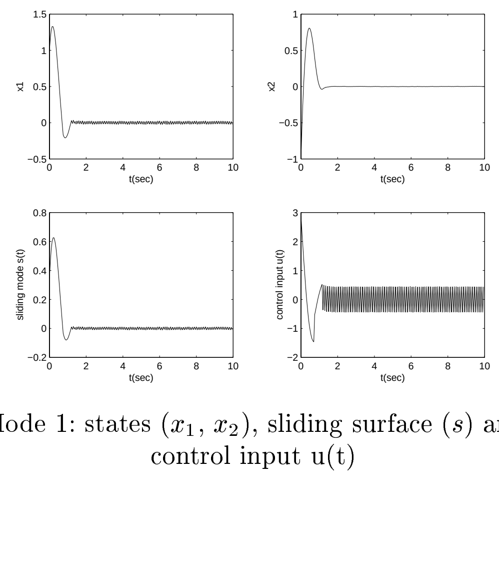
For the two operating conditions (modes), the associated data are:

Mode 1 and Mode 2 as:

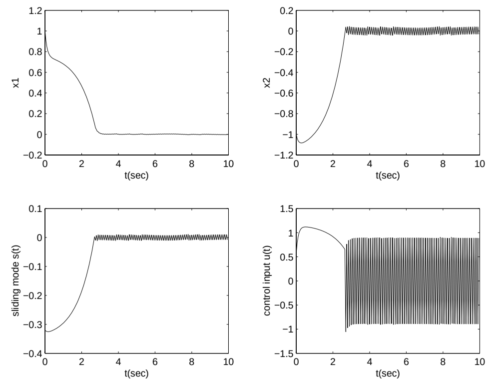
$$A(1) = \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix}, B(1) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, w(t) = 0.01sin(t)$$

$$A(2) = \begin{bmatrix} -1 & 0 \\ 2 & -2 \end{bmatrix}, B(1) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, w(t) = 0.01sin(t)$$

Taking $T(1) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, T(2) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, using Theorem 1 and LMI method, we have $P(1) = 2.3957$, $Q(1) = -0.6727$, $P(2) = 2.2267$, $Q(2) = 0.5243$. Then, $\overline{C}_1(1) = -0.0702, \overline{C}_1(2) = 0.25$, $\overline{C}_2(1) = 0.1059, \overline{C}_2(2) = 0.45$. Taking $\Theta(1) = 0.8$ and $\Theta(2) = 0.54$, by Theorem 2 and LMI techniques, we have $\Omega(1) = 1.9658$ and $\Omega(2) = 2.1037$. Choosing $f(1) = f(2) = 0.01$ and $\epsilon(1) = \epsilon(2) = 0.2$, we have the following simulation results:



Mode 1: states (x_1, x_2), sliding surface (s) and control input u(t)



Mode 2: states (x_1, x_2), sliding surface (s) and control input u(t)

5. CONCLUSIONS

In this paper, the problems of stochastic stability and sliding mode control for a class of linear continuous-time systems with stochastic jumps has been considered. In term of LMI, sufficient conditions are proposed to guarantee the stochastic stability of reduced-order systems. Then, a reaching motion controller is designed such that the resulting closed-loop system can be driven onto the desired sliding surface in a limited time. It has been demonstrated that if a set of coupled linear matrix inequalities has solutions, then the sliding mode control problem can be solved.

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