ON STABILITY CONDITIONS FOR A CLASS OF DYNAMIC FUZZY MODELS

Haishan Ding, Jianqin Mao

The Seventh Research Division, Beijing University of Aeronautics & Astronautics, 100083, Beijing, China. E-mail: chinodhs@163.com

Abstract:Utilizing the design technique of "parallel distributed compensation"(PDC), model-based fuzzy controllers are designed for both continuous-time and discrete-time dynamic fuzzy models. The stability analysis for this class of dynamic fuzzy models is studied in this paper. New stability conditions for this class of dynamic fuzzy models are presented respectively. On comparison with the existing research, the results derived are of less conservative and easy to use relatively, which is test by numerical examples.

Keywords: dynamic fuzzy models, stability analysis, stability conditions

1. INTRODUCTION[1]

A dynamic fuzzy model is presented by S.G.Cao et al first. The main idea is to construct a set of local dynamic systems models to represent the local dynamic behavior of the system, and then to connect the set of local models by membership functions to form a global dynamic model (S.G.Cao, et al. 1997). A dynamic fuzzy model could be considered as the extension of a T-S fuzzy system.

Let a dynamic fuzzy model be the design model (B.Friedland. 1996). Then a model-based fuzzy controller could be designed by utilizing the technique of "parallel distributed compensation" (Hua O.Wang, et al. 1996). And there have been many successful applications, for example, the stable control of a double inverted pendulum is solved in (Yao Hongwei, et al. 2001).

Stability analysis is one of the most basic issues of fuzzy control systems. In this paper new relaxed stability conditions, which only need to calculate eigenvalues, are presented respectively for this class of dynamic fuzzy models whose controllers are designed by the technique of PDC.

The paper is organized as follows: The recast of dynamic fuzzy models is given in Section 2. The main results of this paper are presented in Section 3, and numerical examples are also given. Concluding remarks are collected in Section 4.

2. FROMULATION OF DYNAMIC FUZZY MODELS

A dynamic fuzzy models, which is the extension of a T-S fuzzy model, is described by a set of "If-Then"

[1] Supported by the National Natural Science Foundation of P.R.China (10276005), (90205012), National Key Project for Basic Research of P. R. China (G2002cb312205-04) and the Basic Pre-research Project for National Defense of P. R. China (K1200060301).

fuzzy rules. The premise part of the fuzzy rule includes linguistic information and the consequence part of the rule includes a local dynamic model.

For a multi-input discrete-time dynamic fuzzy model, it can be represented as (S.G.Cao, N.W.Rees and G.Feng. 1997):

R^l_{pd}: If x_1 is F^l_1 and ... x_n is F^l_n,

Then $x(k+1)=A_lx(k)+B_lu(k)$, (1)

$l=1,2,...,m$, where R^l_{pd} denotes the lth fuzzy rule, $A_l\in R^{n\times n}$, $B_l\in R^{n\times p}$, (A_l, B_l) is the lth local model of the dynamic fuzzy model (1), m is the number of the rules, F^l_i is the fuzzy set, $F^l_i(x_i(k))$ denotes the corresponding membership function, $x(k)=[x_1(k), x_2(k),..., x_n(k)]^T\in R^n$ are the state variables of the system, $u(k)\in R^p$ are the input variables of the system. Let $\mu_l(x(k))$ be the normalized membership function of the fuzzy set F^l, where $F^l=\prod^n_{i=1}F^l_i$ and $0\le\mu_l\le1$, $\Sigma^m_{l=1}\mu_l=1$.

Using a singleton fuzzifier, product inference and a center-average defuzzifier, the global dynamic fuzzy model can be expressed as follows:

$x(k+1)=A(\mu(x))x(k)+B(\mu(x))u(k)$, (2)

where $A(\mu(x))=\Sigma^m_{l=1}\mu_l(x)A_l$, $B(\mu(x))=\Sigma^m_{l=1}\mu_l(x)B_l$, $\mu(x)=[\mu_1(x), \mu_2(x),..., \mu_m(x)]$.

For a multi-input continuous-time dynamic fuzzy model, it can be represented as (Sun ZengQi. 1998):

R^l_{pc}: If $x_1(t)$ is M^l_1 and ... $x_n(t)$ is M^l_n,

Then $\dot{x}=A_lx(t)+B_lu(t)$, (3)

$l=1,2,...,m$, where R^l_{pc} denotes the lth fuzzy rule, A_l, B_l denote the same meaning in the discrete time case. M^l_i, $M^l_i(x_i(t))$, $x(t)$, $u(t)$ and $\mu_l(x(t))$ correspond to F^l_i, $F^l_i(x_i(k))$, $x(k)$, $u(k)$ and $\mu_l(x(k))$ in the discrete time case respectively.

As the same, using a singleton fuzzifier, product inference and a center-average defuzzifier, the global dynamic fuzzy model can be expressed as following:

$$\dot{x}=A(\mu(x))x(t)+B(\mu(x))u(t), \quad (4)$$

3. STABILITY ANALYSIS FOR DYNAMIC FUZZY MODELS

If the discrete-time dynamic fuzzy model in Eq.(2) or the continuous-time one in Eq.(4) captures the essential features of the true model, it can be taken as the design model. Then utilizing the technique of "parallel distributed compensation", a model-based fuzzy controller can be designed directly. The main idea is to design a compensator for each rule of the dynamic fuzzy model (1) or (3) (Hua O.Wang, et al. 1996). For each rule, linear control design techniques can be used. The fuzzy controller shares the same fuzzy sets with the dynamic fuzzy model. Suppose the following fuzzy controller has been designed:

For the discrete-time case, it can be expressed by the following m rules:

R^l_{cd}: If $x_1(k)$ is F^l_1 and ... $x_n(k)$ is F^l_n,

Then $u(k)=-L_lx(k)$, (5)

For the continuous-time case, it can be expressed as follows:

R^l_{cc}: If $x_1(t)$ is M^l_1 and ... $x_n(t)$ is M^l_n,

Then $u(t)=-K_lx(t)$, (6)

$l=1,2,...,m$, where $L_l\in R^{p\times n}$, $K_l\in R^{p\times n}$.

There are two strategies to determine the global control. One strategy is that the global control is equal to the local control whose membership is dominant and the other strategy is that the global control is equal to the fuzzy "blending" of local controllers. According to the first strategy, we have

$u(k)=-\Sigma^m_{l=1}\mu_l(x)L_rx(k)=-L_rx(k)$

(the discrete-time case), (7)

or

$u(t)=-\Sigma^m_{l=1}\mu_l(x)K_rx(t)=-K_rx(t)$

(the continuous-time case), (8)

where r satisfies $\mu_r(x)=\max\{\mu_l(x)|1\le l\le m\}$. And with respect to the second strategy, we have

$u(k)=-\Sigma^m_{l=1}\mu_l(x)L_lx(k)$

(the discrete-time case), (9)

or

$u(t)=-\Sigma^m_{l=1}\mu_l(x)K_lx(t)$

(the continuous-time case), (10)

Substitute Eq.(7), (9) into Eq.(2) and substitute Eq.(8), (10) into Eq.(4) respectively, then the closed-loop systems are described by the following models:

$$x(k+1)=\Sigma^m_{i=1}\mu_i(x)(A_i-B_iL_r)x(k) \quad (11)$$

$$\dot{x}=\Sigma^m_{i=1}\mu_i(x)(A_i-B_iK_r)x(t) \quad (12)$$

$$x(k+1)=\Sigma^m_{i=1}\Sigma^m_{j=1}\mu_i(x)\mu_j(x)(A_i-B_iL_j)x(k) \quad (13)$$

$$\dot{x}=\Sigma^m_{i=1}\Sigma^m_{j=1}\mu_i(x)\mu_j(x)(A_i-B_iK_j)x(t) \quad (14)$$

For the stability analysis of dynamic fuzzy models described by Eq.(11) to (14), most results is to find a common positive-definite matrix P and require all or part of local models satisfy certain linear matrix inequations. Because of neglecting the effect of membership functions, these stability conditions are conservative in some sense. In our previous work, the new sufficient conditions for the stability of dynamic fuzzy models described by Eq.(11) and (12) have been presented. They are quoted as following (Ding HaiShan and Mao JianQin. 2003):

Denote $S_{ji}=A_i-B_iL_j$, $Q_{ji}=S^T_{ji}PS_{ji}-P$, $\lambda_{ji}=\max\{\lambda(Q_{ji})\}$ as the largest eigenvalue of the matrix Q_{ji}, where P is an arbitrary fixed n dimension positive-definite matrix, (i, $j=1,...,m$). To divide the m^2 eigenvalues into m parts. Each part is divided into "positive" sub-part

Λ_r^+ and "inpositive" sub-part Λ_r^-. That is: $\{\lambda_{r1}, \lambda_{r2}, ..., \lambda_{rm}\}=\Lambda_r^+\cup\Lambda_r^-=\{\lambda_{ri_1},\ldots,\lambda_{ri_l}\}\cup\{\lambda_{rj_1},\ldots,\lambda_{rj_n}\}$, (r=1,...,m), where $l+n=m$, $1\le i_s, j_t\le m$, (s=1,..., l ; t=1, ..., n).

Theorem 1：The fuzzy dynamic system described by Eq.(11) is asymptotically stable if there exists an n dimension matrix $P>0$, such that $S^T_{rr}PS_{rr}-P<0$ hold for r=1,..., m and $\lambda_{rr}+\lambda_{ri_1}+\ldots+\lambda_{ri_l}<0$ hold for r=1,..., m. (proof see (Ding HaiShan and Mao JianQin. 2003))

Denote $H_{ji}=A_i-B_iK_j$, $\lambda_{ji}=\max\{\lambda(H^T_{ji}P+PH_{ji})\}$ as the largest eigenvalue of the matrix $H^T_{ji}P+PH_{ji}$, where P is an arbitrary fixed n dimension positive-definite matrix, (i, j=1,...,m). To divide the m^2 eigenvalues into m parts. Each part is divided into "positive" sub-part Λ_r^+ and "inpositive" sub-part Λ_r^-. That is:

$$\{\lambda_{r1}, \lambda_{r2}, ..., \lambda_{rm}\}=\Lambda_r^+\cup\Lambda_r^- =\{\lambda_{ri_1},\ldots,\lambda_{ri_l}\}\cup\{\lambda_{rj_1},\ldots,\lambda_{rj_n}\},$$

(r=1,...,m), where $l+n=m$, $1\le i_s, j_t\le m$, (s=1,..., l ; t=1, ..., n).

Theorem 2：The fuzzy dynamic system described by Eq. (12) is asymptotically stable if there exists an n dimension matrix $P>0$, such that $H^T_{rr}P+PH_{rr}<0$ hold for r=1,..., m and $\lambda_{rr}+\lambda_{ri_1}+\ldots+\lambda_{ri_l}<0$ hold for r=1,..., m. (proof see (Ding HaiShan and Mao JianQin. 2003)).

In what follows, the stability conditions of the dynamic fuzzy models described by Eq.(13) and Eq.(14) will be given. For convenience of depiction, divide the m^2 eigenvalues λ_{ji} into "positive" part Λ^+ and "inpositive" set Λ^-. For the discrete-time case, $\lambda_{ji}=\max\{\lambda(Q_{ji})$; for the continuous-time case, $\lambda_{ji}=\max\{\lambda(H^T_{ji}P+PH_{ji})$, i, j=1,...,m.

Theorem 3: The fuzzy dynamic system described by Eq.(13) is asymptotically stable if there exists an n dimension matrix $P>0$, such that $S^T_{rr}PS_{rr}-P<0$ hold for r=1,...,m and $\lambda_{rr}+\sum_{\lambda_{ji}\in\Lambda^+}\lambda_{ji}<0$ hold for r=1,..., m.

Proof: See the Appendix.

Please notice that model (13) can be also written as

$$x(k+1)=\Sigma^m_{i=1}\Sigma^m_{j=1}\mu_i(x)\,\mu_j(x)(A_i-B_iL_j)\,x(k)$$
$$=\Sigma^m_{i=1}\Sigma^m_{j=1}\mu_i(x)\,\mu_j(x)[(A_i-B_iL_j+A_j-B_jL_i)/2]x(k).$$

Denote $G_{ji}=(A_i-B_iL_j+A_j-B_jL_i)/2$ and $\lambda_{ji}=\max\{\lambda(G_{ji}{}^TPG_{ji}-P)\}$, i, j=1,..., m. The following theorem will be derived.

Theorem 4

The fuzzy dynamic system described by Eq.(13) is asymptotically stable if there exists $P>0$, such that $G^T_{rr}PG_{rr}-P<0$ hold for r=1,...,m and $\lambda_{rr}+\sum_{\lambda_{ji}\in\Lambda^+}\lambda_{ji}<0$ hold for r=1,..., m.

Proof: See the Appendix.

For the continue-time case, there is:

Theorem 5: The fuzzy dynamic system described by Eq.(14) is asymptotically stable if there exists $P>0$, such that $H^T_{rr}P+PH_{rr}<0$ hold for r=1,...,m and $\lambda_{rr}+\sum_{\lambda_{ji}\in\Lambda^+}\lambda_{ji}<0$ hold for r=1,...,m.

Proof: See the Appendix.

Notice that model (14) can be also written as

$$\dot{x}=\Sigma^m_{i=1}\Sigma^m_{j=1}\mu_i(x)\,\mu_j(x)(A_i-B_iK_j)\,x(t)$$
$$=\Sigma^m_{i=1}\Sigma^m_{j=1}\mu_i(x)\,\mu_j(x)[(A_i-B_iK_j+A_j-B_jK_i)/2]x(t).$$

If we define $\overline{H}_{ji}=(A_i-B_iL_j+A_j-B_jL_i)/2$ and $\lambda_{ji}=\max\{\lambda(\overline{H}^T_{ji}P+P\overline{H}_{ji})\}$, i, j=1,...,m, The following theorem will be derived.

Theorem 6: The fuzzy dynamic system described by Eq.(14) is asymptotically stable if there exists $P>0$, such that $\overline{H}^T_{rr}P+P\overline{H}_{rr}<0$ hold for r=1,...,m and $\lambda_{rr}+\sum_{\lambda_{ji}\in\Lambda^+}\lambda_{ji}<0$ hold for r=1,...,m.

Proof: See the Appendix.

Remark: Theorems from 1 to 6 only involves of corresponding eigenvalues which are easy to obtain. So they are convenient to be used in stability analysis of dynamic fuzzy models.

Although these stability conditions are not related with membership functions apparently, it can be seen from proofs that they are obtained under the consideration of the properties of membership functions. And the interaction of local dynamic models is considered by means of eigenvalues. So they are more general than some existing results. The advantages of theorem 1 and theorem 2 have been given in (Ding HaiShan and Mao JianQin. 2003). Theorem 4 is more general than theorem 3 in (Hua O.Wang, et al. 1996). Theorem 3 in (Hua O.Wang, et al. 1996) requests that $G_{ij}{}^TP\,G_{ij}-P<0$ hold for $i\le j\le m$. While theorem 4 in this paper only requests that $\lambda_{rr}<0$

hold for r=1,...,m, which is equivalent to $G_{rr}^T P G_{rr} - P<0$ hold for r=1,...,m. So theorem 3 in (Hua O.Wang, et al. 1996) is the special case of theorem 4. For the same reason, theorem 5 also includes theorem 1 in (Sun ZengQi. (1998) as a special case.

In what follows, two numerical examples will be given to illustrate the effectiveness of the theorems.

Example 1: Consider the discrete-time dynamic fuzzy model used in (Hua O.Wang, et al. 1996):

R^1_{pd}: If $x_2(k)$ is $F^1{}_2$ (e.g. "Small")
Then $x(k+1)=A_1x(k)+B_1u(k)$,
R^2_{pd}: If $x_2(k)$ is $F^2{}_2$ (e.g. "Big")
Then $x(k+1)=A_2x(k)+B_2u(k)$,

Where $x(k)=[x_1(k)\ x_2(k)]^T$ and $B_1=[1\ 1]^T$, $B_2=[-2\ 1]^T$,

$$A_1=\begin{bmatrix}1 & -0.5\\ 1 & 0\end{bmatrix},\quad A_2=\begin{bmatrix}-1 & -0.5\\ 1 & 0\end{bmatrix}.$$

Membership functions and the PDC controller are chosen as the same as the ones in (Hua O.Wang, et al. 1996), that is $L_1=[0.65\ -0.5]$ and $L_2=[0.87\ -0.11]$.

If the positive definite matrix P is chosen to be $P=\begin{bmatrix}0.8 & -0.06\\ -0.06 & 2.3\end{bmatrix}$, we will have that $S^T_{12}PS_{12}-P<0$ is not satisfied. So we are not able to use theorem 2 in (Hua O.Wang, et al. 1996). But using theorem 3 in this paper, there is $\lambda_{11}=-0.2923$, $\lambda_{12}=0.1891$, $\lambda_{21}=-0.7475$, $\lambda_{22}=-0.2647$, $\Lambda^+=\{\lambda_{12}\}$ and

$$\lambda_{11}+\sum_{\lambda_{ji}\in\Lambda^+}\lambda_{ji}=\lambda_{11}+\lambda_{12}<0$$

$$\lambda_{22}+\sum_{\lambda_{ji}\in\Lambda^+}\lambda_{ji}=\lambda_{22}+\lambda_{12}<0.$$

Therefore the close-loop dynamic fuzzy model is asymptotically stable. Using theorem 3 in (Hua O.Wang, et al. 1996) or theorem 4 in this paper, the same conclusion is also derived.

But if the positive definite matrix P is chosen to be $P=\begin{bmatrix}2.1 & -0.7\\ -0.7 & 2.3\end{bmatrix}$, theorem 2 and 3 in (Hua O.Wang, et al. 1996) as well as theorem 3 in this paper all lose its effectiveness. However, according to theorem 4 in this paper, we obtain $\lambda_{11}=-0.7487$, $\lambda_{12}=0.3282$, $\lambda_{21}=0.3282$, $\lambda_{22}=-0.6816$ by calculating directly. So $\Lambda^+=\{\lambda_{12},\lambda_{21}\}$ and

$$\lambda_{11}+\sum_{\lambda_{ji}\in\Lambda^+}\lambda_{ji}=\lambda_{11}+\lambda_{12}+\lambda_{21}<0$$

$$\lambda_{22}+\sum_{\lambda_{ji}\in\Lambda^+}\lambda_{ji}=\lambda_{22}+\lambda_{12}+\lambda_{21}<0.$$

So we can conclude that the closed-loop dynamic fuzzy model is asymptotically stable. The simulation results of this example are shown as Fig.1 and Fig. 2 (Hua O.Wang, et al. 1996).

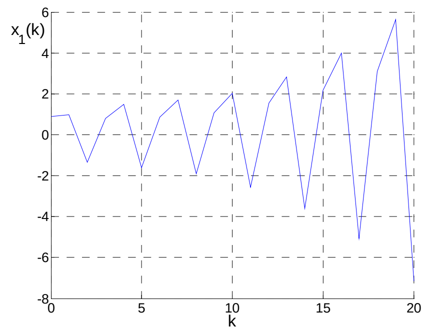

Fig. 1 Open-loop response of $x_1(k)$ at $x(0)=[0.9\ -0.7]^T$

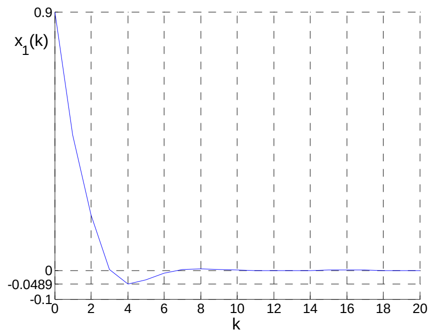

Fig. 2 Close-loop response of $x_1(k)$ at $x(0)=[0.9\ -0.7]^T$

Example 2: Consider the inverted pendulum system used in (G.Feng,S.G.Cao,N.W.Rees,C.K.Chak.1997), where a PDC controller is designed. The system is described by the following five rules:

R^1_{pc}: If $x_1(t)$ is "near 0" and $x_2(t)$ is "near 0"
Then $\dot{x}=A_1x(t)+B_1u(t)$,
R^2_{pc}: If $x_1(t)$ is "near 0" and $x_2(t)$ is "near ±4"
Then $\dot{x}=A_2x(t)+B_2u(t)$,
R^3_{pc}: If $x_1(t)$ is "near ±π/3" and $x_2(t)$ is "near 0"
Then $\dot{x}=A_3x(t)+B_3u(t)$,
R^4_{pc}: If $x_1(t)$ is "near π/3" and $x_2(t)$ is "near 4" or $x_1(t)$ is "near −π/3" and $x_2(t)$ is "near −4"
Then $\dot{x}=A_4x(t)+B_4u(t)$,
R^5_{pc}: If $x_1(t)$ is "near π/3" and $x_2(t)$ is "near −4" or $x_1(t)$ is "near −π/3" and $x_2(t)$ is "near 4"
Then $\dot{x}=A_5x(t)+B_5u(t)$,

where, $x(t)=[x_1(t)\ x_2(t)]^T$ and

$$A_1=\begin{bmatrix}0 & 1\\ 17.2941 & 0\end{bmatrix}\qquad A_2=\begin{bmatrix}0 & 1\\ 14.4706 & 0\end{bmatrix}$$

$$A_3=\begin{bmatrix}0 & 1\\ 5.8512 & 0\end{bmatrix}\qquad A_4=\begin{bmatrix}0 & 1\\ 7.2437 & -0.5399\end{bmatrix}$$

$$A_5=\begin{bmatrix}0 & 1\\ 7.2437 & 0.5399\end{bmatrix}$$

$$\mathrm{B}_1=\begin{bmatrix}0\\ -0.1765\end{bmatrix}\quad \mathrm{B}_2=\begin{bmatrix}0\\ -0.1765\end{bmatrix}$$

$$\mathrm{B}_3=\mathrm{B}_4=\mathrm{B}_5=\begin{bmatrix}0\\ -0.0779\end{bmatrix}$$

The local feedback control gains are as follows (Ding HaiShan and Mao JianQin. 2003):

K_1=[−297.2750 −39.6601],K_2=[−281.2776 −39.6601]

K_3=[−526.6508 −89.8588],K_4=[−544.5263 −82.9281]

K_5=[−544.5263 −96.7895]

In this example, membership functions are as the same as the ones in (Ding HaiShan and Mao JianQin. 2003) and the global control is equal to the fuzzy "blending" of local controllers.

The positive definite matrix P is chosen to be $P=\begin{bmatrix}13.7909 & 0.4378\\ 0.4378 & 0.4938\end{bmatrix}$ for stability analysis.

Calculate λ_{ji} corresponding to H_{ji}. We have λ_{11}=−4.3516, λ_{12}=−3.8136, λ_{13}=−1.1630, λ_{14}=−1.2969, λ_{15}=−0.0442, λ_{21}=−4.8605, λ_{22}=−4.3516, λ_{23}=−0.7276, λ_{24}= −0.7597, λ_{25}=0.4938, λ_{31}= −0.4613, λ_{32}=0.1746, λ_{33}= −4.3516, λ_{34}= −5.0013, λ_{35}= −4.2042, λ_{41}=0.8547, λ_{42}=1.4823, λ_{43}= −3.7019, λ_{44}= −4.3516, λ_{45}= −3.5545, λ_{51}= −0.3315, λ_{52}=0.3033, λ_{53}= −4.4690, λ_{54}= −5.1166, λ_{55}= −4.3516.

So $\sum_{\lambda_{ji}\in\Lambda^+}\lambda_{ji}=\{\lambda_{25}, \lambda_{32}, \lambda_{41}, \lambda_{42}, \lambda_{52}\}$. It is easy to verify that $\lambda_{rr}+\sum_{\lambda_{ji}\in\Lambda^+}\lambda_{ji}<0$ hold for r=1,…,5. According to theorem 5, the pendulum system is asymptotically stable.

When calculate λ_{ji} corresponding to $\overline{H}_{ji}$, it will be found that $\sum_{\lambda_{ji}\in\Lambda^+}\lambda_{ji}=\varnothing$. So according to theorem 6, the pendulum system is also asymptotically stable. However, $H_{31}{}^TP+PH_{31}$ is not a negative definite matrix. So theorem 1 in (Sun ZengQi. 1998) cannot help anything here.

Fig 1 displays the simulation result with initial condition $x(t)$=[0.6 0.2]^T and fig 2 displays the simulation result with initial condition $x(t)$=[1 0]^T, where the angles are in radian. Simulations verify the correctness of the conclusion derived by theorems in this paper.

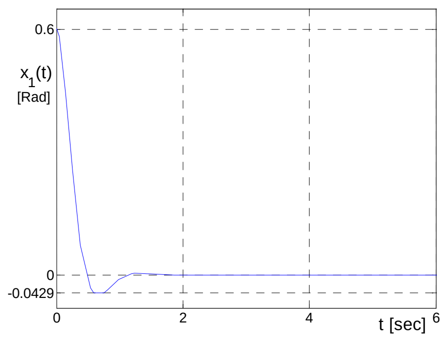

Fig. 3 Response of the angle of the pendulum for x(0)=[0.6 0.2]^T

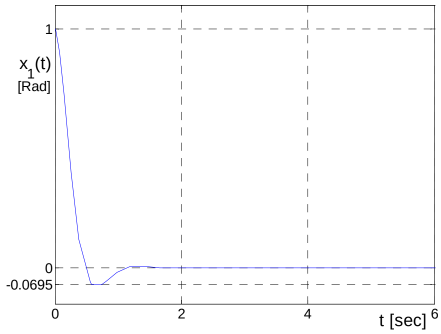

Fig. 4 Response of the angle of the pendulum for x(0)=[1 0]^T

4. CONCLUSION

Stability analysis of dynamic fuzzy models, whose controller is designed by the design technique of PDC, is considered. New relaxed sufficient conditions to guarantee the asymptotical stability are presented. The conditions are simple and they are less conservative compared with existing results. This is shown by theoretic analysis and numerical examples.

APPENDIX−PROOFS

Proof of theorem 3. First, if there exits an n dimension matrix P>0, such that $S^T_{rr}PS_{rr}-P<0$, it is equivalent with $\lambda_{rr}<0$, (r=1,..., m). So we have $\lambda_{rr}\in\Lambda^-$. Choosing as a Lyapunov function candidate $V(x(k))=x^T(k)Px(k)$, hence its difference along the solution of Eq.(13) is

$$\begin{aligned}\Delta V(x(k)) &= V(x(k+1)) - V(x(k))\\ &= x^T(k+1)P\,x(k+1) - x^T(k)P\,x(k)\\ &= x^T(k)\{[\Sigma^m_{i=1}\mu_i(x)(\Sigma^m_{j=1}\mu_j(x)(A_i-B_iL_j)^T)]P[\Sigma^m_{k=1}\mu_k(x)(\Sigma^m_{l=1}\mu_l(x)(A_k-B_kL_l))-P]x(k)\end{aligned}$$

Denote $E_i=\Sigma^m_{j=1}\mu_j(x)(A_i-B_iL_j)$, then we have

$$\begin{aligned}\Delta V(x(k)) &= x^T(k)[(\Sigma^m_{i=1}\mu_i(x)E_i^T)P(\Sigma^m_{k=1}\mu_k(x)E_k)-P]x(k)\\ &= x^T(k)[\Sigma^m_{i=1}\mu^2_i(x)(E_i{}^TPE_i-P)\\ &\quad+\Sigma^m_{i=1}\Sigma^m_{k=i+1}\mu_i(x)\,\mu_k(x)(E_i{}^TPE_k-P+E_k{}^TPE_i-P)]x(k)\\ &= x^T(k)\{\Sigma^m_{i=1}\mu^2_i(x)(E_i{}^TPE_i-P)\\ &\quad+\Sigma^m_{i=1}\Sigma^m_{k=i+1}\mu_i(x)\,\mu_k(x)[E_i{}^TPE_k-P+E_k{}^TPE_i-P-(E_k-E_i)^TP(E_k-E_i)]\}x(k)\\ &\le x^T(k)[\Sigma^m_{i=1}\mu^2_i(x)(E_i{}^TPE_i-P)\end{aligned}$$

$$+ \Sigma^m_{i=1}\Sigma^m_{k=i+1}\mu_i(x)\,\mu_k(x)(E_i^TPE_k - P + E_k^TPE_i - P)]x(k)$$

$$= x^T(k)[\Sigma^m_{i=1}\Sigma^m_{k=1}\mu_i(x)\,\mu_k(x)(E_i^TPE_i - P)]x(k)$$

$$= x^T(k)[\Sigma^m_{i=1}\mu_i(x)(E_i^TPE_i - P)]x(k)$$

$$E_i^TPE_i - P = [\Sigma^m_{j=1}\mu_j(x)(A_i - B_iL_j)^T] P[\Sigma^m_{l=1}\mu_l(x)(A_i - B_iL_l)] - P$$

$$=\Sigma^m_{j=1}\Sigma^m_{l=1}\mu_j(x)\mu_l(x)(S_{ji}^TPS_{li} - P)$$

$$=\Sigma^m_{j=1}\Sigma^m_{l=1}\mu^2_j(x)(S_{ji}^TPS_{ji} - P) +\Sigma^m_{j=1}\Sigma^m_{l=j+1}\mu_j(x)\mu_l(x)(S_{ji}^TPS_{li} - P + S_{li}^TPS_{ji} - P)$$

$$=\Sigma^m_{j=1}\Sigma^m_{l=1}\mu^2_j(x)(S_{ji}^TPS_{ji} - P) +\Sigma^m_{j=1}\Sigma^m_{l=j+1}\mu_j(x)\mu_l(x)[S_{ji}^TPS_{ji} - P + S_{li}^TPS_{li} - P - (S_{ji} - S_{li})^TP(S_{ji} - S_{li})]$$

$$<\Sigma^m_{j=1}\Sigma^m_{l=1}\mu^2_j(x)(S_{ji}^TPS_{ji} - P) +\Sigma^m_{j=1}\Sigma^m_{l=j+1}\mu_j(x)\mu_l(x)(S_{ji}^TPS_{ji} - P + S_{li}^TPS_{li} - P)$$

$$=\Sigma^m_{j=1}\mu_j(x)Q_{ji}$$

Therefore, we have

$$\Delta V(x(k)) < x^T(k)(\Sigma^m_{i=1}\mu_i(x)\,\Sigma^m_{j=1}\mu_j(x)Q_{ji})x(k)$$

$$\le (\textstyle\sum_{\lambda_{ji}\in\Lambda^-}\mu_i(x)\,\mu_j(x)\,\lambda_{ji} + \sum_{\lambda_{ji}\in\Lambda^+}\mu_i(x)\,\mu_j(x)\,\lambda_{ji})\|x(k)\|^2$$

$$\le(\Sigma^m_{i=1}\mu_i(x)^2\lambda_{ii} + \textstyle\sum_{\lambda_{ji}\in\Lambda^+}\mu_i(x)\,\mu_j(x)\,\lambda_{ji})\|x(k)\|^2$$

At arbitrary time k, without generalization, suppose that $\mu_r(x)=\max\{\mu_l(x)|1\le l\le m\}$, then we have

$$\Delta V(x(k))\le\mu_r(x)^2(\lambda_{rr}+\textstyle\sum_{\lambda_{ji}\in\Lambda^+}\lambda_{ji})\|x(k)\|^2<0.$$

Because of the randomicity of t, $\Delta V(x(k))<0$ always holds. So the fuzzy dynamic system described by Eq. (13) is asymptotically stable. □

Proof of theorem 4. Denote $G_{ji}=(A_i - B_iL_j + A_j - B_jL_i)/2$. It is only needed to substitute $\lambda_{ji}=\max\{\lambda(G_{ji}^TPG_{ji} - P)\}$ into the proof of theorem 3. The rest is similar to that of theorem 3. □

Proof of theorem 5. First, similar as the proof of theorem 3, we have $\lambda_{rr}\in\Lambda^-$ and choosing as a Lyapunov function candidate $V(x(t))=x^T(t)Px(t)$, then its derivative along the solution of Eq.(14) is

$$\dot V(x(t)) = \dot x^T(t)\,Px(t) + x^T(t)P\,\dot x(t)$$

$$= x^T(t)\Sigma^m_{i=1}\Sigma^m_{j=1}\mu_i(x)\,\mu_j(x)\,(A_i - B_iK_j)^TP\,x(t) + x^T(t)\,P\,\Sigma^m_{i=1}\Sigma^m_{j=1}\mu_i(x)\,\mu_j(x)\,(A_i - B_iK_j)\,x(t)$$

$$=\Sigma^m_{i=1}\Sigma^m_{j=1}\mu_i(x)\,\mu_j(x)x^T(t)[(A_i - B_iK_j)^TP + P\,(A_i - B_iK_j)]\,x(t)$$

$$\le (\textstyle\sum_{\lambda_{ji}\in\Lambda^-}\mu_i(x)\,\mu_j(x)\,\lambda_{ji} + \sum_{\lambda_{ji}\in\Lambda^+}\mu_i(x)\,\mu_j(x)\,\lambda_{ji})\|x(t)\|^2$$

$$\le(\Sigma^m_{i=1}\mu_i(x)^2\lambda_{ii} + \textstyle\sum_{\lambda_{ji}\in\Lambda^+}\mu_i(x)\,\mu_j(x)\,\lambda_{ji})\|x(t)\|^2$$

At arbitrary time t, without generalization, suppose that $\mu_r(x)=\max\{\mu_l(x)|1\le l\le m\}$, then we have

$$\dot V(x(t)) \le\mu_r(x)^2(\lambda_{rr}+\textstyle\sum_{\lambda_{ji}\in\Lambda^+}\lambda_{ji})\|x(t)\|^2<0.$$

Because of the randomicity of t, $\dot V(x(t))<0$ always holds. So the fuzzy dynamic system described by Eq. (14) is asymptotically stable. □

Proof of theorem 6.Denote $\overline{H}_{ji}=(A_i - B_iK_j + A_j - B_jK_i)/2$.

It is only needed to substitute $\lambda_{ji}=\max\{\lambda(\overline{H}^T_{ji}P + P\overline{H}_{ji})\}$ into the proof of theorem 5. The rest is similar to that of theorem 5. □

REFERENCES

S.G.Cao, N.W.Rees and G.Feng. (1997), Analysis and Design for a Class of Complex Control Systems Part I: Fuzzy Modelling and Indentification, Automatica, Vol.33, No. 6. pp.1017-1028.

B.Friedland. (1996), Advanced Control System Design. Engleward Cliffs, NJ: Prentice-Hall.

Hua O.Wang, Kazuo Tanaka, Michaol F. Griffin. (1996), An Approach to Fuzzy Control of Nonlinear Systems: Stability and Design Issues. IEEE Transactions on Fuzzy Systems. February, Vol.4, No.1, pp.14-23.

Yao Hongwei, Mei Xiaorong, Yang Zhenqiang and Zhuang Xianyi. (2001), Analysis and Design of a Double Inverted Pendulum Based on the Dynamic Fuzzy Model. Control Theory and Applications. Vol.18, No.2, pp.224-233.

S.G.Cao, N.W.Rees and G.Feng. (1997), Analysis and Design for a Class of Complex Control Systems Part II: Fuzzy Modelling and Indentification. Automatica. Vol.33, No. 6. pp.1029-1039.

Sun ZengQi. (1998), Controller Design and Stability Analysis of Time-Continous Systems Based on A Fuzzy State Model. Acta Automatica Sinica. Vol 24, No.2, pp. 212-216.

Ding HaiShan and Mao JianQin. (2003), Stable Sufficient Condition of a Class of Fuzzy Dynamic Systems. The 29th Annual Conference of the Industrial Electronics Society, Virginia, USA. pp.2405-2409.

G.Feng,S.G.Cao,N.W.Rees,C.K.Chak.(1997), Design of fuzzy control systems with guaranteed stability. Fuzzy sets and systems. Vol.85,Issue 1. pp.1-10.